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MATHEMATICAL EXERCISES.

LONDON : PRINTED BY
SPOTTISWOODE AND CO., NEW-STREET SQUARE
AND PARLIAMENT STREET

MATHEMATICAL EXERCISES:

COMPRISING NUMEROUS EXAMPLES IN PURE MATHEMATICS, STATICS, DYNAMICS, AND HYDROSTATICS, TAKEN FROM MILITARY, CIVIL SERVICE, AND OTHER EXAMINATION PAPERS, INTENDED FOR THE USE OF STUDENTS PREPARING FOR EXAMINATION.

WITH TABLES, FORMULÆ, ANSWERS, AND REFERENCES.

BY

SAMUEL H. WINTER.

NEW EDITION.

LONDON:
LONGMANS, GREEN, AND CO.
1877.

187. q. 15.

PREFACE TO THE NEW EDITION.

In this edition, New Examination Papers take the place of the old ones, the questions of the latter being arranged as Miscellaneous Examples.

Plumstead: *July* 1877.

PREFACE TO THE FIRST EDITION.

THE SOURCES from which the following Exercises have been taken are sufficiently indicated by the Table of Contents. This collection differs, in two respects, from any with which the author is acquainted; in the first place, the questions are arranged in such a manner that most of the papers embrace various subjects; secondly, 'book-work' is introduced, in order to assimilate the papers to those set at Examinations.

WOOLWICH COMMON: *August* 1864.

PREFACE TO THE SECOND EDITION.

THIS EDITION contains about 900 additional Examples, selected from Examination Papers which have been set since the publication of the first Edition. The greater part of the new matter will be found in the Appendix; but the fresh questions in Mixed Mathematics commence at page 287. Considerable pains having been taken in correcting errors, it is hoped that they now amount to a comparatively small number.

WOOLWICH COMMON: *January* 1867.

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MATHEMATICAL EXERCISES.

TABLES, FORMULÆ, ETC.

TABLES OF WEIGHTS AND MEASURES.

TROY WEIGHT.

24 grains (grs.)	= 1 pennyweight (dwt.)
20 dwts.	= 1 ounce (oz.)
12 oz.	= 1 pound (lb.)

AVOIRDUPOIS WEIGHT.

16 drams (dr.)	= 1 ounce (oz.)
16 oz.	= 1 pound (lb.)
28 lbs.	= 1 quarter (qr.)
4 qrs. or 112 lbs.	= 1 hundredweight (cwt.)
20 cwt.	= 1 ton.
1 stone (st.)	= 14 lbs.

1 lb. avoirdupois = 7000 grs. troy.
1 lb. troy = 5760 grs. troy.

CLOTH MEASURE.

$2\frac{1}{4}$ in.	= 1 nail (n.)
4 n.	= 1 quarter (qr.)
4 qrs.	= 1 yd.
3 qrs.	= 1 Flemish ell.
5 qrs.	= 1 English ell.
6 qrs.	= 1 French ell.

LINEAL MEASURE.

12 inches (in.)	= 1 foot (ft.)
3 ft.	= 1 yard (yd.)
2 yds. or 6 ft.	= 1 fathom.
$5\frac{1}{2}$ yds. or $16\frac{1}{2}$ ft.	= 1 pole (po.)
40 po. or 220 yds.	= 1 furlong (fur.)
8 fur. or 1760 yds.	= 1 mile (mi.)
3 mi.	= 1 league (lea.)
$69\frac{1}{3}$ mi.	= 1 degree (1°).

1 chain = 22 yds. = 66 ft. = 100 links.
1 hand = 4 inches.
1 line = $\frac{1}{12}$ of an inch.

SUPERFICIAL MEASURE.

144 square inches	= 1 square foot (sq. ft.)
9 sq. ft.	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ sq. yds.	= 1 sq. pole (sq. po.)
40 sq. po.	= 1 rood (ro.)
4 roods, or 4840 sq. yds.	= 1 acre (ac.)

1 square chain = 10000 square links.
1 ac. = 10 ch. = 100000 sq. li.
1 sq. mi. = $(1760)^2$ sq. yds. = 640 acres.

1 rod of brickwork = $272\frac{1}{4}$ sq. ft. $1\frac{1}{2}$ bricks thick.
1 square of flooring = 100 sq. ft.

CUBIC MEASURE.

1728 cubic in. = 1 cubic foot.
27 cub. ft. = 1 cub. yd.

CORN MEASURE.

4 qts. = 1 gal.
2 gal. = 1 pk.
4 pecks = 1 bushel (bu.)
8 bu. = 1 quarter (qr.)

LIQUID MEASURE.

2 pints (pt.) = 1 quart (qt.)
4 qts. = 1 gallon (gal.)
9 gal. = 1 firkin.
18 gal. (Beer.) = 1 kilderkin (fir.)
36 gal. (Beer.) = 1 barrel (bar.)
54 gal. (Beer.) = 1 hogshead (hhd.)
42 gal. = 1 tierce.
63 gal. = 1 hhd.
126 gal. = 1 pipe.
252 gal. = 1 tun.

TIME.

60 seconds (″) = 1 minute (′)
60′ = 1 hour (hr.)
24 hrs. = 1 day (da.)
7 da. = 1 week (1 wk.)

NUMBERS OF FREQUENT OCCURRENCE.

π { = Circumference of circle to diameter 1; = Surface of sphere to diameter 1; = Area of circle to radius 1 } $= 3{\cdot}1415926$; $\log \pi = 0{\cdot}4971499$; $\log \frac{1}{\pi} = \bar{1}{\cdot}5028501$.

$\frac{\pi}{4} = {\cdot}7853986$; $\log \frac{\pi}{4} = \bar{1}{\cdot}8950899$; $\frac{\pi}{6} = {\cdot}5235988$; $\log \frac{\pi}{6} = \bar{1}{\cdot}7189986$

Unit of Circular Measure $= 57^\circ{\cdot}2957795$;
$\log 57{\cdot}2957795 = 1{\cdot}7581226$.

Do. do. $= 206264''{\cdot}8$;
$\log 206264{\cdot}8 = 5{\cdot}3144251$.

$M_{10} = \frac{1}{\log_e 10} = \frac{1}{2{\cdot}3025851} = {\cdot}43429448$; $\log_{10} M = \bar{1}{\cdot}6377843$.

e = base of Napierian system of logarithms $= 2{\cdot}718281828$;
$\log_{10} e = {\cdot}4342945$.

g = force of gravity $= 32{\cdot}19084$ ft.; $\log g = 1{\cdot}5077323$;
$\log \frac{1}{g} = \bar{2}{\cdot}4922677$.

Length of seconds' pendulum = 39·1393 inches;
log 39·1393 = 1·5926130.
No. of cubic in. in 1 gal. = 277·274; log 277·274 = 2·4429091; wt. of 1 cub. in. of water = 252·458 grs.; log. 252·458 = 2·4021892.

Table of the Squares, Cubes, Square and Cube Roots, Reciprocals, Logarithms and Logarithms of Reciprocals of the natural numbers from 1 to 20.

No.	Sq.	Cube	Square Root	Cube Root	Reciprocal	Logarithm	Log. of Reciprocal
1	1	1	1·000000	1·000000	1	0·0000000	0·0000000
2	4	8	1·414213	1·259921	·5	0·3010300	$\bar{1}$·6989700
3	9	27	1·732050	1·442249	·3333333	0·4771213	$\bar{1}$·5228787
4	16	64	2·000000	1·587401	·25	0·6020600	$\bar{1}$·3979400
5	25	125	2·236068	1·709976	·2	0·6989700	$\bar{1}$·3010300
6	36	216	2·449489	1·817121	·1666666	0·7781513	$\bar{1}$·2218487
7	49	343	2·645751	1·912931	·1428571	0·8450980	$\bar{1}$·1549020
8	64	512	2·828427	2·000000	·125	0·9030900	$\bar{1}$·0969100
9	81	729	3·000000	2·080083	·1111111	0·9542425	$\bar{1}$·0457575
10	100	1000	3·162277	2·154434	·1	1·0000000	$\bar{1}$·0000000
11	121	1331	3·316624	2·223980	·0909090	1·0413927	$\bar{2}$·9586073
12	144	1728	3·464101	2·289428	·0833333	1·0791812	$\bar{2}$·9208198
13	169	2197	3·605551	2·351335	·0769230	1·1139434	$\bar{2}$·8860566
14	196	2744	3·741657	2·410142	·0714285	1·1461280	$\bar{2}$·8538720
15	225	3375	3·872983	2·466212	·0666666	1·1760913	$\bar{2}$·8239087
16	256	4096	4·000000	2·519842	·0625	1·2041200	$\bar{2}$·7958800
17	289	4913	4·123105	2·571281	·0588235	1·2304489	$\bar{2}$·7695511
18	324	5832	4·242640	2·620741	·0555555	1·2552725	$\bar{2}$·7447275
19	361	6859	4·358898	2·668402	·0526315	1·2787536	$\bar{2}$·7212464
20	400	8000	4·472136	2·714417	·05	1·3010300	$\bar{2}$·6989700

USEFUL ALGEBRAICAL FORMULÆ.

$(x \pm y)^2 = x^2 \pm 2xy + y^2$; $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy\,(x+y)$.

$(x+y)\,(x-y) = x^2 - y^2$ $\therefore (x^2+xy+y^2)(x^2-xy+y^2)$
$= x^4 + x^2y^2 + y^4$.

$(x^2 - xy + y^2)\,(x+y) = x^3 + y^3$.

$(x^2+xy+y^2)\,(x-y) = x^3 - y^3$.

$(x+a)\,(x+b) = x^2 + (a+b)\,x + ab$.

$\frac{x^n - y^n}{x \pm y}$ is integral when n is an even number.

$\frac{x^n + y^n}{x+y}$ and $\frac{x^n - y^n}{x-y}$ are integral when n is an odd number.

If $ax^2 + bx + c = 0$,

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \frac{-b - \sqrt{b^2-4ac}}{2a}.$$

In an arithmetic progression,

$$l = a + (n-1)\,d;\ s = \frac{n}{2}\,(a+l) = \frac{n}{2}\left\{2a + (n-1)d\right\};$$
$$d = \frac{l-a}{n-1}.$$

In a geometric progression,

$$l = ar^{n-1};\ s = \frac{rl-a}{r-1} = \frac{a\,(r^n-1)}{r-1};\ s_{\frac{1}{0}} = \frac{a}{1-r};\ r = \left(\frac{l}{a}\right)^{\frac{1}{n-1}}.$$

If a and b be any two quantities their

$$\text{A.M.} = \tfrac{1}{2}\,(a+b);\ \text{G.M.} = \sqrt{ab};\ \text{H.M.} = \frac{2ab}{a+b}.$$

No. of shot in a square pile of n courses =

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n\,(n+1)\,(2n+1)}{1.2.3}.$$

No. of shot in a triangular pile of n courses =

$$1+\overline{1+2}+\overline{1+2+3}+\ldots.+\frac{n(n+1)}{2}=$$
$$\frac{n(n+1)(n+2)}{1.2.3}.$$

No. of shot in a rectangular pile of n courses =

$$(1+m)+2(2+m)+3(3+m)+\ldots.n(n+m)=$$
$$\frac{n(n+1)(2n+1+3m)}{1.2.3}.$$

No. of permutations of n things taken r at a time =

$$\frac{\lfloor n}{\lfloor n-r}.$$

No. of combinations of n things taken r at a time =

$$\frac{\lfloor n}{\lfloor r \,\lfloor n-r}.$$

$$(a\pm x)^n = a^n \pm \frac{n}{1}a^{n-1}x+\frac{n(n-1)}{\lfloor 2}a^{n-2}x^2$$
$$\pm\frac{n(n-1)(n-2)}{\lfloor 3}a^{n-3}x^3 + \text{ \&c. \&c. ; where}$$

the $(r+1)$th term is $\frac{\lfloor n}{\lfloor r \,\lfloor n-r}\cdot a^{n-r}x^r$.

General term of the expansion of

$$(a_0+a_1x+a_2x^2+a_3x^3+\ldots.)^n \text{ is}$$

$$\frac{\lfloor n}{\lfloor p \,\lfloor q \,\lfloor r \,\lfloor s \,\lfloor t\ldots}\ a_0^p a_1^q a_2^r a_3^s a_4^t \ldots. x^{q+2r+3s+4t\ldots.}$$

where $p+q+r+s+\ldots\ldots=n$.

$$a^x = 1+(\log_e a)\,x+\frac{(\log_e a)^2x^2}{\lfloor 2}+\frac{(\log_e a)^3x^3}{\lfloor 3}+\text{ \&c.}$$

$$\log_e a = a-1-\frac{(a-1)^2}{2}+\frac{(a-1)^3}{3}-\frac{(a-1)^4}{4}+\text{ \&c.}$$

$$\log_e(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\text{ \&c. \&c.}$$

$$\log_e(n+1) = \log_e n + 2\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \text{\&c. \&c.}\right\}$$

$$\log_a(n+1) = \log_a n + 2M_a\left\{\frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \text{\&c. \&c.}\right\}; \quad M_a = \frac{1}{\log_e a}.$$

$\log MN = \log M + \log N$; $\log M \div N = \log M - \log N$;

$\log N^p = p \log N$; $\log \sqrt[p]{N} = \frac{1}{p} \log N$;

$\log_a N : \log_b N :: \log b : \log a$

$\therefore \log_b N = \log_a N \times \frac{\log_a a}{\log_a b} = \log_a N \div \log_a b.$

$\log_a a = 1$; $\log_a 1 = 0$; $\log_a 0 = -\infty$.

Compound Interest. $A = P(1+r)^t = P\left(1 + \frac{r}{n}\right)^{nt}$, when interest is payable n times a year.

Amount of an annuity of a £ in n years $= \frac{(1+r)^n - 1}{r} \cdot a.$

Present value of same annuity $= \frac{1 - (1+r)^{-n}}{r} \cdot a.$

FORMULÆ IN PLANE TRIGONOMETRY.

Unit of circular measure $= \frac{180°}{\pi} = 57°\cdot2957795$

$= 206264''\cdot8.$

$A° = \frac{180°}{\pi} \times \frac{\text{arc}}{\text{radius}}.$

$\sin(-A) = -\sin A$; $\cos(-A) = \cos A$;

$\tan(-A) = -\tan A.$

$\sin(90° + A) = \cos A$; $\cos(90° + A) = -\sin A$;
$\tan(90° + A) = -\cot A$.

$\sin(180° - A) = \sin A$; $\cos(180° - A) = -\cos A$;
$\tan(180° - A) = -\tan A$.

$\sin(180° + A) = -\sin A$; $\cos(180° + A) = -\cos A$;
$\tan(180° + A) = \tan A$.

$\sin(360° - A) = -\sin A$; $\cos(360° - A) = \cos A$;
$\tan(360° - A) = -\tan A$.

$\sin A = \sin\{n\pi + (-1)^n A\}$; $\cos A = \cos\{2n\pi \pm A\}$;
$\tan A = \tan(n\pi + A)$.

$$\sin A = \sqrt{1 - \cos^2 A} = \frac{1}{\operatorname{cosec} A} = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \sqrt{\frac{1 - \cos 2A}{2}} = \sin(60° + A) - \sin(60° - A).$$

$$\cos A = \sqrt{1 - \sin^2 A} = \frac{1}{\sec A} = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A = 1 - 2 \sin^2 \tfrac{1}{2} A = 2 \cos^2 \tfrac{1}{2} A - 1 = \sqrt{\frac{1 + \cos 2A}{2}}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{1}{\cot A} = \sqrt{\sec^2 A - 1} = \frac{2 \tan \tfrac{1}{2} A}{1 - \tan^2 \tfrac{1}{2} A} = \frac{1 - \cos 2A}{\sin 2A} = \sqrt{\frac{1 - \cos 2A}{1 + \cos 2A}}.$$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$;
$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B};$$

$$\tan(45° \pm B) = \frac{1 \pm \tan B}{1 \mp \tan B}.$$

$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$;
$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$;
$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$;
$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$.

$$\text{Sin}\,(A + B)\sin(A - B) = \sin^2 A - \sin^2 B$$
$$= (\sin A + \sin B)(\sin A - \sin B).$$

$$\text{Cos}\,(A + B)\cos(A - B) = \cos^2 A - \sin^2 B;$$

$$\frac{\sin A + \sin B}{\sin A - \sin B} = \frac{\tan \frac{1}{2}(A + B)}{\tan \frac{1}{2}(A - B)}.$$

$$\text{Sin}\,3A = 3\sin A - 4\sin^3 A; \cos 3A = 4\cos^3 A - 3\cos A;$$

$$\text{Tan}\,3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}.$$

$$\text{Cosec}\,A - \cot A = \tan \tfrac{1}{2}A; \text{ cosec}\,A + \cot A = \cot \tfrac{1}{2}A.$$

If $A + B + C = 90°$;

$$\text{Tan}\,A \tan B + \tan A \tan C + \tan B \tan C = 1;$$

$$\text{Cot}\,A + \cot B + \cot C = \cot A \cot B \cot C;$$

$$\text{Sin}\,2A + \sin 2B + \sin 2C = 4\cos A \cos B \cos C.$$

If $A + B + C = 180°$;

$$\text{Sin}\,A + \sin B + \sin C = 4\cos \tfrac{1}{2}A \cos \tfrac{1}{2}B \cos \tfrac{1}{2}C;$$

$$\text{Cos}\,A + \cos B + \cos C = 1 + 4\sin \tfrac{1}{2}A \sin \tfrac{1}{2}B \sin \tfrac{1}{2}C;$$

$$\text{Tan}\,A + \tan B + \tan C = \tan A \tan B \tan C;$$

$$\text{Cot}\,A \cot B + \cot A \cot C + \cot B \cot C = 1.$$

$$2\sin A = \pm\sqrt{1 + \sin 2A} \pm \sqrt{1 - \sin 2A}.$$

$$2\cos A = \pm\sqrt{1 + \sin 2A} \mp \sqrt{1 - \sin 2A}.$$

Numerical Values of Functions of some Angles.

$$\text{Sin}\,0° = 0 = \cos 90°; \cos 0° = 1 = \sin 90°;$$
$$\tan 0° = 0 = \cot 90°; \tan 90° = \infty = \cot 0°;$$

$$\text{Sin}\,15° = \frac{\sqrt{3} - 1}{2\sqrt{2}} = \cos 75°; \sin 45° = \frac{1}{\sqrt{2}} = \cos 45°;$$

$$\text{Sin}\,18° = \frac{\sqrt{5} - 1}{4} = \cos 72°; \sin 60° = \frac{\sqrt{3}}{2} = \cos 30°;$$

$$\text{Sin}\,30° = \tfrac{1}{2} = \cos 60°; \sin 75° = \frac{\sqrt{3} + 1}{2\sqrt{2}} = \cos 15°;$$

Tan 15° $= 2 - \sqrt{3} = \cot 75°$; tan 60° $= \sqrt{3} = \cot 30°$;

Tan 30° $= \frac{1}{\sqrt{3}} = \cot 60°$; tan 75° $= 2 + \sqrt{3} = \cot 15°$;

Tan 45° $= 1 = \cot 45°$.

In any Plane Triangle ABC

$$\text{Sin A} = \frac{a \sin \text{B}}{b} = \frac{a \sin \text{C}}{c} = \sin (\text{B} + \text{C}) =$$
$$\frac{2}{bc}\left\{s\,(s-a)\,(s-b)\,(s-c)\right\}^{\frac{1}{2}};\ s = \frac{a+b+c}{2}.$$

$$\text{Cos A} = \pm \frac{\sqrt{c^2 - a^2 \sin^2 \text{C}}}{c} = \frac{b^2 + c^2 - a^2}{2bc} =$$
$$\frac{c \sin^2 \text{B} \mp \cos \text{B}\ \sqrt{b^2 - c^2 \sin^2 \text{B}}}{b}.$$

$$\text{Sin } \tfrac{1}{2} \text{A} = \sqrt{\frac{(s-b)\,(s-c)}{bc}};\ \cos \tfrac{1}{2} \text{A} = \sqrt{\frac{s\,(s-a)}{bc}};$$
$$\tan \tfrac{1}{2} \text{A} = \sqrt{\frac{(s-b)\,(s-c)}{s\,(s-a)}};$$
$$\tan \tfrac{1}{2} (\text{A} - \text{B}) = \frac{a-b}{a+b} \cot \tfrac{1}{2} \text{C}.$$

$$a = \frac{b \sin \text{A}}{\sin \text{B}} = \frac{c \sin \text{A}}{\sin \text{C}} = b \cos \text{C} + c \cos \text{B} =$$
$$\sqrt{b^2 + c^2 - 2bc \cos \text{A}} = b \cos \text{C} \pm \sqrt{c^2 - b^2 \sin^2 \text{C}}.$$

If s = area of triangle; $\text{s} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)} =$
$$\frac{ab}{2} \cdot \sin c = \frac{c^2}{2} \cdot \frac{\sin \text{A} \sin \text{B}}{\sin (\text{A} + \text{B})}.$$

Radius of circumscribed circle $= \frac{abc}{4\,\text{s}} = \frac{a}{2 \sin \text{A}}$.

Radius of inscribed circle $= \frac{S}{s} = \frac{a \sin \frac{1}{2} B \sin \frac{1}{2} C}{\cos \frac{1}{2} A}$.

Radius of escribed circle touching the side $a = \frac{S}{s-a}$.

Area of quadrilateral inscribed in circle $=$

$$\sqrt{(s-a)(s-b)(s-c)(s-d)}\ ;\ s = \frac{a+b+c+d}{2}.$$

Area of polygon of n sides inscribed in circle whose radius is $r = \frac{1}{2} nr^2 \sin \frac{360^\circ}{n}$; perimeter $= 2nr \sin \frac{180^\circ}{n}$.

Area of polygon circumscribing the same circle $= nr^2 \tan \frac{180^\circ}{n}$; perimeter $= 2nr \tan \frac{180^\circ}{n}$.

FORMULÆ IN SPHERICAL TRIGONOMETRY.

The sides and angles of the polar triangle are the supplements respectively of the angles and sides of the primitive triangle.

$A + B + C > \pi < 3\pi$.

In any spherical triangle;

$\cos a = \cos b \cos c + \sin b \sin c \cos A$.

$\sin A : \sin B : \sin C :: \sin a : \sin b : \sin c$.

$\cot a \sin b = \cot A \sin C + \cos b \cos C$; $\cos A = -\cos B \cos C + \sin B \sin C \cos a$.

$$\sin \tfrac{1}{2} A = \sqrt{\left\{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}\right\}};\ \cos \tfrac{1}{2} A = \sqrt{\left\{\frac{\sin s \sin (s-a)}{\sin b \sin c}\right\}};\ \tan \tfrac{1}{2} A = \sqrt{\left\{\frac{\sin (s-b) \sin (s-c)}{\sin s \sin (s-a)}\right\}}$$

$$\sin A = \frac{2}{\sin b \sin c} \left\{\sin s \sin (s-a) \sin (s-b) \sin (s-c)\right\}^{\frac{1}{2}}$$

$$\text{Sin } \frac{a}{2} = \sqrt{-\frac{\cos s \cos (s-A)}{\sin B \sin C}};$$

$$\cos \frac{a}{2} = \sqrt{\frac{\cos (s-B) \cos (s-C)}{\sin B \sin C}};$$

$$\tan \frac{a}{2} = \sqrt{-\frac{\cos s \cos (s-A)}{\cos (s-B) \cos (s-C)}}.$$

Napier's Analogies.

$$\text{Tan } \tfrac{1}{2}(A+B) = \frac{\cos \frac{1}{2}(a-b)}{\cos \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C;$$

$$\text{Tan } \tfrac{1}{2}(A-B) = \frac{\sin \frac{1}{2}(a-b)}{\sin \frac{1}{2}(a+b)} \cot \tfrac{1}{2} C.$$

$$\text{Tan } \tfrac{1}{2}(a+b) = \frac{\cos \frac{1}{2}(A-B)}{\cos \frac{1}{2}(A+B)} \tan \tfrac{1}{2} c;$$

$$\text{Tan } \tfrac{1}{2}(a-b) = \frac{\sin \frac{1}{2}(A-B)}{\sin \frac{1}{2}(A+B)} \tan \frac{c}{2}.$$

Area of lune $= \frac{A}{2\pi} \cdot 4\pi r^2 = 2Ar^2$.

Area of spherical triangle $= \frac{A+B+C-\pi}{2\pi} \cdot 2\pi r^2 =$

$\frac{\text{spherical excess}}{\text{4 right angles}} \times$ area of hemisphere.

FORMULÆ IN MENSURATION.

Area of rectangle $= ab$; a and b being adjacent sides.

Area of parallelogram $= ab \sin \theta$, a and b being adjacent sides including an angle θ.

Area of triangle $= \frac{pc}{2} = \frac{ab}{2} \sin C =$

$$\sqrt{\{s(s-a)(s-b)(s-c)\}} = \frac{c^2}{2} \cdot \frac{\sin A \sin B}{\sin (A+B)}$$

p being the $\perp$r drawn from $\angle$ A to side c ; $s = \frac{1}{2}(a+b+c)$.

Area of trapezoid $= \frac{(a+b)\,h}{2}$;

h being the $\perp$r distance between the parallel sides a, b.

Area of trapezium inscribed in circle $=$
$\sqrt{\{(s-a)(s-b)(s-c)(s-d)\}}$, $s = \frac{1}{2}(a+b+c+d)$.

Area of regular polygon $= \frac{na^2}{4} \times \cot \frac{180^\circ}{n}$;

n = number of sides each equal to a.

Area of regular polygon of n sides inscribed in circle, radius $= r$, $= \frac{nr^2}{2} \sin \frac{360^\circ}{n}$.

Area of regular polygon of n sides circumscribing same circle $= nr^2 \cdot \tan \frac{180^\circ}{n}$.

Circumference of circle $= 2\pi r$; $\pi = 3{\cdot}14159265$ etc. $=$ $3{\cdot}1416$ nearly.

Length of arc subtending $\angle$ A° at centre $= \frac{\pi r \mathrm{A}^\circ}{180^\circ}$.

Area of circle $= \pi r^2$; area of sector $= \pi r^2 \times \frac{\mathrm{A}^\circ}{360^\circ}$.

Area of segment of circle $= \frac{r}{2}(a-p)$,

p being the perpendicular drawn from the end of the arc a to radius through its origin.

Area of ring between two concentric circles $= \pi (r_1^2 - r_2^2)$
$= \pi (r_1+r_2)(r_1-r_2)$.
r_1 and r_2 being the radii of the outer and inner circles.

Surface of solid bounded by rectilineal figures $=$ sum of areas of bounding surfaces.

Curve surface of cylinder $= 2\pi r h$; r = rad. of base, $h = \perp$ height;

Whole surface of cylinder $= 2\pi r (h+r)$.

Curve surface of cone $= \pi r l$; $l =$ slant height.

Curve surface of frustum of cone $= \pi (r_1 + r_2) l$
r_1, r_2, radii of ends; $l =$ slant height.

Surface of sphere $= 4\pi r^2$.

Curve surface of segment of sphere $= 2\pi r h$; $h =$ height of segment.

Curve surface of spherical zone $= 2\pi r h$; $h =$ height of zone.

Volume of cylinder $= \pi r^2 h$.

Volume of cone $= \frac{\pi r^2 h}{3}$.

Volume of pyramid $= \frac{h}{3} \times$ area of base.

Volume of frustum of pyramid or cone

$$= \frac{h}{3}\left\{\Delta + \sqrt{\Delta \delta} + \delta\right\}$$

h, perpendicular height; Δ, δ areas of ends.

Volume of sphere $= \frac{4}{3}\pi r^3$.

Volume of segment of sphere $= \frac{\pi h}{6}(h^2 + 3r^2)$, r being radius of base, h, the height, of the segment; or volume $= \pi h^2 (r - \frac{h}{3})$

r being radius of sphere; h, height of segment.

Volume of zone of sphere $= \frac{\pi h}{6}\left\{h^2 + 3(p^2 + q^2)\right\}$

p, q being radii of ends.

FORMULÆ IN CO-ORDINATE GEOMETRY.

Equation to straight line, axes rectangular, $y = mx + c$.

Equation to straight line, axes rectangular, through (x', y'),
$y - y' = m(x - x')$.

Equation to straight line, axes rectangular, through
(x', y') and (x'', y''); $y - y' = \frac{y'' - y'}{x'' - x'}(x - x')$.

Equation to straight line, axes rectangular, through
(x', y') & $\perp$ to $y = mx + c$; $y - y' = -\frac{1}{m}(x - x')$.

Equation in terms of the intercepts of the axes, $\frac{x}{a} + \frac{y}{b} = 1$.

Equation in terms of perpendicular from origin,
$x \cos \alpha + y \sin \alpha - p = 0$.

Equation when the axes are oblique, $y = \frac{\sin \alpha}{\sin(\omega - \alpha)} \cdot x + c$.

Condition of perpendicularity of
$y = mx + c$ and $y = m_1 x + c_1$; $mm_1 = -1$.

Length of perpendicular, from
(x', y') upon $y = mx + c = \pm \frac{y' - mx' - c}{\sqrt{1 + m^2}}$.

Length of perpendicular, from
(x', y') upon $x \cos \alpha + y \sin \alpha - p, = x' \cos \alpha + y' \sin \alpha - p$.

Equation to circle, origin anywhere, $(x - a)^2 + (y - b)^2 = r^2$.

Equation to circle, origin at centre, $x^2 + y^2 = r^2$.

Equation to tangent at (x', y'), $xx' + yy' = r^2$;
$y = mx + r\sqrt{1 + m^2}$.

Equation to normal at (x', y'), $y = \frac{y'}{x'} \cdot x$.

Equation to chord of contact, $xh + yk = r^2$.

Equation to parabola, origin at vertex, $y^2 = 4ax$.

Equation to tangent at (x', y'), $yy' = 2a(x+x')$; $y = mx + \frac{a}{m}$.

Equation to normal at (x', y'), $y - y' = -\frac{y'}{2a}(x - x')$;

or $y = mx - 2am - am^3$.

Equation to ellipse, origin at centre, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Equation to tangent at (x', y'), $a^2yy' + b^2xx' = a^2b^2$,

or $y = mx + \sqrt{a^2m^2 + b^2}$.

Equation to normal at (x', y'), $y - y' = \frac{a^2y'}{b^2x'}(x - x')$,

or $y = mx - \frac{(a^2 - b^2)\,m}{\sqrt{b^2m^2 + a^2}}$.

Equation to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Equation to tangent at (x', y'), $a^2yy' - b^2xx' = -a^2b^2$,

or $y = mx + \sqrt{m^2a^2 - b^2}$.

Equation to normal at (x', y'), $y - y' = -\frac{a^2y'}{b^2x'}(x - x')$

or $y = mx - \frac{(a^2 + b^2)\,m}{\sqrt{a^2 - b^2m^2}}$.

Polar equation to straight line, $p = r\cos(\theta - \alpha)$.

Polar equation to circle, $c^2 = r^2 + l^2 - 2lr\cos(\theta - \alpha)$.

Polar equation to parabola, focus, the pole, $r = \frac{2a}{1 + \cos\theta}$;

vertex, the pole, $r = \frac{4a\cos\theta}{\sin^2\theta}$.

Polar equation to ellipse, focus, the pole, $r = \frac{a(1-e^2)}{1+e\cos\theta}$;

centre, the pole, $r^2(a^2\sin^2\theta + b^2\cos^2\theta) = a^2b^2$.

Polar equation to hyperbola, focus, the pole, $r = \frac{a(e^2-1)}{1+e\cos\theta}$;

centre, the pole, $r^2(a^2\sin^2\theta - b^2\cos^2\theta) = -a^2b^2$.

FORMULÆ IN DIFFERENTIAL CALCULUS.

$u = x^n$, $\frac{du}{dx} = nx^{n-1}$; $u = \log_a x$, $\frac{du}{dx} = \frac{1}{x\log_e a}$; if $a = e$, $\frac{d(\log_e x)}{dx} = \frac{1}{x}$.

$u = a^x$, $\frac{du}{dx} = a^x \log_e a$; if $a = e$, $\frac{d(e^x)}{dx} = e^x$.

$u = z^n$, z a function of x, $\frac{du}{dx} = az^{n-1}\frac{dz}{dx}$.

$u = a^z$, z a function of x, $\frac{du}{dx} = a^z \log_e a \frac{dz}{dx}$.

$u = z^v$, z and v functions of x, $\frac{du}{dx} = z^v\left\{\frac{dv}{dx}\log_e z + \frac{v}{z}\cdot\frac{dz}{dx}\right\}$

$u = f(z)$, z a function of x, $\frac{du}{dx} = \frac{du}{dz}\cdot\frac{dz}{dx}$.

$u = \sin\frac{x}{a}$, $\frac{du}{dx} = \frac{1}{a}\cos\frac{x}{a}$; $u = \cos\frac{x}{a}$, $\frac{du}{dx} = -\frac{1}{a}\sin\frac{x}{a}$.

$u = \tan\frac{x}{a}$, $\frac{du}{dx} = \frac{1}{a}\sec^2\frac{x}{a}$; $u = \cot\frac{x}{a}$, $\frac{du}{dx} = -\frac{1}{a}\operatorname{cosec}^2\frac{x}{a}$.

$u = \sec\frac{x}{a}$, $\frac{du}{dx} = \frac{1}{a}\sin\frac{x}{a}\sec^2\frac{x}{a}$;

$u = \operatorname{cosec}\frac{x}{a}$, $\frac{du}{dx} = -\frac{1}{a}\cos\frac{x}{a}\operatorname{cosec}^2\frac{x}{a}$.

$u=\sin^{-1}\frac{x}{a}$; $\frac{du}{dx}=\frac{1}{\sqrt{a^2-x^2}}$; $u=\cos^{-1}\frac{x}{a}$, $\frac{du}{dx}=-\frac{1}{\sqrt{a^2-x^2}}$.

$u=\tan^{-1}\frac{x}{a}$, $\frac{du}{dx}=\frac{a}{a^2+x^2}$; $u=\cot^{-1}\frac{x}{a}$, $\frac{du}{dx}=-\frac{a}{a^2+x^2}$.

$u=\sec^{-1}\frac{x}{a}$, $\frac{du}{dx}=\frac{a}{x\sqrt{x^2-a^2}}$;

$u=\operatorname{cosec}^{-1}\frac{x}{a}$, $\frac{du}{dx}=-\frac{a}{x\sqrt{x^2-a^2}}$.

$u=\operatorname{vers}^{-1}\frac{x}{a}$, $\frac{du}{dx}=\frac{1}{\sqrt{2ax-x^2}}$.

Leibnitz's Theorem.

If $u=yz$, y and z functions of x, $\frac{d^nu}{dx^n}=y\frac{d^nz}{dx^n}+n\frac{dy}{dx}\cdot$

$$\frac{d^{n-1}z}{dx^{n-1}}+\frac{n(n-1)}{\lfloor 2}\cdot\frac{d^2y}{dx^2}\cdot\frac{d^{n-2}z}{dx^{n-2}}+\ldots.+\frac{\lfloor n}{\lfloor r\,\lfloor n-r}\cdot$$

$$\frac{d^ry}{dx^r}\cdot\frac{d^{n-r}z}{dx^{n-r}}+\frac{\lfloor n}{\lfloor r+1\,\lfloor n-r-1}\cdot\frac{d^{r+1}y}{dx^{r+1}}\cdot\frac{d^{n-r-1}z}{dx^{n-r-1}}\cdots$$

$$+\frac{d^ny}{dx^n}z.$$

Maclaurin's Theorem.

$$f(x)=f(0)+xf'(0)+\frac{x^2}{\lfloor 2}f''(0)+\ldots.$$

$$+\frac{x^n}{\lfloor n}f^n(0)+\frac{x^{n+1}}{\lfloor n+1}f^{n+1}(\theta x).$$

Taylor's Theorem.

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{\lfloor 2} f''(x) + \ldots + \frac{h^n}{\lfloor n} f^n(x) + \frac{h^{n+1}}{\lfloor n+1} f^{n+1}(x+\theta h).$$

Equation to tangent, $y' - y = \frac{dy}{dx}(x' - x)$.

Equation to normal, $y' - y = -\frac{1}{\frac{dy}{dx}}(x' - x)$.

Subtangent $= y\frac{dx}{dy}$; subnormal $= y\frac{dy}{dx}$.

Differential coefficient of an arc, $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$$\therefore 1 = \left(\frac{dx}{ds}\right)^2 + \left(\frac{dy}{ds}\right)^2.$$

Differential coefficient of an area, $\frac{dA}{dx} = y$.

Differential coefficient of volume of solid of revolution, $\frac{dV}{dx} = \pi y^2$.

Differential coefficient of surface of solid of revolution, $\frac{dS}{dx} = 2\pi y \cdot \frac{ds}{dx}$.

Radius of curvature, $\rho = \frac{\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$

Parabola, $\rho = \frac{2(a+x)^{\frac{3}{2}}}{\sqrt{a}}$; Ellipse, $\rho = \frac{(a^2 - e^2x^2)^{\frac{3}{2}}}{ab}$;

Hyperbola, $\rho = \frac{(e^2x^2 - a^2)^{\frac{3}{2}}}{ab}$.

Cycloid, $\rho = 2\sqrt{2ay}$.

FORMULÆ IN INTEGRAL CALCULUS.

$$\int ax^m dx = \frac{ax^{m+1}}{m+1}; \quad \int \frac{dx}{x} = \log x; \quad \int a^x dx = \frac{a^x}{\log_e a};$$

$$\int e^x dx = e^x.$$

$$\int \sin x\, dx = -\cos x; \quad \int \cos x\, dx = \sin x;$$

$$\int \sec^2 x\, dx = \tan x; \quad \int \operatorname{cosec}^2 x\, dx = -\cot x.$$

$$\int \sin mx\, dx = -\frac{1}{m}\cos mx; \quad \int \cos mx\, dx = \frac{1}{m}\sin mx;$$

$$\int \frac{dx}{\sin x} = \log \tan \frac{x}{2}.$$

$$\int \frac{dx}{\cos x} = \log \tan \left(\frac{\pi}{4} + \frac{x}{2}\right); \quad \int \frac{dx}{\sin x \cos x} = \log \tan x;$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a}; \quad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}.$$

$$\int \frac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1}\frac{x}{a}; \quad \int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\sec^{-1}\frac{x}{a};$$

$$\int \frac{dx}{x\sqrt{2ax - x^2}} = \operatorname{vers}^{-1}\frac{x}{a}.$$

$$\int \frac{dx}{x^2-a^2} = \frac{1}{2a} \cdot \log \frac{x-a}{x+a};$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \log \left\{ x + \sqrt{x^2 \pm a^2} \right\};$$

$$\int \frac{dx}{x\sqrt{a^2 \pm x^2}} = \frac{1}{a} \cdot \frac{\log x}{a + \sqrt{a^2 \pm x^2}}.$$

Lengths of Curves.

$$s = \int \sqrt{\left\{ 1 + \left(\frac{dy}{dx}\right)^2 \right\}} dx.$$

Parabola, $s = \sqrt{ax + x^2} + a \log \frac{\sqrt{x} + \sqrt{a+x}}{\sqrt{a}}$;

Cycloid, $s = \sqrt{8ax}$; Catenary, $s = \frac{c}{2}\left(e^{\frac{x}{c}} - e^{-\frac{x}{c}}\right)$.

Ellipse, $s = 2\pi a \left\{ 1 - \frac{e^2}{2^2} - \frac{1 \cdot 3e^4}{2^2 \cdot 4^2} - \text{etc.} \right\}$

Areas of Curves.

$$A = \int f(x) dx.$$

Circle, $A = \pi r^2$; Sector, $A = \frac{ra}{2}$; Ellipse, $A = \pi ab$;

Cycloid, $A = 3\pi r^2$.

Parabola, $A = \frac{2}{3}$ of circumscribing rectangle.

Surfaces of Solids.

$$S = \int 2\pi y ds = \int 2\pi y \frac{ds}{dy} dy = \int 2\pi y \frac{ds}{dx} dx.$$

Sphere, $S = 4\pi r^2$; Cylinder, $S = 2\pi r (x_2 - x_1)$;

Frustum of cone $S = \pi \tan \alpha \sec \alpha (x_2^2 - x_1^2)$.

Cone, $S = \pi \operatorname{cosec} \alpha r^2$.

Paraboloid, $S = \frac{8\pi\sqrt{a}}{3}\left\{ (x+a)^{\frac{3}{2}} - a^{\frac{3}{2}} \right\}$.

Oblate spheroid, $s = 2\pi a^2\left\{1 + \frac{1-e^2}{2e}\log\frac{1+e}{1-e}\right\}$;

Prolate spheroid, $s = 2\pi a^2\left\{\sqrt{1-e^2}\ \frac{\sin^{-1}e}{e} + 1 - e^2\right\}$

Volumes of Solids.

$$v = \int \pi y^2 dx + c\,;\ v_2 - v_1 = \pi\int_{x_1}^{x_2} y^2 dx.$$

Sphere, $v = \frac{4\pi r^3}{3}$; Pyramid, $v = \frac{1}{3}a^2h$; Cone, $v = \frac{1}{3}\pi r^2 h$.

Paraboloid $v = \frac{1}{2}$ circumscribing cylinder;

Prolate spheroid, $v = \frac{4}{3}\pi ab^2$;

Oblate spheroid, $v = \frac{4}{3}\pi a^2 b$; Solid generated by revolution of cycloid about its axis, $v = \pi a^3\left\{\frac{3\pi^2}{2} - \frac{8}{3}\right\}$.

CIVIL SERVICE PAPERS.

I.

1. Write in figures one million four hundred and eighty-seven thousand six hundred and twenty-three; and express in words 70600804016003.

2. From £4768954 13*s.* 7½*d.* take £2989767 18*s.* 11¾*d.*

3. (1) Multiply £2487 14*s.* 2¼*d.* by 3076: (2) divide £7652194 0*s.* 9*d.* by 3076.

4. (1) Add ⅓, ¾, ⅝, ⅞: (2) divide ⅗ of ⅘ by ⅞ of ¾: (3) Multiply 332·1 by 44·1: (4) divide 2·5 by ·32.

5. Calculate the duty at £15 per cent. on £963 15*s.*

6. What is the income of a person who pays £22 7*s.* 5*d.* for income-tax at the rate of 7*d.* in the pound?

7. Find the value of 136 at £1 7*s.* 2*d.*

8. What is the expense of digging a ditch of which the solid content is 57$\dot{5}$5 cub. yds. at 15*s.* 7¾*d.* per yard.

9. At what rate will the interest on £4127 10*s.* amount to £92 17*s.* 4½*d.* in a year?

10. Find the compound interest on £364 for 3 years at 3¼ per cent.

11. Assuming that 32 bricks will make 9 sq. ft. of pavement, how many will be required to pave a yard 63 ft. long by 36 ft. wide?

12. How much time, in the course of 33 yrs., will a person, who rises at 5 o'clock in the morning, gain over another, who rises at 8 o'clock? supposing both to go to bed at 10 P.M. and computing 365¼ days to the year.

II.

1. Express in figures (1) Nine millions ten thousand and nineteen; (2) Five thousand and six million eight thousand and ninety-four.

2. Divide £89075 12*s.* 6*d.* by 9; and £713202 12*s.* by 576.

3. Reduce (1) £527 16*s.* 8½*d.* to farthings: (2.) 47 cwts. 1 qr. 16 lbs. 2 oz. to drams.

4. (1) Reduce $\frac{7040}{7360}$ to its lowest terms: (2) add $\frac{1}{4}$, $\frac{5}{12}$, $\frac{7}{15}$, $\frac{3}{20}$.

5. (1) Add 35·2176, 201·00541, 3·1482, ·054, 7543·4: (2) take 5·7124 from 3017·215: (3) multiply 5412·384 by 1·0023: (4.) divide 21546·8306 by ·0542.

6. Find the income-tax on £875 15*s.* at 14*d.* in the pound.

7. If a rental of £8050 a year be charged with land-tax at £11 5*s.* per £100, find the net income.

8. Find the value (1) of 373 at 9*s.* 7¾*d.*; (2.) of 821 at 13*s.* 10¾*d.* by practice.

9. Find the interest on £1158 17*s.* 6*d.* for 1 yr. 115 da. at 2½ per cent.

10. What sum of money put out at interest, for 6 yrs., at 3½ per cent., will amount to £259 2*s.* 10*d.*?

11. A ship is worth £16,000, and a person possessing $\frac{3}{8}$ of her, sells $\frac{5}{16}$ of his share, how much is the remainder of his share worth?

12. How many yards of paper 29 inches wide, will paper a room which is 22 ft. 9 in. long, 17 ft. 3 in. wide, and 9 ft. 8 in. high?

III.

1. Multiply 53786417276 by 5846, and divide 632799181 by 7243.

2. (1) Multiply £3001 6*s.* 4½*d.* by 351, and (2) divide £20954 17*s.* 2¼*d.* by 9.

3. How many lbs. of silver are there in 270 spoons, each weighing 1 oz. 13 dwts. 8 grs.? and how many seconds in 35 d. 8 h. 48 m. 29 sec.?

4. Add $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{11}$ and $\frac{2}{13}$ of $7\frac{4}{5}$, and subtract the result from $3\frac{3}{35}$.

5. (1) Subtract 239·89759 from 1314·9 ; and (2) multiply 95·376 by ·0283.

6. If a tax on an income of £1132 amount to £124 19*s.* 10*d.*, what is that in the pound ?

7. If the carriage of 107 cwts. 10 lbs. cost £37 9*s.* 7½*d.*, what would the carriage of a ton cost?

8. Find the price of 329 yds. 3 qrs. 2 nls. at 5*s.* 2½*d.* per quarter.

9. Find the discount on £517 10*s.* due 8 months hence at 5½ per cent. simple interest.

10. If telegraph posts are 66 yards apart, and a train passes one every 3″, how fast is the train moving?

11. If $\frac{5}{7}$ of 1 qr. cost 54*s.*, find the cost of $\frac{4}{9}$ of 1 bushel.

12. A coach goes 9 mi. an hour, and a train goes 21 mi. whilst the coach goes 11 ; how much time will be saved on a journey of 378 miles, by taking the train instead of the coach ?

IV.

1. Multiply 567039884276 by 8, and 403692158 by 396.

2. Divide 5802476484 by 6, and 294683010 by 865.

3. From £40132 7*s.* 2¼*d.* take £37849 15*s.* 7½*d.*

4. Multiply £25841 16*s.* 5¼*d.* by 7, and £1854 7*s.* 7½*d* by 465.

5. Divide £85432 19*s.* 5½*d.* by 7, and £27543 12*s.* 8*d* by 352.

6. A person having £5704 18*s.* 4½*d.* lays out ⅓ in goods which he sells for £2316 5*s.* 10*d.* ; how much has he gained?

7. Reduce (1) 375 cwts. 2 qrs. 1 st. 13 lbs. to oz., and (2) 36 mi. 3 fur. 36 po. 5 yds. to feet.

8. If £59 10*s.* will buy 25 cwts. 2 qrs. of sugar, what weight will £6 14*s.* 2*d.* buy ?

9. If the 1*d.* loaf weighs 5 oz. when wheat is 62*s.* per qr., what ought it to weigh when wheat is 64*s.* per quarter ?

10. Find, by Practice, the value of; (1) 373 articles at £2 16*s.* 10¼*d.* each; (2) the rent of 67 acres of land for 2 yrs. 9 mo. 11 da., at £2 5*s.* an acre per annum.

11. Find the interest on £1275 15*s.* for 5¾ years at 4 per cent. per annum.

12. At what rate per cent., simple interest, will £951 7*s.* 6*d.* amount to £1141 13*s.* in 5 years?

V.

1. The Norman Conquest took place in 1066 A.D., the accession of Queen Victoria in 1837; how many years elapsed between these events?

2. (1) Multiply £36758 17*s.* 5¼*d.* by 7, and (2) £1563 17*s.* 9¾*d.* by 357.

3. Divide £13854 9*s.* 2½*d.* by 158.

4. (1) How many solid yards, feet and inches, are there in 175893 solid inches? And (2) how many square yds., ft. and in. are there in the top of a table, the sides of which are 21 ft. 7½ in. and 19 ft. 5 in.?

5. (1) Multiply $\frac{3}{5}$ of $\frac{7}{4\frac{3}{3}}$ of 15 by $\frac{1\frac{4}{5}}{1\frac{5}{7}}$ of $12\frac{2}{7}$; (2) what number multiplied by $\frac{3}{5}$ will produce $15\frac{3}{5}$?

6. What fraction of half-a-crown is $\frac{2}{9}$ of 6*s.* 8*d.*?

7. If 24 men, working 8 hrs. a day, can build a house in 70 days, in how many days might it be built by 42 men, working 10 hrs. a day?

8. Find, by Practice, (1) the price of 329 yds. 3 qrs. 2 nls. at 5*s.* 2½*d.* per quarter; (2) the dividend on £2734 16*s.* 8*d.* at 9*s.* 4½*d.* in the pound.

9. (1) Express $\frac{2}{15}\frac{4}{4}\frac{1}{24}$ as a decimal; (2) reduce 8*s.* 11¼*d.* to the decimal of 1 guinea.

10. Find the sum of ·3211 guin. $\frac{3}{8}$ cr., ·6925*s.*, and reduce the result to the decimal of £1.

11. Find the interest on £4000, in 6 months, at 5 per cent., less income-tax at 1*d.* in the £.

12. What sum will amount to £798 15*s.* in 1 year at 6½ per cent.?

VI.

1. (1) Reduce 6 ac. 1 ro. 4 po. to the fraction of $2\frac{1}{2}$ roods; (2) if $\frac{3}{5}$ lb. cost 8*s*. 2*d*. find the cost of $5\frac{1}{7}$ lb.

2. Multiply 41·018 by 200, and 1·02 by 3·067; divide 69·814 by ·00521 and by 52100.

3. Reduce (1) $\frac{7}{8}$ hf. cr. to the decimal of 1 guin.; (2) 2·0445, (3) ·000625, (4) $\cdot 1\dot{4}\dot{4}$, (5) $\cdot 34\dot{2}1\dot{8}$ to vulgar fractions; (6) $\frac{3}{32}$, $\frac{40}{33}$ to decimal fractions.

4. The shares of a railway are at $59\frac{1}{2}$ when consols are at $93\frac{1}{2}$; what should be their value when consols are at $71\frac{1}{2}$?

5. If 49 men can do a piece of work in 130 days of 8 hrs. each, how many hrs. a day must 196 men work to do as much in 26 days?

6. Convert 22 lbs. 5 oz. 15 dwts. 20 grs. troy into avoirdupois weight, 1 lb. avoirdupois being to 1 lb. troy as 175 to 144.

7. Find, by Practice, the value of 5 lbs. 9 oz. 7 dwts. 12 grs. of gold at £3 17*s*. 10*d*. an ounce.

8. What sum of money put out at interest for 3 years at $4\frac{1}{2}$ per cent. will amount to £1602 $1\frac{1}{4}$*d*.?

9. In what time will £1337 12*s*. 1*d*. amount to £1698 15*s*. $1\frac{3}{4}$*d*., at 6 per cent. simple interest?

10. If £3 = 20 thalers, 25 thalers = 93 francs, 62 francs =25 gulden: how many gulden are equal in value to £1?

11. If 20 horses and 196 sheep can be kept 18 days for £151 10*s*., what sum will keep 15 horses and 72 sheep for 8 days, if 5 horses eat as much as 76 sheep?

12. Find the amount of £13333 6*s*. 8*d*. for 4 years at 5 per cent. compound interest.

VII.

1. Reduce $\frac{34}{35}\frac{32}{74}$ to its lowest terms and divide the result by $1\frac{1}{2}$.

2. Simplify (1) $\frac{3}{4} + \frac{2}{3} + \frac{7}{12} + \frac{5}{96} + \frac{11}{100}$ and take the result from $2\frac{1}{6}$; (2) $3\frac{4}{7}$ of $\frac{4}{95}$ of $\frac{38}{20\frac{1}{7}} \times 6\frac{2}{3}$; (3) $\frac{5}{6}$£ + $\frac{7}{9}$ of £2 10*s.* 3*d.* + $3\frac{1}{3}$ of 14*s.*

3. Take the sum of ·004, 2·08, 314·6073, ·19874, 1000, from 21118·0116.

4. (1) Multiply ·671 by ·42 and by ·0420; (2) divide 738·952716 by ·07 and by 70.

5. How many francs at $9\frac{1}{2}$*d.* each are there in £693 14*s.* 9*d.*?

6. The area of a rectangular plot is 101 yds. 0 ft. 81 in.; its length is 36 ft. 9 in. Find the breadth.

7. If when wheat is 11*s.* 3*d.* a bushel, the sixpenny loaf weighs 28 oz., how much should it weigh when wheat is 84*s.* a quarter?

8. If the carriage of 65 lbs. for 120 miles costs 6*s.*, how much may I have carried 750 miles for a guinea?

9. Find, by Practice, the value of 2 roods 19 poles 12 yards of land at £80 13*s.* 4*d.* an acre.

10. What sum of money put out at interest for $3\frac{1}{2}$ years at 4 per cent. will amount to £296 8*s.*?

11. In what time will £171 7*s.* 6*d.* amount to £217 1*s.* 6*d.* at $3\frac{1}{3}$ per cent.?

12. Find the discount on £1511 7*s.* 6*d.*, payable $3\frac{1}{4}$ years hence, at 4 per cent. simple interest.

VIII

1. Reduce £125 10*s.* into francs and centimes, exchange at 23 francs 6 centimes per £1 (1 franc = 100 centimes).

2. What salary is due for 95 days at £32 17*s.* a year?

3. An officer's pay is 12*s.* 3*d.* a day; how much is that a year?

4. A banker owes £24680, and can pay 15*s.* 6*d.* in the pound, what is the value of his assets?

5. What is the amount of the weekly wages of 377 men at 1*s.* 8*d.* a day?

6. A coach goes from London to Liverpool, at the rate of 9 miles an hour, in 24 hrs.; in what time would the same distance be traversed by a train moving at the rate of 24 miles an hour?

7. A besieged town containing 22400 inhabitants has provisions for 3 weeks; how many must leave in order that the provisions may last 7 weeks?

8. If 5 men receive £18 15*s.* for 12 weeks' wages, what will be the wages of 16 men for 20 weeks?

9. If 6 iron bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh 288 lbs., how much will 15 weigh each $6\frac{1}{2}$ ft. long, 4 in. broad, and 3 in. thick?

10. Find the value of 345 tuns 2 hhds. 42 gals. at £$4\frac{1}{2}$ 10*s.* per tun.

11. Reduce (1) ·00175 and (2) $\cdot 1\dot{3}0\dot{9}$ to fractions in their lowest terms.

12. $44\frac{1}{2}$ guineas used to be coined out of 1 lb. of standard gold, how many sovereigns are now coined out of this weight?

IX.

1. Reduce (1) 13 lbs. 7 oz. 16 grs. to grains; (2) 1847638 in. to miles &c., (3) 138297 cubic inches to yards.

2. Find, by Practice, the value of 3 lbs. 5 oz. 14 dwts. 12 grs. at 17*s.* 6*d.* an oz.

3. Find the value of (1) $\frac{2}{7} \times \frac{\frac{5}{11} \text{ of } 5\frac{1}{2} \times 2\frac{1}{3}}{\frac{4}{9} \text{ of } \frac{3}{13} \times 8\frac{2}{3}}$;

(2) £$\frac{2}{3}$ + $\frac{3}{7}$ of £2 10*s.* 2*d.* + $2\frac{1}{3}$ of 16*s.*

4. Reduce $\frac{5}{64}$, $\frac{3}{14}$ to decimals; and express 23·038125, ·0006875 as vulgar fractions in their lowest terms.

5. Reduce $\frac{5}{8}$ of 1 cr. to the decimal of 21*s.*; and $6\frac{2}{3}$ cwt. to the dec. of 1 ton.

6. Reduce $\cdot 1\dot{3}\dot{5}$, $\cdot 012\dot{3}\dot{6}$, to their equivalent vulgar fractions.

7. A bankrupt's effects amount to 2548 guineas, and his debts to £3057 12*s.*, what will he pay in the £?

8. If 5 men can do a piece of work in 27 days, how long would 6 men be in doing double the quantity?

9. Find the interest on £260 10*s.* for $3\frac{1}{3}$ years, at $4\frac{1}{4}$ per cent.

10. At what rate per cent. will £320 amount to £360 16*s.* in 3 years?

11. Find how many yards of paper $\frac{7}{8}$ yd. wide, will paper a room whose length is 26 ft. 4 in., breadth 18 ft. 8 in., height 12 ft. 3 in.?

12. Extract the square root, (1) of 531441; (2) of $32\frac{1}{9}$.

X.

1. (1) Reduce $\frac{6}{7}\frac{3}{3}\frac{9}{1}\frac{8}{2}$ to its lowest terms; (2) divide the sum of $\frac{2}{5}$, $\frac{4}{5}$, $\frac{3}{10}$, $\frac{7}{18}$ by $7\frac{5}{6}$; (3) find the value of $(\frac{2}{3}$ of $\frac{7}{8} + \frac{3}{7}$ of $\frac{14}{15})$ of 1*s.*

2. (1) Multiply 3·05 by ·25 and ·32 by ·231 : (2) divide 721·41885 by 21·9, and ·1 ÷ ·001 by ·2.

3. The price of consols is $88\frac{3}{8}$; how many railway shares, at £8 15*s.* each, can be purchased for £1000 stock?

4. The rates of the express and mail trains on a railway are 40 and 28 miles an hour respectively: what time is saved by taking the faster for a journey of 192 miles?

5. If 21 men take 8 days to dig 20 ac., how many acres will 16 men dig in 12 days?

6. Find, by Practice, the cost (1) of 11 mi. 3 fur. 55 yds. of railway at £32500 a mile; (2) of 716 ac. at £44 11*s.* $3\frac{1}{4}$*d.* an acre.

7. Find the amount of £322 13*s.* for $10\frac{1}{4}$ years at $3\frac{1}{2}$ per cent. simple interest.

8. Find the discount on £1452, paid 2 months before it is due, at 5 per cent.

9. Find the amount of £240 in 4 years at 3 per cent., compound interest.

10. If 23 lbs. at 2*s.* are mixed with 27 lbs. at 3*s.*, and the mixture is sold at 2*s.* 9*d.* per lb., what is the total gain; and what the gain per cent.?

11. Extract the square root of 106929 ; also of 803·7 to 3 places of decimals.

12. Explain the meaning of the term *local* value. Have Roman numerals any local value ?

XI.

1. Multiply (1) 85001997 by 5, and (2) 73299863 by 497.

2. Divide 2975069895 by 35, and 628010082036 by 709.

3. From £19813 7*s.* 7½*d.* take £2009 6*s.* 10¾*d.*

4. Multiply (1) £35087 17*s.* 7½*d.* by 9, and (2) £9003 19*s.* 1½*d.* by 39.

5. How many yards of paper 1 yd. wide are required to paper a room 15 ft. long, 12 ft. wide, and 10 ft. high ?

6. How often will a wheel 3¼ yds. in circumference turn in a distance of 208 miles ?

7. If by paying down £70 17*s.* 6*d.* a person becomes entitled to £2 9*s.* 7*d.* a year, what income should he gain by paying down £2126 5*s.* ?

8. Find the value of ; (1) 9 tons 4 cwts. 3 qrs. 21 lbs. at £14 5*s.* 9*d.* a ton ; (2) of 217¼ yards of lace at £2 17*s.* 7½*d.* a yard.

9. Find the interest on £194, at 5 per cent. per annum, from the 3rd of March to the 20th of December following.

10. When the 3 per cents. are at £85 17*s.* 6*d.*, what is the rate of interest?

11. Find the sum of (1) £3⅝, 9¾*s.* and 2¼*d.* : (2) of ·130055, 900, 57·1, 13·34 and ·00000397.

12. Divide the quotient of 16 ÷ 2 by ·0002.

XII.

1. (1) Multiply £3467 5*s.* 7½*d.* by 346 ; (2) divide 2665 miles 2 furl. 31 poles ½ yard by 58.

2. A train is travelling at the rate of 32½ miles per hour ; over how many feet does it pass per second?

3. If 84 men can do a piece of work in two months, how long will it take 12 men to do the same?

4. If the carriage of 18 tons 13 cwts. 3 qrs. 14 lbs. cost 18*s.* $7\frac{3}{4}d.$, what will be the cost of carriage of 255 tons, 9 cwts. 2 qrs. 14 lbs.?

5. What is the value of $\frac{\frac{2}{5} \text{ of } 5\frac{3}{4} - \frac{3}{8} \text{ of } 4\frac{1}{4}}{\frac{7}{8} \text{ of } \frac{3}{7} \text{ of } 5\frac{1}{4} - \frac{5}{8} \text{ of } \frac{4}{15} \text{ of } 3\frac{2}{3}}$

6. Find, by Practice, the value of 1389 tons of coal at £1 13*s.* 8*d.* per ton.

7. What decimal of a pound troy is 16 dwts.?

8. What are the values of ·346 of a pound sterling, and of ·058 of a year?

9. (1) Multiply 32·47 by ·0033; (2) divide 1·0226816 by ·0436.

10. What sum placed out at interest at 4 per cent. would amount to £560 in 3 years, allowing simple interest.

11. Divide £350496 among 4 persons in the proportion of 3, 7, 9, and 11.

12. A sum of £250 17*s.* 6*d.* is transmitted through Paris to New York. One sovereign English is worth 24 francs 79 cent. at Paris, and 6 francs 20 cent. French are worth 2 dollars 25 cents American. What is the value of the sum at New York in American currency?

COLLEGE OF PRECEPTORS.

XIII.

1. What sum added to £43 14*s.* $3\frac{3}{4}d.$ will make 48000 farthings?

2. Multiply 3 tons 14 cwts. 2 qrs. 14 lbs. 12 oz. by 38, and prove the result.

3. A person bought 240 yards of cloth at 14*s.* 9*d.* a yard, and sold it at 15*s.* 7*d.* a yard; how much did he gain?

4. How long a time would it take a column of men, extending half a mile in length, to pass by, at the rate of 75 steps a minute, reckoning each step as 2 feet 8 inches?

5. Explain clearly the meaning of a vulgar fraction. State the rule for the addition of vulgar fractions, and prove it by an example.

6. (1) Add together $\frac{3}{4} + \frac{5}{6} + \frac{11}{12} + \frac{14}{15}$.

(2) From $2\frac{1}{3}$ of $4\frac{1}{19}$ take $2\frac{1}{3} + 4\frac{1}{19}$.

(3) Divide $5\frac{1}{4} + 3\frac{9}{11}$ by $(\frac{3}{7} \div 1\frac{4}{7})$.

7. How often is $\frac{3}{5}$ of £1 11*s.* 8*d.* contained in 19 guineas?

8. If $5\frac{1}{2}$ yards make one pole, show that there are $30\frac{1}{4}$ square yards in a square pole.

9. A man working 10 hours a day completes a piece of work in 12 days; in how many days could he do it, working 8 hours a day?

10. If a garrison of 1500 men, having provisions for 5 weeks, be reinforced with 500 men, how long will the provisions last?

11. Find, by Practice, the rent of $209\frac{1}{4}$ acres, at £2 17*s.* 6*d.* per acre.

12. Find the area of a path 6 feet wide, which surrounds a court 30 yards long and 24 yards broad.

XIV.

1. Multiply 591863 by 907; and divide 10546279 by 84 by short division, (two factors).

2. What will be the cost of relieving 165000 poor persons at 1*s.* $3\frac{1}{2}$*d.* per head?

3. If £1279 13*s.* $8\frac{3}{4}$*d.* be divided into sums of £55 12*s.* $9\frac{1}{4}$*d.* each, how many such sums will there be?

4. A building society purchases three estates containing 35 a. 2 r. 23 p. $12\frac{1}{2}$ square yds., 9 a. 2 r. 36 p. $27\frac{1}{4}$ sq. yds., and 40 a. 0 r. 35 p. $20\frac{3}{4}$ sq. yds. respectively. If the property be divided into 128 equal parts, of how much does each part consist?

5. Reduce to their simplest forms—

(1) $4\frac{3}{4} + \frac{3}{7}$ of $10\frac{1}{2} + 3\frac{3}{10}$. (2) $\frac{2\frac{1}{2}-\frac{1}{3}}{2\frac{1}{3}-1\frac{1}{4}}$. (3) $\frac{13667}{14186}$.

Is $\frac{13}{77}$ greater or less than $\frac{14\frac{1}{2}}{88}$, and by how much ?

6. Reduce $\frac{3}{10}$ of £2 0*s.* $3\frac{3}{4}$*d.* to the fraction of £1 4*s.* $2\frac{1}{4}$*d.* ; and 2 qrs. 3 lbs. 1 oz. to the fraction of 1 cwt. 3 qrs. 0 lbs. 14 oz.

7. Find, by Practice, the price of 1298 yards at 17*s.* $9\frac{3}{4}$*d.* per yard, and the price of 15 yds. 2 ft. 8 in. at £1 19*s.* per yard.

8. Find the value of a wedge of gold weighing 14 lbs. 3 oz., if 2 oz. 3 dwts. be worth £6 9*s.*

9. If the carriage of 169 cwts. 2 qrs. for 130 miles cost £34, what weight may I have carried 78 miles for the same money ?

10. What length of paper 1 ft. 9 in. wide will be required to cover a wall 17 ft. 3 in. long, and 11 ft. 8 in. high ?

11. Divide £154 10*s.* among 3 railway companies in the proportions of 27, 31, 45 ; and show that the result is correct.

12. Extract the square root of 8450649.

XV.

1. Add together 72 tons 5 cwts. 15 lbs., 18 tons 6 cwts. 3 qrs., 14 cwts. 18 lbs., and multiply the sum by 56.

2. How many days, hours, minutes, and seconds are there in 1213459 seconds ?

3. In 1000 guineas how many crowns, florins, and three-penny pieces are there ?

4. If 144 tons of coals cost £181 16*s.*, find the price of one ton, and of 245 tons.

5. How many half-crowns, shillings, and farthings, an equal number of each, are there in £169 ?

6. What is meant by the expression $\frac{7}{9}$?

Give the rules for reducing whole numbers and mixed numbers to improper fractions; and for multiplying and dividing a fraction by a whole number. Give examples in each case.

7. (1) Add together $2\frac{1}{16}$, $3\frac{17}{18}$, $\frac{26}{27}$, and 4.

(2) Subtract $\frac{5}{6} + \frac{7}{12} + \frac{3}{4}$ from $\frac{2}{3} + \frac{3}{4} + \frac{11}{12}$.

(3) Multiply $2\frac{1}{4}$ of $10\frac{2}{3}$ of 15 by $7\frac{1}{5}$ of $\frac{1}{144}$.

(4) Divide $2\frac{1}{5}$ of $3\frac{1}{4}$ by $5\frac{1}{2}$ of $4\frac{1}{3}$.

8. Find the value of $12\frac{4}{15}$ of £3 15*s.* $3\frac{3}{4}$*d.*

9. How much money will pay for 196 qrs. of wheat at £2 15*s.* 6*d.* per quarter, and $109\frac{1}{2}$ qrs. of oats at £1 17*s.* 11*d.* per quarter?

10. If 161 lbs. of tea are given in exchange for 12 cwts. 21 lbs. of sugar at $5\frac{3}{4}$*d.* per lb., what is the price of tea per lb.?

11. If a staff 5 feet high casts a shadow 7 ft. 2 in. long, what is the height of a steeple which casts a shadow 215 ft. long?

12. How many planks $15\frac{1}{4}$ feet long $1\frac{1}{4}$ feet wide will floor a room 60 feet long by $30\frac{1}{2}$ feet wide?

XVI.

1. Reduce 53 tons 4 cwts. 77 lbs. to ounces; and 1 mile 5 furl. 13 poles 2 inches, to inches.

2. What is the value of 687 Napoleons, a Napoleon being worth 15*s.* $11\frac{1}{2}$*d.*?

3. A person, after paying 10*d.* in the pound income-tax upon a yearly income of £400, laid by £85 17*s.*; what was his average weekly expenditure?

4. What sum must be added to £31 14*s.* $6\frac{1}{2}$*d.* to make it exactly divisible by £2 11*s.* 4*d.*?

5. What is meant by the Least Common Multiple of two or more numbers? Find that of 18, 51, 34, 7, 30.

6. Reduce $\frac{21609}{25}$, $\frac{28857}{1423}$, to their lowest terms.

7. Add together (1) $5\frac{14}{15}$, $3\frac{19}{20}$, $11\frac{44}{45}$. (2) $\frac{7}{10}$, $\frac{11}{15}$, $\frac{13}{18}$, $\frac{5}{36}$.

8. Simplify $\frac{6\frac{1}{19}\frac{3}{}-4\frac{1}{19}}{\frac{2}{3}\div 15\frac{6}{11}}$

9. Find the value of $14\frac{1}{15}\frac{4}{}$ times £2 6*s.* $4\frac{3}{4}d.$

10. What is the cost of 31 oz. 15 dwts. 13 grs., at 7*s.* $7\frac{1}{2}d.$ per oz.?

11. If the carriage of 3 cwts. 50 lbs. for 135 miles cost 15*s.*, how far ought 5 cwts. 19 lbs. to be carried for the same money?

12. Divide 241 sq. yds. 8 sq. ft. 112 sq. inches by 46 sq. ft. 8 sq. in.

XVII.

1. If 177 be the quotient, 379 the divisor, and 85 the remainder, what is the dividend?

2. Divide 6287644 by 121 by short division (two factors); and find the least number which added to 152181255 will make it exactly divisible by 3854.

3. Find the total weight of a luggage train of 23 carriages, the average weight of each carriage being 3 tons 5 cwts. 1 qr. 23 lbs.

4. From £1076 4*s.* $3\frac{3}{4}d.$, 5*s.* $1\frac{3}{4}d.$ is subtracted, and the remainder is divided equally among 527 people: how much will each have?

5. Reduce $27\frac{3}{4}$ guineas to three-pennies, and 1234567 inches to miles.

6. What is meant by the G. C. M. of two or more numbers? Find the G. C. M. of 13536 and 23148.

7. Simplify (1) $\frac{5}{18}\frac{0}{3}\frac{1}{7}$. (2) $\frac{2}{3}+\frac{5}{6}+\frac{11}{12}+\frac{17}{18}$.
(3) $[(2\frac{1}{3}+\frac{1}{6})\times(3\frac{1}{2}+\frac{3}{8})]\div(2-\frac{1}{16})$.
(4) $\cfrac{1}{2+\cfrac{3}{4+\frac{5}{6}}}$

8. Express 15 sq. yds. 6 sq. ft., as the fraction of 1 acre 6 poles; and find the value of $\frac{3}{4}$ of a guinea $+\frac{3}{8}$ of a crown $-\frac{3}{8}$ of 7*s.* 6*d.*

9. Find, by Practice, the rent of 75 acres 2 roods 12 poles, at £1 11*s.* 6*d.* per acre.

10. If 5 pieces of cloth, each containing 12 yards, cost £17 10*s.*, what will 22 yards cost?

11. If 15 men in 9 days can reap a field, how long will 10 men require to reap a field of twice the area?

12. A cistern 8 ft. long, 3 ft. wide, and 9 inches deep, is filled with pulp for making paper. How long a sheet 2 feet 6 inches wide can be made from it, the thickness of the paper being $\frac{1}{250}$ of an inch, supposing $\frac{1}{2}$ the volume of the pulp to be lost in the process of drying?

XVIII.

1. Reduce 4 tons 5 cwts. 3 qrs. 27 lbs. to ounces, and 495 yards to ells. Prove the results.

2. Divide £395 14*s.* $0\frac{3}{4}$*d.* by 125, and prove the result.

3. Divide £20 among 3 persons, giving to the first £2 more than to the second, and to the second £1 more than to the third.

4. Find the G. C. M. of 17255 and 14161; and the L. C. M. of 40, 42, 44, 48. Give reasons for the method employed in finding the L. C. M.

5. Show that the value of a fraction is not altered by multiplying the numerator and denominator by the same number. Reduce $\frac{13455}{11385}$ to its lowest terms.

6. Reduce to simple fractions—

(1) $3\frac{2}{5} + 4\frac{3}{8} + \frac{1}{12} + 2\frac{19}{20}$.

(2) $5\frac{1}{11}$ of $1\frac{1}{14}$ of $4\frac{8}{9}$ of 3.

(3) $10\frac{1}{2} \times 3\frac{1}{7} - 4\frac{2}{3} \div 3\frac{1}{3}$.

7. Find the value of $\frac{3}{16}$ of a guinea + $1\frac{1}{6}$ of a crown + $\frac{2}{15}$ of half-a-crown, and reduce the result to the fraction of £1.

8. Find, by Practice, the value of—

(1) 397 things, at £3 2*s.* $9\frac{3}{4}$*d.* each.

(2) 135 sq. yds. 6 ft. 72 in., at 2*s.* 3*d.* per yard.

9. A grocer buys 200 lbs. of sugar at $5\frac{1}{4}d.$ per lb.; what does it cost him? At what price per lb. must he sell it to clear £1 9*s.* 2*d.*?

10. If 1 lb. 2 oz. 9 dwts. of silver cost £5 8*s.* $4\frac{1}{2}d.$, what is the price of an ounce?

11. In what time will 40 men do a piece of work which 36 men can do in 30 days?

12. If 11,000 copies of the *Times* be issued daily, each copy consisting of 2 sheets, and each sheet being 4 ft. by 3 ft., how many acres will one edition cover?

XIX.

1. How many half-guineas are there in a thousand half-crowns; and in 28624 inches, how many furlongs, poles, yards, feet, and inches?

2. How many steps of 2 ft. 9 in. each must be taken in a minute, in order to walk at the rate of four miles an hour?

3. Simplify (1) $\frac{1}{3\frac{1}{9}}$ of $6\frac{1}{2}$ of $114-3\frac{8}{13}$.

(2) $\dfrac{7\frac{1}{2}\times\frac{1}{5}\text{ of }\frac{4}{9}}{13\frac{5}{12}-12\frac{7}{9}}$.

(3) Compare the values of $\dfrac{2\frac{1}{2}}{27}$, $\dfrac{8}{81}$, and $\dfrac{20}{243}$; and resolve 30 and 42 into their prime factors.

4. Express the difference between $\frac{1}{8}$ of 18*s.* 4*d.*, and $\frac{3}{8}$ of a crown, as a fraction of 2*s.* 6*d.*; and express $2\frac{2}{3}$ of 5 a. $3\frac{5}{8}$ r. as the fraction of 19 a. $2\frac{3}{4}$ r.

5. A bankrupt's debts amount to £869 10*s.*, and he can pay only 11*s.* 6*d.* in the pound. What is his estate worth? and how much will a creditor, to whom he owes £57 6*s.* 8*d.*, receive?

6. Find, by Practice, the rent of a farm of 225 a. 1 r. 19 p., at £2 12*s.* 10*d.* per acre per annum.

7. Divide the product of ·035 and ·0056 by ·00007. Also divide 5·04 by ·012, and 420 by ·420.

8. Find the difference between ·70323 of a pound, and 3·5646 of a shilling; and reduce 14 hrs. 15 min. to the decimal of $3\frac{1}{2}$ days.

9. What will a piece of plate cost, which weighs 3 lbs. 4 oz. 5 dwts., if the price of silver be £3 6*s.* per lb., and the charge for workmanship be 2*s.* 8*d.* per ounce?

10. How many hours a day must 42 boys work, to do in 45 days what 27 men can do in 28 days of 10 hours each, the work of a boy being half that of a man?

11. If 8 guineas be expended in purchasing Brussels carpet $\frac{3}{4}$ yd. wide, at 3*s.* 6*d.* per yard, for a room 20 ft. long and 16 ft. 9 in. broad; how much of the floor will remain uncovered?

12. At what rate per cent. will the interest on £375 amount to £48 2*s.* 6*d.* in 3 years 8 months?

XX.

1. How many acres, roods, poles, yards, feet, and inches are there in 40253798 square inches?

2. Simplify (1) $\frac{5}{7}-1\frac{3}{5}+2\frac{6}{45}+\frac{4}{7}$ of $2\frac{1}{5}-2\frac{3}{7}$.

(2) $819_{5\,6\,1}^{1\,3}-815_{1\,8\,7}^{\,7}$.

3. Find the value of £17 2*s.* 9*d.* $\div 11\frac{5}{12}+$£1 16*s.* 9*d.* $+\frac{63}{99}$.

What part of half-a-crown is $\frac{1}{7}$ of $\frac{2\frac{1}{4}-\frac{2}{3}\text{ of }1\frac{5}{6}}{\frac{1}{5}\times 3\frac{1}{3}+\frac{1}{36}}$ of $\frac{2}{11}$ of a guinea?

4. What is the value of $\frac{1}{11}$ of $\frac{1}{12}$ of a vessel, if a person who owns $\frac{3}{11}$ of it sells $\frac{1}{9}$ of $\frac{7}{8}$ of his share for £350?

5. Standard gold consists of 22 parts pure gold with two parts copper, and from a pound Troy of this metal are coined $46\frac{29}{40}$ sovereigns. Find the quantity of pure gold in 100 sovereigns.

6. Divide ·027 by 14·4; 1632 by ·0008; and 7 by ·091.

7. Reduce 7*s.* $8\frac{1\,9\,4\,2}{1\,0\,0\,0\,0}$*d.* to the decimal of half a guinea; and express the difference between 2·778125 of 6*s.* 8*d.* and ·06$\dot{3}$ of £13 2*s.* 6*d.* as the decimal of 2*s.* 3*d.*

8. If an ounce of gold be worth £4·0099, what is the value of ·561 of a lb. ?

9. If 12 men can dig a trench 40 yards long and 4 feet wide, in 10 days of 8 hours each, how many hours a day must 55 men work, in order to dig a trench of the same depth, 220 yards long and 5 feet wide, in 18 days ?

10. When the three per cents. are at 92⅝, what must be given for £533 6*s.* 8*d.* stock ? What rate of interest will be obtained by purchasing three per cents. at 91¼ ?

11. Define simple and compound interest. Find the present value of £252 19*s.* 3*d.* due a year hence at 3½ per cent. ; and the amount of £850 in 3 years at 6 per cent. compound interest.

12. The content of a solid block of stone 12 ft. 6 in broad, and 3 ft. 9 in. thick, is 27 cu. yds. 1 cu. ft. 810 cu. in. Required its length, and its price at 6*d.* per cubic foot.

XXI.

1. (1) Add together 369 tons 13 cwts. 1 qr. 14 lbs. ; 127 tons 1 cwt. 3 qrs. 17 lbs.; 258 tons 19 cwts. 2 qrs. 23 lbs.; and 1 ton 5 cwts. 3 qrs. 2 lbs. (2) Multiply £628 17*s.* 4½*d.* by 1407. (3) Divide 375 miles 2 fur. 7 po. 2 yds. 1 ft. by 39.

2. (1) How many seconds are there in a leap year ? (2) Divide £6 16*s.* 9*d.* between two persons so that one may have double the sum which the other receives. (3) How many times will a coach wheel of 18½ ft. in circumference turn between London and York, being 197 miles ?

3. (1) If £4 15*s.* 7½*d.* purchase 3 cwts. 17 lbs., what must be given for 16 cwts. 3 qrs. 12 lbs. ? (2) A owes B £56 5*s.*, but B consents to remit 15 per cent. for immediate payment ; what sum does A pay B ? (3) If 4 cwts. 1 qr. 7 lbs. cost £5 per cwt., what will be the cost per lb. when the cost of the whole is reduced by £5 9*s.* 3*d.* ?

4. Divide $1\frac{3}{8}$ of $2\frac{1}{3}$ by $1\frac{3}{4}$ of $5\frac{1}{2}$.

5. Reduce $\frac{667}{1102}$ to its lowest terms.

6. Reduce $\frac{3}{4}$ of $\frac{1}{2}$ guinea to the fraction of £1.

7. Multiply 5·6 by ·00457.

8. Divide 816 by ·0004.

9. Reduce 7*s.* 4½*d.* to the decimal of £1.

10. A man after doing $\frac{3}{8}$ of a piece of work in 30 days is assisted by another, with whom he completes it in 6 days; how long would each be about it separately?

11. What number is that from which if there be taken $\frac{2}{7}$ of $\frac{3}{5}$, and to the remainder $\frac{8}{15}$ of $\frac{5}{16}$ be added, the sum will be 10?

12. How many 3-inch cubes can be cut from a 12-inch cube? (Royal Naval College.)

NAVAL CADETSHIP PAPERS.

XXII.

1. The moon's distance from the earth is 240,000 miles, and the sun's is 95,000,000 miles; how much farther is the sun from the earth than the moon is?

2. (1) Reduce 540,697 inches to miles, furlongs, &c.

(2) Multiply 15 lbs. 9 oz. 17 dwts. 18 grs. by 23.

3. If 4 cwt. 1 qr. 26 lbs. of tobacco cost £43 18*s.* 6*d.* what quantity can be purchased for £4 19*s.* 9*d.*?

4. What quantity of silver can be bought for £36, when 14 lbs. 6 oz. 15 dwts. cost £45?

5. A person's income is £598 10*s.*, his weekly expenses are £7 10*s.*, and he gives away £20 annually in charity; how much will he have remaining?

6. When coals cost 11*s.* 6*d.* a ton, what will be the annual expense of fuel for an engine whose daily consumption averages 2½ tons?

7. How many days of 15 hrs. each would 60 men take to perform a piece of work, if 45 men can do it in 30 days of 12 hrs. each?

8. If 1 lb. troy of gold be coined into £46 14*s.* 6*d.* what is the weight of a sovereign ?

9. (1) Reduce $\frac{6}{7}\frac{4}{3}\frac{0}{9}\frac{2}{5}$ to its lowest terms.

(2) From $10\frac{2}{21}$ take $7\frac{5}{14}$.

(3) Multiply together $\frac{5}{6}$, $\frac{9}{10}$ and $2\frac{2}{3}$.

10. (1) Multiply 7·51 by 0016.

(2) Divide 201·4095 by ·703.

(3) Reduce $100\frac{4}{125}$ to a decimal.

11. How many square ft. of glass are required to glaze 5 windows, each containing 14 panes of glass, each measuring 18 in. by 6 in. ?

12. A man mixes 20 gal. of ale which cost him 16*d.* a gallon, with 65 gal. of beer which cost him 9*d.* a gallon ; at what price must he sell the mixture to gain 20 per cent. ?

XXIII.

1. Express in figures five hundred and nine millions six thousand and one.

2. Divide 7,890,567 by 42, first, by long division, and secondly, by its factors 6 and 7.

3. If a land mile consists of 1,760 yds., and a knot, or sea mile, of 6,080 feet, how many knots correspond to 1,400 land miles ?

4. (1) Divide 121,260 c. yds. 22 c. ft. 392 c. in. by 584.

(2) Find the value of 17 pieces of cloth each 41 yds. 1 qr. long, at £1 8*s.* 0½*d.* for 2 yds. 3 qrs.

5. A coach-wheel 7 ft. 8 in. in circumference revolves 3,762 times in performing a certain journey ; what would be the circumference of a wheel which made 3,496 revolutions in the same distance ?

6. Standard gold being worth £3 17*s.* 10½*d.* per oz. troy, what will be the price of 2 lbs. 9 oz. 6 dwts. 16 grs.?

7. If 11 men can reap a field of corn of 7½ ac. in 3 days of 12 hrs. each, how long will it take 8 men to reap 9 ac. working 16 hrs. a day ?

8. A can perform a piece of work in 4 days, which B can perform in 5 days; in what time can they perform it together ?

9. (1) From $\frac{11}{14} \times \frac{9}{113}$ take $\frac{1}{56}$.

(2) Divide $5\frac{1}{3}$ of $4\frac{1}{8}$ by $7\frac{1}{3}$.

(3) Reduce 14*s.* $10\frac{1}{2}$*d.* to the fraction of a pound.

10. (1) Multiply ·1 by ·1, and divide the product by ·00625.

(2) Reduce $\frac{3}{8}\frac{2}{5}\frac{1}{5}$ to a decimal.

(3) Find the sum of ·125 of 30*s.* and ·375 of 13*s.* 4*d.*

11. How much water must be added to a cask containing 80 gal. of spirits at 14*s.* a gal. to reduce the price to 10*s.* 6*d.* a gallon ?

12. If by selling a chronometer for £40, I lose 20 per cent., what ought I to sell it for to gain 10 per cent. ?

13. Add together $\frac{15\frac{2}{3}}{7\frac{5}{6}}$ of £2, $\frac{4}{15}$ of £140 10*s.* 6*d.* and $\frac{3}{5}$ of 4*s.* 2*d.*

XXIV.

1. What number divided by 48,762 will give 6,285 for a quotient and 26,108 for a remainder ?

2. How many dollars are there in £289 17*s.* at 4*s.* 3*d.* per dollar ?

3. A stationer buys 36 reams of paper at 16*s.* 6*d.* per ream, and 84 reams at 17*s.* 9*d.* per ream; find his entire outlay.

4. If 2 cwt. 1 lb. cost £116 19*s.* $0\frac{3}{4}$*d.* what is the cost of 1 lb.?

5. An equal number of men, women, and boys, earned £8 5*s.* in 6 days; each man earned 1*s.* 4*d.*, each woman 10*d.*, and each boy 4*d.*, a day; how many were there of each ?

6. If the duty on hops at $1\frac{1}{2}$*d.* per lb. amounts to £26,357 9*s.* $10\frac{1}{2}$*d.*, on what quantity is it paid ?

7. (1) What will 1 ton 11 cwt. 2 qrs. 14 lbs. cost at £1 13*s.* 7*d.* per cwt. ?

(2) Extract the square root of 22,071,204.

8. In what time will 25 men do a piece of work, which 12 men can do in 15 days ?

9. (1) From $\frac{2}{5}+\frac{5}{6}$ take $\frac{3}{8}+\frac{7}{12}$.

(2) Find the value of $\frac{3}{7}$ of $1\frac{2}{5}$ of $12\frac{1}{2}$ of $\frac{3}{20}$.

(3) Reduce $\frac{4}{7}$ lb. to the fraction of a cwt.

10. (1) Divide ·0068 by 340, and 314 by ·0005.

(2) Extract the square root of 3·026.

(3) Reduce 16 cwt. 1 qr. 21 lbs. to the decimal of a ton.

11. I bought 40 oxen for £500, and sold 12 at a loss of 3 per cent.; how must I sell the rest per head so as to be free from loss on the whole?

12. Five hundred boys are distributed in 3 houses; the smallest house contains $\frac{7}{25}$ of the whole number, and the largest contains $\frac{9}{14}$ of the smallest: what is the number in each ?

13. Standard gold is formed by mixing 11 parts of fine gold with one part of copper; how many sovereigns will be coined from 4 oz. avoirdupois of fine gold, when 1 lb. troy of standard gold is coined into $46\frac{23}{40}$ sovereigns? (1 *lb. av.*=7,000 *grs. troy.*)

XXV.

1. What number divided by 9,612 gives for a quotient 58,647 ?

2. A gold watch and chain cost £21, the watch being £11 10*s.* dearer than the chain ; find the price of each ?

3. (1) Required the cost of 25 cwt. of tobacco at £7 15*s.* 8*d.* per cwt.

(2) What is the weight, in lbs., of ten anchors, which weigh altogether 46 tons 1 qr. 17 lbs. ?

4. If A has £3 3*s.* $3\frac{3}{4}$*d.*, and B has £2 0*s.* $10\frac{1}{4}$*d.*; how many farthings have they between them, and how much money must A give to B that the latter may have 2,000 farthings ?

5. A farmer buys 14 sheep at £1 11*s.* 4*d.* each: three of them having died, at what price must he sell the rest to gain 38*s.* by the bargain?

6. What quantity of silver may be bought for £72, if the cost of 14 lbs. 6 oz. 15 dwts. be £45?

7. Find the cost of 2 qrs. 24 lbs. at the rate of 20 guas. a ton.

8. If the carriage of 6 cwt. 2 qrs. for 124 miles cost £3 4*s.* 8*d.*, what weight will be carried 93 miles for £3 0*s.* 7½*d.*?

9. (1) Add together $\frac{3}{7}$, $\frac{5}{21}$ and $\frac{17}{24}$.
(2) Divide $5\frac{5}{14}$ by $12\frac{1}{2}$.
(3) What is the value of $2\frac{2}{7}$ of £6 3*s.* 4*d.*?

10. (1) Add together 14·1254, 11·5, 16·0004 and 17·112.
(2) Multiply ·0625 by 1·74.
(3) Reduce $1\frac{1}{2\,8}$ to a decimal.

11. Having sold 12 yards of cloth for £5 14*s.*, I thereby gained 14 per cent.; what was the prime cost of a yard?

12. What number is that to which if $\frac{2}{7}$ of $\frac{5}{9}$ be added, the sum will be 1?

13. How many 3-inch squares can be cut out of a 12-inch square?

XXVI.

1. From one hundred tons subtract 10 cwt. 3 qrs. 16 lbs. 10 oz.

2. What is the cost of 8,630 yards of cloth at £1 9*s.* 3*d.* per yard?

3. Divide 355 tons 13 cwt. 2 qrs. 13 lbs. 12 oz. by 132.

4. A person bought 11 cwt. 1 qr. of sugar for £48 19*s.* 0½*d.*, and sold it at 10*d.* per lb.; did he gain or lose, and how much?

5. What is the pay of 548 labourers for 15 weeks at 2*s.* 9½*d.* per day?

6. A bankrupt pays 14*s.* 7½*d.* in the £ off a debt of £9,655; *how much do the* creditors lose?

7. If 4 oz. 15 dwts. of silver plate cost £1 11*s.* 6*d.*, what will five articles, each 3 oz. 12 dwts., cost?

8. If 5 yds. of linen cost 11*s.* 2*d.*, what must be given for 9 pieces, each containing 21 yds. 1 qr. ?

9. (1) Extract the square root of 172,265,625.
(2) From $1\frac{1}{15}$ take $\frac{1}{30}$.
(3) Reduce $\frac{41}{56}\frac{23}{88}$ to its lowest terms.
(4) Multiply $10\frac{1}{8}$ by $5\frac{1}{9}$.

10. (1) From 7·00001 take ·700001.
(2) Multiply ·0409 by 3·417.
(3) Divide 1138·47 by 125,000.

11. A gardener has a piece of matting 73 yds. 1 ft. 8 in. long, and 3 ft. 9 in. wide, to cover a wall 94 ft. long and 10 ft. high; how many square feet of wall will remain uncovered?

12. Express as a vulgar fraction $7\dot{2}85714\dot{2}$.

13. Find the least fraction which, when added to the sum of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{5}{12}$, will make the result a whole number.

XXVII.

1. Find the difference between 17 tons 14 cwt. 1 qr. 17 lbs. and 19 tons 12 cwt. 3 qrs., and multiply the result by 175.

2. Find the G.C.M. of 742, 795, and 901.

3. Convert £426 17*s.* 9*d.* into dollars, at 4*s.* $7\frac{1}{2}$*d.* per dollar.

4. Divide £814 among 3 persons, in the ratios $\frac{2}{5} : \frac{3}{7} : \frac{5}{8}$.

5. How many yards of cloth, at 9*s.* per yard, must be given in exchange for 3 cwt. 3 qrs. 12 lbs. of sugar at 7*d.* per pound?

6. If the expenses of a family of 10 persons for 3 weeks be £22 1*s.*, what will be the expenses of a family of 15 persons for 2 weeks 1 day?

7. Find the simple interest on £312 6*s.* 8*d.* for 1 year 120 days, at $4\frac{1}{2}$ per cent. per annum.

8. A person spending annually £240, saves £2 15*s.* of it

quarterly by ready payment: what is the rate of discount? If he by this means make an increase of $20\frac{5}{8}$ per cent. upon his annual saving, what is his annual income?

9. (1) Add together $\frac{2}{3}$, $\frac{1}{7}$ of $3\frac{1}{3}$, and $\frac{5\frac{3}{7}}{6\frac{2}{3}}$.

(2) Find the difference between $\frac{2}{3}$ of 12*s.* 6*d.* and $\frac{3}{5}$ of 15*s.* 4*d.*; and reduce the result to the decimal of 4*s.* 6*d.*

10. If $\frac{3}{5}$ of a ship cost £5,427, what will be the cost of $\frac{1}{8}$ of her?

11. Extract the square root of $3\frac{1}{16}$.

12. (1) Reduce ·0225 and $1{\cdot}03\dot{4}\dot{7}$ to vulgar fractions.

(2) Divide 17·28 by ·0144.

13. Find the sum of 2·024 of 3*s.* 8*d.*, ·128 of 12*s.* 9*d.*, and $1{\cdot}02\dot{3}$ of £1·28. (*Assistant Clerks'.*)

SANDHURST AND LINE PAPERS.

XXVIII.

1. Add together $6\frac{3}{5}$, $2\frac{1}{4}$, $\frac{3}{10}$, and $2\frac{1}{2}$.
2. Subtract $22\frac{5}{13}$ from $30\frac{7}{30}$.
3. Multiply $21\frac{7}{15}$ by $4\frac{2}{7}$.
4. Divide $5\frac{1}{4}$ by $19\frac{3}{8}$.
5. Add together 42·307, ·05869, 57·933, and 439·2865.
6. Subtract 47·3825 from 150·293104.
7. Multiply 56·413 by ·0285.
8. Divide 11·83 by 26.
9. Reduce ·275 of £1 13*s.* 4*d.* to pence.
10. Reduce 1 ton 3 cwt. 2 qrs. 9 lbs. to ounces.
11. If 3 acres of land cost £465 18*s.*; how much may be bought for £6716 14*s.* 6*d.*
12. Find the simple interest on £635 for 9 years at $3\frac{1}{4}$ per cent. per annum.
13. Add together $2\frac{1}{16}$, $5\frac{1}{9}$, $\frac{2}{3}$, and $6\frac{3}{8}$.

14. Subtract $19\frac{2}{39}$ from $45\frac{1}{60}$.
15. Multiply together $6\frac{5}{18}$, $1\frac{2}{25}$, $16\frac{2}{3}$, and $\frac{3}{4}$.
16. Divide $3\frac{9}{20}$ by $2\frac{3}{10}$.
17. Add together ·0625 of a shilling and ·0375 of a pound.
18. Subtract 2·25 pints from ·315 of a gallon.
19. Multiply ·087654 by ·385.
20. Divide 688·2 by ·555.
21. Express 4 hours as the decimal of a week.
22. In 234247 grains troy, how many lbs., ozs., dwts., &c.?
23. If 14 grains of gold cost 2*s.* $2\frac{1}{4}$*d.*; what is the price per ounce?
24. If I pay £19 13*s.* 9*d.* per annum for income tax, being at the rate of 5 pence in the £; what is my income?
25. In what time will £445 amount to £486 14*s.* $4\frac{1}{2}$*d.* at $2\frac{1}{2}$ per cent. per annum (simple interest)?
26. Find by practice the value of 5 cwt. 3 qrs. 21 lbs. at £15 per ton.
27. The value of unwrought silver being 4*s.* 11*d.* per ounce and the workmanship equal to $\frac{3}{5}$ths of the value of the metal; what will be the whole cost of a salver weighing 45 ounces?

XXIX.

1. Add together $\frac{4}{7}$, $1\frac{5}{6}$, $\frac{8}{21}$, and $\frac{11}{42}$.
2. Subtract $6\frac{4}{5}$ from $8\frac{3}{7}$.
3. Multiply $4\frac{1}{5}$ by $\frac{10}{63}$.
4. Divide $1\frac{7}{8}$ by $3\frac{5}{12}$.
5. Add together 70·152, ·00628, 1·905, and 3281·6013.
6. Subtract 47·6125 from 350·013.
7. Multiply 501·32 by ·0063.
8. Divide 17·465 by 98·6 to three places of decimals.
9. Find the value of 6·125 of £1 13*s.* 4*d.*
10. In 3706529 inches how many miles, furlongs, poles, &c. are there?
11. If 69 lbs. of salt cost 7*s.* $2\frac{1}{4}$*d.*, what will be the value of 4 cwt. 1 qr. 7 lbs.?

12. Find the simple interest on £5,781 for 10 years at $4\frac{1}{2}$ per cent. per annum.

13. Add together $5\frac{2}{9}$, $3\frac{1}{12}$, and $1\frac{4}{27}$.

14. Subtract $7\frac{8}{11}$ from $10\frac{12}{143}$.

15. Multiply together $10\frac{1}{8}$, $\frac{6}{13}$, $\frac{2}{27}$, and $1\frac{1}{3}$.

16. Divide $6\frac{4}{9}$ by $2\frac{4}{63}$.

17. Add together 1·375 gallons and 3·5 quarts.

18. Subtract 33·75 of a minute from 1·95 of an hour.

19. Multiply ·401296 by 8·05.

20. Divide 2054·95 by ·0563.

21. Express 7*s.* $9\frac{3}{4}$*d.* as the decimal of £8.

22. In 57625318 square inches how many acres, roods, &c. are there ?

23. A person after paying an income tax on £515 has £484 19*s.* 2*d.* left ; how much did he pay in the £?

24. A bankrupt's estate amounts to £910 3*s.* $1\frac{1}{2}$*d.*, and his debts to £1875 ; what can he pay in the pound ?

25. At what rate per cent. will £425 amount to £573 15*s.* in 5 years at simple interest?

26. Find by practice the value of 5 cwt. 3 qrs. 21 lbs. at £1 5*s.* 8*d.* per cwt.

27. A person completes a journey of 1056 miles in 12 days, travelling 11 hours a day ; in how many days would he complete 480 miles, going 6 hours a day at the same rate ?

XXX.

1. Add together $4\frac{3}{16}$, $\frac{2}{9}$, $2\frac{5}{12}$, and $\frac{1}{2}$.

2. Subtract $11\frac{6}{11}$ from $15\frac{1}{10}$.

3. Multiply $13\frac{5}{9}$ by $3\frac{3}{61}$.

4. Divide $12\frac{5}{17}$ by $3\frac{1}{34}$.

5. Add together 3281·6013, 42·307, ·05869, and 8·42.

6. Subtract 19·28945 from 200·1076.

7. Multiply 169·35 by ·00265.

8. Divide 16·1 by 64·4.

9. Reduce ·0545 of £2 7*s.* 6*d.* to pence and the decimal of a penny.

10. Reduce 1 lb. 2 ozs. 3 dwts. to grains troy.

11. If £39 19*s.* will buy 47 tons; how much may be bought for 11*s.* 8¼*d.*?

12. Find the simple interest on £950 for 8 years at 5½ per cent. per annum.

13. Add together $3\frac{5}{12}$, $\frac{3}{25}$, $2\frac{1}{6}$, and $4\frac{7}{10}$.

14. Subtract $24\frac{6}{55}$ from $30\frac{1}{70}$.

15. Multiply together $6\frac{1}{3}$, $\frac{5}{42}$, $\frac{9}{38}$, and $1\frac{2}{5}$.

16. Divide $10\frac{7}{38}$ by $9\frac{1}{4}$.

17. Add together 1·025 of a minute and ·0625 of an hour, and give the answer in seconds.

18. Subtract 2·25 ounces from ·015 of a cwt.

19. Multiply ·42881 by 5·63.

20. Divide 242 by ·55.

21. Express 1*s.* 9*d.* as the decimal of 3 guineas.

22. In 3424768 inches how many miles, furlongs, poles, &c.?

23. If 1 cwt. 1 qr. 21 lbs. of copper cost £4 6*s.* 3*d.*, what is the price per ton?

24. If a clock is two minutes and a half fast at noon on the 1st of January, and gains three and a half minutes a week, what time will it indicate at noon on January 31st?

25. At what rate per annum (simple interest) will £850 amount to £1136 17*s.* 6*d.* in 7½ years?

26. Find by practice the value of 2 bushels 1 peck 1 gallon 2 quarts at £3 16*s.* per bushel.

27. If railway fares in Prussia are only two-thirds of railway fares in England, and it costs £1 2*s.* 6*d.* to travel 180 miles in England; what will it cost to travel 160 miles in Prussia?

XXXI.

1. Add together $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$, and $\frac{7}{8}$.

2. Subtract $2\frac{1}{4}$ from $20\frac{1}{8}$.

3. Multiply $\frac{4}{11}$ by $1\frac{4}{7}$.

4. Divide $2\frac{3}{4}$ by $1\frac{1}{16}$.

5. Add together 16·41215, 9·376, ·00403, and 270·3.

6. Subtract 17·2398 from 27·06.

7. Multiply 46·2375 by ·0074.

8. Divide 92·3784 by ·623 to three places of decimals.

9. Find the value of 1·25 of £1 13*s.* 4*d.*

10. Reduce 4 tons 3 cwt. 1 qr. 9 lbs. to ounces.

11. Find the income tax on £356 10*s.* at 5*d.* in the £.

12. Find the simple interest on £576 for 6 years at $4\frac{1}{2}$ per cent. per annum.

13. Add together $1\frac{7}{12}$, $3\frac{7}{24}$, and $4\frac{1}{3}$.

14. Subtract $5\frac{3}{20}$ from $10\frac{1}{12}$.

15. Multiply together $\frac{9}{14}$, $\frac{5}{27}$, $9\frac{4}{5}$, and $2\frac{3}{7}$.

16. Divide $2\frac{13}{14}$ by $4\frac{1}{7}$.

17. Add together 2.6 of a day and ·85 of an hour, and give the answer in minutes.

18. Subtract 4·42 of a cwt. from 3·64 of a ton.

19. Multiply 62·5 by ·579.

20. Divide ·473928 by 24·18.

21. Express ·0425 of a mile in yards.

22. In 156704 square inches how many square yards and square feet are there?

23. What is the income of a man whose income tax at 4*d.* in the pound amounts to 23 guineas?

24. At what rate per cent. will £230 15*s.* amount to £305 14*s.* $10\frac{1}{2}$*d.* in 13 years at simple interest?

25. Find the dividend on £274 10*s.* at 8*s.* 3*d.* in the £.

26. If 2 horses can plough 7 acres of ground in a day, how many horses will be required to plough 161 acres in $11\frac{1}{2}$ days?

XXXII.

1. Add together $1\frac{1}{13}$, $2\frac{2}{5}$, $1\frac{1}{26}$, $2\frac{1}{6}$.

2. Subtract $3\frac{9}{14}$ from $4\frac{1}{21}$.

3. Multiply $14\frac{11}{15}$ by $2\frac{6}{17}$.

4. Divide $3\frac{9}{28}$ by $5\frac{19}{21}$.

5. Add together 1338·465, 1·096, 2·001, and 231·0103.

6. Subtract 13·0003 from 16·7.

7. Multiply 312·14 by 1·012.

8. Divide 9·462 by ·83.

9. Express ·315 of 13*s.* 4*d.* in pence.

10. Reduce 2 qrs. 5 bus. 3 pks. 1 gal. to quarts.

11. If 67 yards of cloth cost £36 17*s.*, how many can be bought for £111 7*s.* 6*d.*

12. Find the simple interest on £1248 for 6 years, at $3\frac{1}{4}$ per cent. per annum.

13. Add together $2\frac{1}{15}$, $3\frac{1}{5}$, $\frac{1}{20}$, and $1\frac{1}{24}$.

14. Subtract $2\frac{7}{12}$ from $5\frac{3}{16}$.

15. Multiply together $3\frac{27}{28}$, $18\frac{2}{3}$, $\frac{3}{19}$, and $7\frac{3}{5}$

16. Divide $7\frac{21}{68}$ by $4\frac{3}{17}$.

17. Add together 2·6 of a ton and 1·3105 of a cwt., subtract 14·35 of a lb. from the result, and give the answer in pounds, ounces, and the decimal of an ounce.

18. Subtract 11·351 of a shilling from 1·231 of a guinea.

19. Multiply 11·43231 by ·028.

20. Multiply together 2·1, ·03, and 10·8.

21. Divide 22287·662 by 183·74.

22. Divide 2902·11892 by 34·831.

23. Reduce 5 lbs. 10 oz. 11 dwts. to the decimal of 14 lbs. 2 oz. troy.

24. If 2 cwt. 2 qrs. 14 lbs. of copper cost £9 14*s.* 3*d.*, what is the price per ton?

25. A sum of £3024 lent at simple interest produces £378 in 5 years. What is the rate per cent.?

26. If pasturage for 28 oxen for 11 days can be obtained for £7 10*s.*, what should be paid on account of a herd of 33 feeding for 63 days?

XXXIII.

1. Add together $2\frac{2}{9}$, $\frac{8}{11}$, $5\frac{1}{3}$, and $\frac{7}{22}$.

2. Subtract $6\frac{5}{8}$ from $11\frac{1}{40}$.

3. Multiply $8\frac{7}{13}$ by $1\frac{2}{37}$.

4. Divide $14\frac{3}{7}$ by $9\frac{1}{21}$.

5. Add together ·05869, 57·933, 439·2865, and 37·6849.

6. Subtract 56·293104 from 400·0738.

7. Multiply ·0285 by 6·259.

8. Divide 17·6 by 275.

9. Reduce ·35 of 2 days 13 hours to minutes.

10. Reduce 2 lbs. 3 ozs. 4 dwts. to grains troy.

11. A bankrupt's debts amount to £7563 10*s.*, and his assets to £5042 6*s.* 8*d.*; what dividend can he pay?

12. Find the simple interest on £1305 15*s.* for 7 years at $4\frac{1}{3}$ per cent. per annum.

13. Add together $12\frac{3}{5}$, $1\frac{1}{4}$, $\frac{7}{10}$, and $2\frac{1}{25}$.

14. Subtract $12\frac{11}{54}$ from $19\frac{5}{432}$.

15. Multiply together $3\frac{4}{9}$, $6\frac{5}{14}$, $\frac{3}{124}$, and $2\frac{1}{3}$.

16. Divide $19\frac{13}{18}$ by $7\frac{3}{8}$.

17. Add together ·0375 of a guinea and ·435 of a crown.

18. Subtract 2·125 perches from ·425 of an acre.

19. Multiply ·004865 by 327.

20. Multiply together 7·3, ·65, and ·04.

21. Divide 238·7 by ·55.

22. Divide ·3008096 by 41·32.

23. Express 6 lbs. as the decimal of a cwt.

24. In 3234247 ounces, how many lbs., cwt., qrs., &c.?

25. If 37 tons 9 cwt. 3 qrs. cost £337 7*s.* 9*d.*, what is the price per ton?

26. Four reapers begin to cut a field of barley, and it is found after working for 5 days they have cut 10 acres. Two additional men are then employed who work at the same rate, and the whole is cut in 3 days more. How many acres does the field contain?

XXXIV.

1. Add together $\frac{1}{6}$ of $\frac{3}{5}$, $\frac{1}{3}$ of $\frac{2}{7}$, and $1\frac{1}{2}$ of $\frac{5}{6}$.

2. Subtract $7\frac{5}{6}$ from $20\frac{3}{10}$.

3. Multiply together $4\frac{2}{3}$, $1\frac{5}{13}$, $\frac{2}{45}$, and $1\frac{7}{72}$.

4. Divide $2\frac{7}{16}$ by $3\frac{1}{8}$.
5. Add together 12·07043, ·7131, 754·5, 6·3, and ·07385.
6. Subtract 82·6874 from 701·212.
7. Multiply ·0263 by 2·36.
8. Divide ·4336218 by 7362.
9. Reduce 16·08 pennyweights to the decimal of a pound troy.
10. Add together $\frac{14}{15}$, $1\frac{4}{25}$, and $5\frac{3}{20}$.
11. Subtract $3\frac{9}{11}$ from $8\frac{2}{3}$.
12. Multiply together $2\frac{3}{7}$ of $1\frac{4}{11}$, $2\frac{5}{8}$ of $\frac{4}{5}$, and $3\frac{1}{2}$ of $\frac{1}{10}$.
13. Divide $4\frac{5}{9}$ by $12\frac{1}{3}$.
14. Add together 5·2 of a day and ·0265 of an hour, and give the answer in minutes and the decimal fraction of a minute.
15. Subtract 6·42 of a furlong from 3·64 of a mile, and give the answer in yards and the decimal fraction of a yard.
16. Multiply ·382 by ·148.
17. Divide 81·27 by ·481 to 3 places of decimals.
18. Reduce ·0325 of £5 to the decimal of £3 6*s*. 8*d*.
19. Reduce 1 mile 3 furlongs 10 poles 2 feet to feet.
20. If 1 ton 5 cwt. of iron cost £1 11*s*. 3*d*., what will 1 cwt. 2 qrs. cost?
21. Find (by Practice) the dividend on £8976 5*s*. at 13*s*. 3*d*. in the pound.
22. Find the simple interest on £1760 for 9 years at $3\frac{3}{4}$ per cent. per annum.

XXXV.

1. Add together $4\frac{2}{3}$, $\frac{2}{3}$, $1\frac{7}{15}$, and $3\frac{5}{6}$.
2. Subtract $\frac{4}{7}$ from $6\frac{5}{21}$.
3. Multiply $13\frac{1}{3}$ by $1\frac{4}{5}$.
4. Divide $7\frac{3}{10}$ by $1\frac{4}{5}$.
5. Add together ·006, 1·27, 39·567, and 140·806.
6. Subtract 159·643 from 240·321.
7. Multiply 537·6 by 40·56.
8. Divide 16·0134 by ·39.

9. Express 6·545 of 12*s.* 6*d.* in pence and the decimal of a penny.

10. Reduce 2 qrs. 3 bus. 2 pks. to pints.

11. If 37 articles cost £8 11*s.* $1\frac{1}{2}d.$; how many can be bought for £35 3*s.*?

12. Find the simple interest on £480 for 5 years at $3\frac{1}{2}$ per cent. per annum.

13. Add together $3\frac{2}{7}$, $4\frac{1}{3}$, $\frac{1}{8}$, and $\frac{14}{21}$.

14. Subtract $2\frac{17}{39}$ from $5\frac{2}{13}$.

15. Multiply together $5\frac{4}{7}$ of $2\frac{2}{13}$, $\frac{5}{8}$, and $1\frac{2}{15}$.

16. Divide $4\frac{9}{10}$ by $12\frac{5}{6}$.

17. Add together 12·36 of a ton and ·03475 of a cwt., and give the answer in pounds, ounces and decimal of an ounce.

18. Subtract 18·52 of a shilling from 21·43 of a crown.

19. Multiply 4·0576 by ·0068.

20. Multiply together ·01, ·17, and 1·06.

21. Divide ·01461201 by 357.

22. Divide ·75973 by ·00205.

23. Reduce 13 yds. 27 ins. to the decimal of 5 miles.

24. If 6 yards of silk cost £1 7*s.* 9*d.*, what will 177 yards cost?

25. At what rate per cent. will £2630 amount to £3090 5*s.* in $3\frac{1}{2}$ years?

26. If the price of 3 per cent. Consols is 96, what interest does it afford to an investor?

XXXVI.

1. Add together $\frac{2}{3}$, $\frac{4}{5}$, $\frac{5}{6}$, and $\frac{8}{9}$.

2. Subtract $3\frac{1}{6}$ from $10\frac{1}{9}$.

3. Multiply $\frac{3}{13}$ by $1\frac{5}{8}$.

4. Divide $3\frac{2}{3}$ by $1\frac{1}{15}$.

5. Add together 8·376, ·06703, 37·04, and 28·064015.

6. Subtract 28·4532 from 320·6.

7. Multiply 4·2635 by ·068.

8. Divide 8·4605 by ·248 to three places of decimals.

9. Find the value of 3·75 of £2 14*s.* 8*d.*

10. In 1250784 seconds how many weeks, days, &c., are there ?

11. If 3 tons of iron cost £16 10*s.*, what will be the value of 3 cwt. 2 qrs. 14 lbs. ?

12. Find the simple interest on £732 for 8 years at $3\frac{1}{4}$ per cent. per annum.

13. Add together $1\frac{7}{10}$, $3\frac{7}{20}$, and $2\frac{1}{25}$.

14. Subtract $5\frac{3}{16}$ from $10\frac{1}{8}$.

15. Multiply together $1\frac{2}{9}$, $\frac{5}{12}$, $8\frac{1}{10}$, and $2\frac{1}{22}$.

16. Divide $2\frac{13}{16}$ by $4\frac{1}{8}$.

17. Add together 2·16 of a yard and ·05 of a mile.

18. Subtract 4·42 of an hour from 3·64 of a day.

19. Multiply ·506 by ·632.

20. Divide 13·915 by 575.

21. Express 13*s.* 4*d.* as the decimal of £4.

22. In 8156704 cubic inches how many cubic yards and cubic feet are there ?

23. When the income tax was reduced from 6*d.* to 4*d.* in the pound, a person found that he had £42 3*s.* 9*d.* less income tax to pay. What was his income ?

24. In what time will £230 15*s.* amount to £305 14*s.* $10\frac{1}{2}d.$ at $2\frac{1}{2}$ per cent. simple interest ?

25. Find the dividend on £2560 5*s.* at 12*s.* 8*d.* in the £.

26. If 7 men received £17 10*s.* wages for 15 days, what will be the wages of 30 men for 6 days ?

XXXVII.

1. State and prove the rule for pointing in the divisior of decimal fractions.

Divide 4·8 by 12 and by ·12.

2. Find the value of

$$(x+y)\sqrt{x^2+y^2}-(x-y)\sqrt{\tfrac{1}{4}(x^2-y^2)}$$

when $x=4$ and $y=3$.

3. Divide $x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc$ by $x - c$, and divide the quotient by $x - b$. To the product of what three factors is the first dividend equal?

4. Simplify—

$$(x-y)^3+(x+y)^3+3(x+y)^2(x-y)+3(x+y)^2(x-y).$$

5. Show that if $2s=a+b+c$

$$(s-a)^2+(s-b)^2+(s-c)^2+s^2=a^2+b^2+c^2.$$

6. Find the greatest common measure of—

$x^4-ax^3-a^2x^2-a^3x-2a^4$ and $3x^3-7ax^2+3a^2x-2a^3$.

7. State and prove the rule for the extraction of the square root, and find the square root of—

$$x^6+\frac{1}{x^6}+6x^4+\frac{6}{x^4}+15x^2+\frac{15}{x^2}+20.$$

8. Solve the equations—

$$\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$$

$$x^3=8+4(x+3)(x-2)$$

$$\begin{cases} x+y+z=a+b+c \\ bx+cy+az=cx+ay+bz=ab+bc+ca. \end{cases}$$

9. A, B and C travel from the same place at the rates of 4, 5, and 6 miles an hour respectively; and B starts two hours after A. How long after B must C start in order that they may overtake A at the same instant?

10. The following is an extract from a logarithm book :—

N.	0.	1.	2.	3.	4.	5.	6.	7.	8.	9.	Dif.
1905	2798950	9178	9406	9634	9862	$\bar{0}$090	$\bar{0}$317	$\bar{0}$545	$\bar{0}$773	$\bar{1}$001	228

What is the logarithm of $190\frac{1}{2}$ and of 190595·7? Divide the product of these two quantities by 19057·5, using logarithms to obtain the result to two places of decimals.

11. What are the characteristics of the logarithms of 10 and $\frac{1}{10}$ to the bases 3, $\frac{3}{2}$, $\frac{2}{3}$, and $\frac{1}{2}$ respectively?

12. What is the cube root of $\frac{1}{19{\cdot}053}$ to six places of decimals, having given log. $3{\cdot}74408 = {\cdot}5733455$, and the logarithms in Question 10?

13. An officer proceeding to India in a troop ship is allowed 60 cubic feet of baggage. A lieutenant had two cases and a basket, one case being 4 feet $2\frac{1}{2}$ inches long, 2 feet $3\frac{3}{4}$ inches wide, and 2 feet $4\frac{1}{4}$ inches deep, and the other being half those dimensions in length, width, and depth, and the cylindrical basket being $3\frac{1}{2}$ feet high by 2 feet in diameter. By how much did he exceed or fall short of his allowance, actual contents being taken for the cylinder, and fractions of a cubic foot being omitted in the result?

14. The chord of an arc is 8 feet and the height of the arc is 2 feet, what is the radius of the circle?

15. The great pyramid of Egypt was 481 feet high, when complete, and its base was 764 feet in length; find the volume to the nearest number of cubic yards.

XXXVIII.

1. Write down the quotient of $\frac{x^4-y^4}{x-y}$, and show that the quotient of $\frac{x^m-y^m}{x-y}$ is always finite, if m is any positive integer.

2. Prove the rules for the multiplication and division of decimals. When a fraction is converted into a decimal, in what cases will a terminating decimal be obtained? If a recurring decimal is obtained, what limit is there to the number of figures in the recurring period?

3. Divide $a+4b$ by $a^3-2a^2b^2+2b^3$.

4. Reduce to its simplest form—

$$\frac{x^2-xy}{x^2+y^2}\times\left(\frac{x}{x+y}-\frac{y}{x+y}\right)\times\frac{x^2-y^2}{x^2+xy}\times\left(1-\frac{x^2-y^2}{x^2+y^2}\right).$$

5. Find the greatest common measure of

$$15x^4-19x^3y-22x^2y^2+51xy^3-27y^4$$

and $21x^3+4x^2y-44xy^2+39y^3$.

6. Show that the cube of the expression

$$\left(-\frac{r}{2}+\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}}+\left(-\frac{r}{2}-\sqrt{\frac{r^2}{4}+\frac{q^3}{27}}\right)^{\frac{1}{3}},$$

(which expression may be represented by x) is $-r-qx$.

7. Solve the equations—

$$(1.)\ \frac{x+3\frac{1}{2}}{9}-\frac{5x-24}{27}=\frac{13x-12}{15}-6\frac{7}{18}.$$

$$(2.)\ 5x^2-2y^2=-x$$
$$x^2+y=y^2+x.$$

8. A certain number consisting of two digits is equal to six times the sum of the digits; and if 117 be subtracted from three times the number, the digits are reversed. Find the number.

9. If α, β be the roots of $x^2+px+q=0$, show that $\alpha+\beta=-p$, and $\alpha\beta=q$.

If they be in the ratio of 1 : 2, show that one of them satisfies the equation $6px^2+(5p^2-6q)\,x+p\,(p^2-2q)=0$, and find the other root of this equation.

10. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, prove that each of these fractions

$$=\frac{x+y+z}{a+b+c}=\sqrt{\frac{x^2+y^2+z^2}{a^2+b^2+c^2}}.$$

11. In what kind of questions would you chiefly seek the assistance of logarithmic arithmetic?

Explain clearly the method of finding the number corresponding to a given logarithm which does not appear *exactly* in the tables.

12. Define a logarithm.

If $2^{2x}.\ 7^{3x}=1882384$, what is the value of x?

Given that $\log_{10} 2=\cdot3010300$ and $\log_{10} 3=\cdot4771213$,

find $\log_{10}3{\cdot}75$, $\log_{10}{\cdot}5625$, and $\log_{10}5\sqrt[3]{\frac{3}{4}}$; find also $\log 25$ to the base $\sqrt[3]{5}$.

13. Find the square root of $62\frac{1}{8}\frac{3}{6}\frac{3}{7}$; also find the number of yards in the side of a square park containiug 439 acres, 37 perches, and $5\frac{3}{4}$ square yards.

14. The contents of two cubes are respectively 5359·375 and 5·359375 cubic feet; find the difference of the lengths of their edges in inches.

15. A semi-circular area, whose diameter is 15 yards, is covered with carpet two feet wide; what will it cost at 3*s*. 6*d*. per yard (allowing 11·6425 square yards for waste). Given $\pi=3{\cdot}1416$.

16. Show how to reduce an irregular polygon of five sides to a triangle of equal area.

17. The cost of planting a hexagonal field at £5 10*s*. an acre was £7 15*s*. $7\frac{1}{4}d$.; find what would be the cost of fencing round it with iron railing at 2*s*. $4\frac{1}{2}d$. per yard.

(It may be assumed that $\sqrt{3}=\frac{97}{56}$.)

XXXIX.

1. Prove that $a^m \times a^n = a^{m+n}$ when m and n are positive integers: and hence deduce the meaning of a^{-m} and $a^{\frac{m}{n}}$.

2. Reduce to their simplest forms—

$$(1.)\ \frac{1+x^3}{1+2x+2x^2+x^3}$$

$$(2.)\ \frac{1}{4x+4z\ \Big/\ 1+2\,\frac{(x+y)}{6-x}}$$

$$(3.)\ \frac{2}{x^3+x^2+x+1}-\frac{1}{x^3-x^2+x-1}.$$

3. Find the greatest common measure of $5x^3-29x^2+19x+5$, and $3x^3-13x^2-15x+25$.

4. Extract the square root of—

$$(1.)\ x^4+x^3y+\frac{x^2y^2}{4}-\frac{3x^2}{2}-\frac{3xy}{4}+\frac{9}{16}. \quad (2.)\ 4-\sqrt{7}.$$

and the cube root of—

$$x^6+6x^5-40x^3+96x-64.$$

5. Find the fourth power of $1\pm\sqrt{-1}$, and the square root of $1+a^2+\{1+a^2+a^4\}^{\frac{1}{2}}$.

6. Prove that an equation of the second degree has two roots and no more; and show in what cases the roots are equal, rational, irrational or imaginary.

7. Solve the equations—

$$(1.)\ \frac{6}{\sqrt{2x+8}}=\frac{7}{\sqrt{3x+7}}.$$

$$(2.)\ \frac{48}{x+3}=\frac{165}{x+10}-5.$$

$$(3.)\ xy=6. \quad x^4+y^4=97.$$

$$(4.)\ x^y=y^x. \quad x^2=y^3.$$

What value of a gives the value 3 $(2\frac{1}{4})^{-\frac{1}{2}}$ to the quantity $2\sqrt[3]{\frac{3a}{2}}$.

8. Show that numbers are divisible by 11, when the sum of their odd digits equals the sum of their even; and by 9, when the sum of their digits is divisible by 9.

9. If 2 cubical blocks of stone contain together 8 cubic feet, and the side of the less is to that of the greater as 3 to 4; find the side of each.

10. A rectangular garden contains 1,200 square yards, and the length is to the breadth as 4 to 3; what will the fencing cost at 3*s*. 6*d*. per yard?

11. What is the weight of a hollow sphere of metal, whose inside diameter is 18 inches, and the thickness 2 inches?

Given $\pi=\frac{22}{7}$, and a cubic foot of the metal weighs 7776 ounces.

12. The area of a triangle is 6 acres 2 roods and 8 perches, and a perpendicular from one angle on the base measures 524 links; find the length of the base in chains.

Given 1 chain=100 links=22 yards.

13. Find the area in acres of a triangle, whose sides are 25, 20, and 15 chains.

14. Explain the following terms in logarithms: base, mantissa, characteristic, modulus, and prove that

$$\log_a b \times \log_b a = 1.$$

15. Show that $\log\left(\frac{m}{n}\right)^r = r\log m - r\log n$, and find the logarithms of ·0625 and 14·4;

having given log 2=·3010300, log 3=·4771213.

16. Given log 8·5615=·9325499
and log 8·5614=·9325448
find log 856·1427 and log ·008561427.

XL.

1. Show that the decimal fraction equivalent to a vulgar fraction is either terminated, or, if not terminated, recurring.

2. What is the value of the following expressions?

$$10-[4-3-2-1],$$
$$10-[4-(3-2-1)],$$
$$10-[4-(3-\overline{2-1})].$$

3. Resolve $4a^2b^2-(a^2+b^2-c^2)^2$ into four factors.

4. Prove the rule for finding the greatest common measure of two algebraical expressions.

5. Find the greatest common measure of

$$3x^5-10x^3+15x-8, \text{ and } x^4-2x^2+1.$$

6. If $c+\frac{1}{a}=1$, and $a+\frac{1}{b}=1$, prove that $b+\frac{1}{c}=1$.

7. Solve the equations—

(1.) $\frac{a}{x}+\frac{b}{a}-\frac{a}{b}=0.$

(2.) $\frac{x}{a+b}+\frac{y}{a-b}=2a$, and $\frac{x-y}{4ab}=1.$

(3.) $x^{-3}+\frac{1}{x}=2.$

8. If the minute hand of a clock be four feet long and the hour hand three feet long, find the times between four and five o'clock at which their ends are five feet apart.

9. Define the characteristic of a logarithm, and from your definition show what are the characteristics of log 10 and log ·1 to the base $\frac{3}{2}$.

10. Given the logarithms to the base 10 of 2 and 3, find the logarithms of 72, 135 and $\sqrt[3]{\frac{1}{6}}$.

log 2=·3010300.
log 3=·4771213.

11. The distance between two towns is 54 miles, and the distance between their places on a map is $6\frac{3}{4}$ inches. Find the scale on which the map is drawn.

12. A cube contains 11 cubic feet 675 cubic inches. Find the length (1) of its edge, (2) of its diagonal.

13. In order to find the distance of an enemy's position from his battery an artillery officer riding at the rate of ten miles an hour goes, in his reconnoitre, owing to the nature of the ground, two minutes to the east, three minutes to the north, one and a half to the north-west, and four to the north-east, from which last point the enemy's position is one mile to the north. For what range should the guns be sighted?

14. Show how to find the distance between two points, one of which is inaccessible on account of a river or some other obstacle.

Any of the following questions may be substituted for any of the preceding.

A. Find the number of terms of an arithmetical progression in which the first term, the common difference, and the sum of the series is given.

Ex. How many terms of the series $15+14+13+$ .....must be taken in order that the sum of the series may be 120 ?

Explain the double answer in this case : When one of the integral roots is negative interpret the meaning.

B. Find the number of combinations of n things taken r together (assuming the number of permutations).

The number of permutations of n things taken r together is to the number of permutations taken $r-1$ together as 10 : 1, and the corresponding combinations as 5 : 3. Find the values of n and r.

C. Given $\log 648 = 2{\cdot}81157501$
$\log 864 = 2{\cdot}93651374$
Find $\log 3$ and $\log 5$.

D. An English imperial gallon is a measure which will hold 10 lbs. avoirdupois of pure water. A cubic foot of pure water weighs 997·137 ounces avoirdupois. The French litre is a cubic decimetre, that is, a cube on a side=3·937 inches. Find to two places of decimals how many litres there are in a gallon.

XLI.

1. Multiply $x^3-3x^2+11x-15$ by x^2-5x+3, and find the value of the product when $x=8$.

Resolve into factors the expressions—

(α) $(5x+4a)^2-(4x+5a)^2$;
(β) $x^2-(a+b)x+ab$; and
(γ) $x^2-18x+77$.

2. Divide—

(α) $x^4-x^3+3x^2-71x+44$ by $x-4$;

(β) $x^3+y^3+z^3-3xyz$ by $x+y+z$; and

(γ) $x^4-7x^2y^2+y^4$ by $x^2+3xy+y^2$.

3. Find the highest common factor and the least common multiple of the expressions—

$x^3-7x^2+33x-63$ and $x^3-6x^2+30x-63$.

Find the value of r when the two expressions $x^2-9x+20$, and $x^2-(r+3)\,x+3r$ have a common factor.

4. If $\frac{a}{b}=\frac{c}{d}$, prove that $\frac{a+b}{a-b}=\frac{c+d}{c-d}$, and that

$$\frac{3a^4+4b^4}{3a^4-4b^4}=\frac{3c^4+4d^4}{3c^4-4d^4}.$$

5. Find the square of $\sqrt{3}-4\sqrt{5}+\sqrt{15}$, and the square root of $152-30\sqrt{15}$.

Calculate to four places of decimals the value of the expression—

$$\frac{24\sqrt{5}-5\sqrt{3}}{2\sqrt{5}-\sqrt{3}}.$$

6. Prove that the sum of the cubes of three consecutive numbers is divisible by the sum of the numbers, and that the difference between the fourth powers of the greatest and least of the three numbers is divisible by eight times the middle number.

7. Solve the following equations—

(α) $\frac{7x+4}{9}-\frac{4x+5}{7}+\frac{2}{5}(x+1)=\frac{3x+1}{5}$.

(β) $\frac{x^2+5}{x}=\frac{4x+5}{4}$.

(γ) $x^2-88x+1911=0$.

(δ) $x^2+4\sqrt{x^2+4}=41$.

(ϵ) $\left.\begin{matrix}x^2+xy+y^2=331\\ x^2-xy+y^2=111\end{matrix}\right\}$

·8. A number of two digits is such that, if the digits be inverted, the new number is less by 9 than the original number, and the sum of the squares of the digits is 113; find the number.

9. A body of 684 soldiers is to be drawn up in the form of a hollow square, three deep; find the number of men in the front of each side of the square.

10. Define the logarithm of a number to a given base, and find the logarithms of 10000 to the base 10 and to the base $\sqrt[3]{10}$, and also of 2401 to the base 7, and to the base $\frac{1}{\sqrt[3]{7}}$

Find from the tables $\log_{10}487{\cdot}9634$, and, by help of the tables, find an approximate value of $(3{\cdot}7452)^{\frac{1}{2}} \div (3{\cdot}7121)^{\frac{1}{3}}$.

11. The sides of a right-angled triangular field, adjacent to the right angle, are 357 feet and 476 feet; find the length of the third side of the field, and the number of square feet in its area.

12. The three different edges of a rectangular parallelopiped are respectively 3 feet, 2·52 feet, and 1·523 feet in length; find the number of cubic feet in the parallelopiped.

Find also the cubic space inside a box of the same external dimensions, constructed of a material one-tenth of a foot in thickness.

13. A river 20 feet deep and 100 yards wide is flowing at the rate of 3 miles an hour; find how many tons of water run into the sea per minute, it being given that a cubic foot of water weighs 1,000 ounces.

14. Assuming that the ratio of the perimeter of a circle to its diameter is that of 3·1416 to unity, find the volume and the surface of a cylinder, the radius of which is 4 feet and the height 100 feet.

Supposing that a wedge is cut out of this cylinder

by two planes through its axis inclined to each other at an angle, which is half a right angle, what will be the volume left ?

XLII.

1. Find the numerical value of $x^2-2ax+a^2$ when $x=1$ and $a=\frac{1}{2}$; find also the value of x, if $(10000)^{\frac{1}{x}}=10$.

2. Perform the operations indicated in the following examples :—

(α) $\{2(a+b)-3(c-d)\}-\{2(a-b)+3(c+d)\}$.

(β) $(a^5-a^4b+ab^4-b^5)\times(a+b)$.

(γ) $(16x^4-72x^2a^2+81a^4)\div(4x^2+12ax+9a^2)$.

(δ) $\dfrac{a^4-a^2b^2+b^4}{a^2-ab\sqrt{3}+b^2}$.

(ε) $\sqrt{1-4x^{\frac{1}{2}}+10x-12x^{\frac{3}{2}}+9x^2}$

3. Find the least common multiple of $(1-x)$, $(1-x^2)^2$, $(1+x)^3$.

Reduce to its lowest terms—

$$\frac{3x^3-27ax^2+78a^2x-72a^3}{2x^3+10ax^2-4a^2x-48a^3}.$$

4. If $3x=\sqrt[3]{9}$, what is x?

Prove $\dfrac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}\times\dfrac{3^{\frac{1}{2}}+2^{\frac{1}{2}}}{3^{\frac{1}{2}}-2^{\frac{1}{2}}}=5.$

5. Solve the following simple equations :—

(α) $\dfrac{x-2}{3}-\dfrac{x-4}{6}=7-\dfrac{x+3}{4}$.

(β)
$$\left.\begin{aligned} x+y+z&=36\\ x+3y-2z&=12\\ x+\frac{y}{3}+\frac{z}{2}&=20 \end{aligned}\right\}$$

6. A boat's crew row 9 miles with the tide in $\frac{3}{4}$ of an hour, and when the tide is flowing at half its former rate, the same crew row 9 miles against the tide in an hour and a half. Required the rate of the strongest tide and the rate at which the crew will row in still water.

7. Solve the following quadratic equations :—

(α) $(x+3)^2=6x+58$.

(β) $\frac{3x-1}{2x}-\frac{2x}{3x-1}=\frac{24}{35}$.

(γ) $x^3-8=4(x-2)(x+7)$.

8. If the lengths of two straight lines be each expressed in feet and inches, show how the product of their lengths may represent superficial measurement. Apply the rule of duodecimals to find the area of a room 32 feet 8 inches long by 18 feet 4 inches wide, and explain the different terms in the product.

9. Define the logarithm of a number. If the base be 3, of what numbers are 1, 2, 3, -1, -2, -3 respectively the logarithms? Why is it found most convenient to calculate logarithms to a base 10? Of what number is ·5 the logarithm to a base 10?

Find by the aid of the tables $\frac{(24\cdot76)^{\frac{3}{2}}}{(\cdot0045)^{\frac{1}{3}}}$.

10. A right angled triangle has a base of 240 feet, and the hypothenuse is 400 feet; find the area of the triangle.

11. A pyramid has a regular hexagon for its base, each side being 20 feet; find the cubical content of the pyramid, if its altitude is 12 feet.

12. The three edges of a rectangular parallelopiped that meet in an angle are respectively 25, 54, 160 feet. Find the side and diagonal of a cube which shall have the same volume as the parallelopiped.

13. A field has 5 sides; show by what measurements the field may be divided into triangles and its area found.

XLIII.

1. Prove that a^n+b^n is divisible by $a+b$ without remainder, when n is an uneven number.

Resolve into their simple factors the expressions

(1.) $6(x^2-1)+5x$.

(2.) $(a^2-1)x^2+(3a-1)x+a-a^2$.

2. State and prove the rule for finding the highest common divisor of two algebraical quantities.

Find the highest common divisor of

$9x^3+53x^2-9x-18$, and $x^3+10x^2+19x-30$.

3. Reduce to their simplest forms—

(1.) $\dfrac{a^2+b^2+c^2+2ab+2bc+2ac}{a^2-b^2-c^2-2bc}$,

(2.) $\dfrac{a^3+a(1+a)y+y^2}{a^4-y^2}$;

and add together

$$\frac{3}{4(1-x)^2}+\frac{3}{8(1-x)}+\frac{1}{8(1+x)}+\frac{x-1}{4(1+x^2)}.$$

4. Prove that if the sum of two square numbers be multiplied by the sum of two other square numbers, the product is the sum of two square numbers.

5. Extract the square roots of—

(1.) $\dfrac{9}{4}+6x-17x^2-28x^3+49x^4$;

(2.) $87-12\sqrt{42}$; and simplify

$$\frac{\sqrt{1-x}+\dfrac{1}{\sqrt{(1+x)}}}{1+\dfrac{1}{\sqrt{1-x^2}}}$$

6. Establish the relations between the coefficients and the roots of a quadratic equation.

Under what circumstances are the roots of $x^2+2px+q=0$ both real, and both positive? If α and β are the two roots of this equation, express $\alpha^2-\alpha\beta+\beta^2$ and $\alpha^3+\beta^3$ in terms of p and q.

7. Solve the equations—

(1.) $\frac{48}{x+3}=\frac{165}{x+10}-5$;

(2.) $\sqrt{(2x+7)}+\sqrt{(3x-18)}=\sqrt{(7x+1)}$;

(3.) $xy+x+y=11$;
$x^2y+xy^2=30$;

(4.) $\frac{x}{2}+\frac{y}{3}=7=\frac{20}{x}+\frac{30}{y}$.

8. Prove that if N is a whole number, which is divisible by any prime number other than 2 and 5, the decimal fraction equivalent to $\frac{1}{N}$ is a circulator.

9. A rectangular court is 20 yards longer than it is broad, and its area is 4524 square yards. Find its length and breadth.

10. The weights of two spheres, which are solid and made of the same material, are 512 lbs. and 729 lbs. respectively. If the radius of the first sphere is 16 inches, what will it cost to gild the surface of the second sphere at a penny and three farthings the square inch? $\left[\pi=\frac{22}{7}\right]$.

11. The perimeters of a circle, a square, and an equilateral triangle are each of them one foot. Find the area of each of these figures to the nearest hundredth of a square inch.

12. The sides of a triangular field are respectively 10 chains, 8 chains, and 12 chains. The chain being 22 yards, find the acreage of the field, and the perpendicular distance of its longest side from the opposite corner.

13. A cubical cistern contains, when completely full, 2,000 cubic feet of water. Find the length of one of its sides to the nearest tenth of an inch.

14. Define the logarithm of a given number to a given base.

The logarithm of 2 to base 10 is ·30103, find the logarithms of $2^{-\frac{1}{10}}$, of ·002, of $5^{-\frac{1}{5}}$, and of 6250.

15. Prove that the logarithm of the product of two factors is the sum of the logarithms of the two factors.

Solve the equation $2^x=5$.

16. The mantissas of the logarithms of 79531 and 79532 are respectively ·9005364 and ·9005419; find the logarithm of 795·314, and find the number of which the logarithm is $\bar{2}$·900539.

XLIV.

1. Multiply $4x-5a-2b$ by $3x-2a+5b$, and divide $x^3-x^2z\sqrt{z}$ by $x-x\sqrt{z}$: and find the cube of

$$\sqrt[3]{a+x}-\sqrt[3]{a-x}.$$

2. Prove the rule for finding the square root of a binomial surd, and extract the square root of $5-2\sqrt{6}$.

3. Find the least whole number, which, when divided by 3 and 5, has its respective remainders 1 and 3.

4. Extract the sixth root of $x^6+6x^5+15x^4+20x^3+15x^2+6x+1$.

5. Solve the following equations:

(1.) $$\sqrt{x}-4=\frac{259-10x}{4+\sqrt{x}}.$$

(2.) $$\frac{10}{x}-\frac{14-2x}{x^2}=2\tfrac{4}{9}.$$

(3.) $$x+y+\sqrt{xy}=19$$
$$x^2+y^2+xy=133.$$

6. The sides of three cubes have equal differences, their sum is 15 inches, and the solid content of the three together is 495 inches: required the side of each.

7. Reduce ·PQ to a vulgar fraction, where P contains p, and Q contains q decimal places; and divide ·4$\dot{8}$ by 1·2$\dot{3}$.

8. Given log 8·1213= ·9096256, write down the logarithms of 81·213, 812·13, 81213, and ·081213.

9. Given log 20=1·30103000, log 30=1·47712125, find the logarithm of 360.

10. If the side of a cube be 2, required the side of another cube exactly double the former.

Given log 2=·30103, log 25=1·39794,
log 36=1·41497.

11. Find the number of cubic feet in a room which is 36 feet long, 20 feet wide, 10 feet high to the spring of the roof, and 16 feet from the floor to the pitch of the roof.

12. How many marbles, each one inch in diameter, can be packed in a box, whose internal dimensions are an exact cubic foot, and how much space will be left vacant? Given $\pi=3{\cdot}1416$.

13. The area of a triangle is 5 acres, 3 roods, 5·12 perches, and its perpendicular measures 826 links; what will be the expense of making a ditch the whole length of its base, at 2*s.* 6*d.* for every 7 yards?

14. Find the weight of a ball of ash wood, whose specific gravity is $\frac{4}{5}$ that of water, the diameter of the ball being 10 inches, and the weight of a cubic foot of water being 1000 oz.

XLV.

1. Prove that $x^{mn}-1$ is divisible by x^n-1 without remainder; and divide

$$a^3(b-c)+b^3(c-a)+c^3(a-b) \text{ by } a+b+c.$$

2. Reduce to their simplest forms—

(1.) $\dfrac{4x^2-8x+3}{4x^2+4x-3}$.

(2.) $\dfrac{n}{1-x^n}+\dfrac{n}{1-x^{-n}}$.

(3.) $\dfrac{1}{x}-\dfrac{1}{(x+1)^2}-\dfrac{2}{x+1}+\dfrac{x}{1+x+x^2}$.

3. If m and n are two whole numbers, each of which leaves the remainder 1 when divided by 4, show that $m \times n$ leaves the remainder 1 when divided by 4.

Prove also that every uneven square number leaves either the remainder 1, or the remainder 9, when divided by 16.

4. Find the highest common divisor of

$3x^4-10x^3+9x^2-2x$ and $2x^4-7x^3+2x^2+8x$;

and the least common multiple of x^2-1, x^2-x-2, x^2+x-2.

Show that if d is the greatest common divisor of two numbers a and b, their least common multiple is $\frac{ab}{d}$.

5. Extract the square root of—

(1.) $9a^2-30ab-3ab^2+25b^2+5b^3+\frac{b^4}{4}$.

(2.) $87-12\sqrt{42}$.

and prove that a quadratic surd cannot be equal to the sum or difference of two dissimilar quadratic surds.

6. Solve the equations—

(1.) $\frac{4x+3}{9}+\frac{7x-29}{5x-12}=\frac{8x+19}{18}$.

(2.) $\sqrt{(x-3)(2x-3)}+\sqrt{(x-1)(2x-5)}=\sqrt{2}$.

(3.) $\frac{1}{x}+\frac{1}{y}=2$, $\frac{1}{x^3}+\frac{1}{y^3}=14$.

7. What are the conditions that the roots of the equation $ax^2+2bx+c=0$ should be (1) real, (2) equal, (3) imaginary? What do you know about the roots of this equation, (1) if $a=c$, (2) if a and c have opposite signs?

8. If I pay one guinea for a cubical block of marble, of which the side is one foot, what ought I to pay for another cubical block of the same marble, of which the side is equal in length to the diagonal of the first block?

9. Determine to the nearest hundredth of an inch the

radius (1) of a sphere whose volume is one cubic foot, (2) of a sphere whose surface is one square foot.

[In this and the following question $\pi=3{\cdot}1416$.]

10. If a room be 40 feet long by 20 feet broad, and contain 12,800 cubic feet; what addition will be made to its cubic contents by throwing out a semicircular bow at one end?

11. Given a circle, show how to find the radii of the n concentric circles which divide the area of the given circle into $n+1$ equal parts.

If the radius of the given circle is one foot, find to the nearest hundredth of an inch the radii of the two concentric circles which divide its area into three equal parts.

12. A field is in the form of a trapezoid; its parallel sides are respectively 10 chains 30 links, and 7 chains 70 links; the distance between them is 7 chains 50 links; find the acreage.

13. If x is the logarithm of a to the base b, what is the logarithm of a^m to the base b^n?

Write down the logarithms of 256 and ·0625 to the base $\frac{1}{2}$; and of $\frac{1}{27}$ and 81 to the base 243.

14. Given log $18=1{\cdot}2552725$, log $125=2{\cdot}0969100$, log $21=1{\cdot}3222193$, find the logarithms of the numbers from 2 to 9, inclusively.

If $\log\frac{1025}{1024}=\alpha$, and $\log 2=\beta$, show that

$\log 4100=\alpha+12\beta$.

15. Find the seventh root of 23, given $\log 23=1{\cdot}3617278$, $\log 15650=4{\cdot}1945143$, $\log 15651=4{\cdot}1945421$.

XLVI.

1. Prove that in Euc. ii. 11, the given straight line has been divided so that the squares on the whole and one of

the parts are together equal to three times the square on the other part.

2. If two parallel chords AB, CD be drawn in a circle then AC and BD are equally distant from the centre.

3. Describe a circle about a given triangle.

If the triangle be right angled, prove that the sum of the two sides containing the right angle is equal to the sum of the diameters of the inscribed and circumscribed circles.

4. A and B are fixed points, and P is a movable point such that PA always bears a fixed ratio to PB.

Prove that P lies on a circle, and find the centre of that circle.

5. Define a degree, a grade, and the unit of circular measure.

The number that denotes an angle when expressed in circular measure is less by 1·5 than that denoting the number of degrees in the same angle. Find the angle.

6. Express all the trigonometical ratios in terms of the sine.

If the cosecant be twice the sine, find the angle.

7. Prove that

$$\cos(A+B)=\cos A \cos B-\sin A \sin B.$$

stating the limitations assumed in the proof as to the values of A and B.

8. Find cos 2 A and sin 3 A.

9. If any arc of a circle be divided into two equal and also into two unequal parts, prove that the rectangle of the chords of the two unequal arcs, together with the square of the chord of the arc between the points of section, is equal to the square on the chord of half the arc.

10. Prove that in any triangle

$$\text{Sin } A=\frac{2}{bc}\sqrt{s(s-a)(s-b)(s-c)}.$$

11. In the triangle ABC

$A=26°26'$, $b=127$ and $a=85$. Find B.

Given log $1{\cdot}27=\cdot1038037$

Log $8{\cdot}5=\cdot9294189$

Log sin $26°\ 26'=9{\cdot}6485124$

Log sin $41°\ 41'=9{\cdot}8228302$

Log sin $41°\ 42'=9{\cdot}8229721$.

12. If one angle of a triangle be 65°, and one of the sides containing this angle be four times the other, find the remaining angles.

Given log $2=\cdot3010300$ Log $3=\cdot4771213$

Log cot $32°30'=10{\cdot}1958127$

Log tan $43°17'=9{\cdot}9739602$

Log tan $43°18'=9{\cdot}9742133$.

13. The angle of elevation of a tower at a distance of 20 yards from its foot is three times as great as the angle of elevation 100 yards from the same point.

Show that the height of the tower in feet is $\frac{300}{\sqrt{7}}$.

XLVII.

1. Produce a given straight line so that the rectangle contained by the whole line thus produced, and the part produced, shall be equal to the square on the given line.

2. If any number of equal straight lines are placed in a circle, show that a circle can be described which shall touch them all, and that each of them is bisected at its point of contact with this circle?

3. If two circles touch each other externally, and if through the point of contact a straight line be drawn meeting both the circles, show that the segments of the circles, which lie on opposite sides of this line, are similar to each other.

4. If a circle be inscribed in a triangle, show that each

side of the triangle is divided at the point of contact into two parts, of which the difference is equal to the difference between the other two sides of the triangle.

5. Divide a right angle into three, and also into five equal parts.

6. When is the first of four magnitudes said to have the the same ratio to the second which the third has to the fourth ?

Prove that triangles which have the same altitude are to one another as their bases.

Show also that triangles which have the same base, are to one another as their altitudes.

7. Define duplicate ratio, and prove that similar triangles are to one another in the duplicate ratio of their homologous sides.

On the side AB of a triangle ABC, AD is taken equal to one third of AB; and on AC, AE is taken equal to one third of AC; DE and DC are joined. What is the ratio (1) of DE to BC, (2) of the triangle ADE to the triangle ABC, (3) of the triangles CDA, CDB to each other and to the whole triangle ABC ?

8. Define the sine, cosine, and tangent of an angle: and trace the variation of the tangent, in sign and magnitude, through the four quadrants.

Given $\cos A = -\frac{5}{13}$, find sin A and cot A; A being supposed less than 180°.

9. Prove that $\cos(A-B) = \cos A \cos B + \sin A \sin B$, and employ this formula to find the value of cos 15°.

10. Prove the following equalities:

(1.) $\text{Sin}(60°+A) - \sin(60 - A) = \sin A$.

(2.) $(\text{Cos}\, A + \sin A \sin 2A)^3 - (\sin A + \cos A \sin 2A)^3 = (\cos 2A)^3$.

(3.) $\text{Tan}\,(B-C) + \tan(C-A) + \tan(A-B) = \tan(B-C)\tan(C-A)\tan(A-B)$.

11. Show that in any triangle—

(1.) $\text{Cos } A = \frac{b^2+c^2-a^2}{2bc}$.

(2.) $\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$,

where s is half the sum of the sides; and find an expression for the radius of the circle which touches one side of the triangle and the other two sides produced.

12. Show how to solve a triangle when two sides and the included angle are given.

Given in a triangle ABC, AB=4517 yards, AC=150 yards, A°=31°30′, find the remaining parts.

13. A, B are two points, 200 yards apart, on the bank of a river, and C is a point on the opposite bank. The angles ABC, BAC are respectively 54°30′, and 65°30′; find the breadth of the river.

14. Find an expression for the area of a regular polygon of n sides inscribed in a given circle; and show that the square of the side of the inscribed pentagon is equal to the sum of the squares of the sides of the inscribed hexagon and decagon.

XLVIII.

1. If the side BC of a triangle ABC is bisected in D, prove that $AB^2+AC^2=2AD^2+2BD^2$.

2. Prove that of all triangles which have a given base and and vertical angle, the greatest is that in which the two sides are equal.

3. Divide a given straight line so that the rectangle contained by its segments shall be equal to a given square, not greater than the square on half the line.

4. If in any triangle the centres of the inscribed and circumscribed circles coincide, the triangle is equilateral.

5. When are two magnitudes said to be of the same kind? When are two magnitudes of the same kind said to be in-

commensurable? Does Euclid's definition of the sameness of ratios apply to incommensurable magnitudes?

6. Define compound ratio; and prove that equiangular parallelograms have to one another the ratio which is compounded of the ratios of their sides.

Show that any two parallelograms have to one another the ratio which is compounded of the ratios of their bases and altitudes.

7. Each of three circles cuts the other two: prove that the three common chords, or these chords produced; pass through one and the same point.

Given a circle, and two points in its plane; describe a circle which shall pass through the two given points, and meet the given circle in two points diametrically opposite to one another.

8. Show that there are in general two different angles between 0° and 360° which have a given sine; or a given cosine; but that there is only one angle between 0° and 360°, which has both a given sine, and a given cosine.

Given the tangent of an angle, express in terms of it the other trigonometrical ratios of the angle.

9. Establish the formulæ—

(1). $\cos A - \cos B = 2 \sin \frac{A+B}{2} \sin \frac{B-A}{2}$.

(2.) $\cos 3A = 4 \cos^3 A - 3 \cos A$;

and employ (2) to obtain the values of cos 18° and sin 18°.

10. Prove the equalities—

(1) $\frac{\sin 2A}{1+\cos 2A} \cdot \frac{\cos A}{1+\cos A} = \tan \frac{1}{2} A$.

(2) $\sin (B-C) + \sin (C-A) + \sin (A-B)$
$+ 4 \sin \frac{1}{2}(B-C) \sin \frac{1}{2}(C-A) \sin \frac{1}{2}(A-B) = 0$;

and solve the equation—

$\sin(A+30°) + \cos(A+30°) = \sin(A-30°) + \cos(A-30°)$.

11. Prove that in any triangle—

(1.) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ = diameter of circumscribing circle.

(2.) $(b+c) \sin \frac{1}{2} A = a \cos \frac{1}{2} (B-C)$.

12. Given the three sides of a triangle; find the logarithmic sines of its half-angles.

The sides of a triangle are 525 feet, 650 feet and 777 feet respectively. Determine its three angles.

13. A, B, C, are three points in a straight line on a level piece of ground. A vertical pole is erected at C; the angle of elevation of its top, as observed from A, is 5° 30′; and, as observed from B, is 10° 45′; the distance from A to B being 100 yards. Find the distance BC, and the height of the pole.

14. Prove that the area of a circle is equal to the product of its semi-circumference by its radius.

Three circles, each of radius 1 foot, touch one another externally. Find the area of the curvilinear figure included between them.

XLIX.

1. If a square and a rectangle have equal areas, prove that the perimeter of the square is less than that of the rectangle.

2. AB and CD are two chords in a circle which intersect when produced in the point E without the circle. Prove that the difference of the angles subtended at the centre by the arcs AC and BD is double of the angle AEC.

3. The diameter AB of a circle is produced to C, and through C the line CD is drawn at right angles to the line ABC: Prove that if the straight line AD cuts the circle in E, the rectangle AE, AD will be equal to the rectangle AB, AC.

4. ABCD is a rectangle, and E, F, G, and H are the feet of the perpendiculars let fall upon the diagonals from the angular points: Prove that EFGH is also a rectangle.

5. Define the circular measure of an angle.

If the number of degrees in an angle be equal to the number of grades in the complement of the same angle, prove that the circular measure of the angle is $\frac{5\pi}{19}$.

6. Define the principal trigonometrical ratios, and express each of them in terms of the tangent.

Find sin 60° and cot 30°.

7. Find sin (A+B) in terms of the sines and cosines of A and B, stating the limitations in the values of A and B assumed in the proof. Apply to find sin 75°.

8. Find tan 2A and sin 3A.

9. Given $\tan \frac{A}{2} = \frac{a}{b}$, find sin A, and sin 2A.

10. In any triangle of which the semi-perimeter is s, prove the formula, $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$.

11. From B and A the acute angles of the right-angled triangle ABC the lines BD and AE are drawn to the opposite sides of the triangle such that the angles DBC and EAC are each equal to α.

Prove that DE=AB tan α.

12. Given A=40°, a=140·5, b=170·6. Find B and C.

Given that log sin 40°=9·8080675.
log 1405 =3·1476763.
log 1706 =3·2319790.
Log sin 51° 18′=9·8923342.
Log sin 51° 19′=9·8924354.

13. Two sides of a triangle are 85 and 75 yards respectively, and the included angle is 70°; find the remaining angles.

Given that

Log 160 = 2·20412.
Log cot 35° = 10·15147732.
Log tan 5° 6′ = 8·9505967.
Log tan 5° 7′ = 8·9520211.

L.

1. Given a square and one side of a rectangle whose area is equal to that of the square; find the other side of the rectangle.

2. If two circles cut one another, and the tangents to one circle at the points where it meets the other circle intersect upon the circumference of the latter, prove that this latter circle passes through the centre of the former.

3. If ABC be a triangle, and if O be the point of intersection of AD and BE, the perpendiculars from A and B upon the sides BC and CA respectively, and if AD produced meet the circumscribing circle in F, prove that OD is equal to DF.

4. If ABC be a triangle right-angled at A, and DAB and EAC be the triangles similar to ABC having their right angles at D and E respectively, prove that the triangles DAB and EAC will be together equal to the triangle ABC.

5. Define circular measure and show how to convert degrees into circular measure.

In the pentagon ABCDE all the angles are equal, and AF and BF are drawn perpendicular to EA and BC, meeting in the point F. Find the circular measure of the angle AFB.

6. Express each of the trigonometrical functions in terms of the tangent.

If $\operatorname{cosec} A = \frac{2}{\sqrt{3}}$ find sec A, supposing that A lies between 90° and 180°.

7. Find tan (A+B) in terms of the tangents of A and B.

Prove that tan 15°+tan 75°=4.

8. Find sin 3A and cos 3A.

Prove that $\frac{\sin 3A}{\sin A}-\frac{\cos 3A}{\cos A}=2$.

9. In any triangle ABC whose sides are a, b, and c, prove that—

$$a^2=b^2+c^2-2bc\cos A.$$

10. Assuming the result of the last question, find a formula for $\cos\frac{A}{2}$ in terms of the sides adapted to logarithmic computation.

11. In any triangle if A=47°, a=180, b=215, find B,

Given that—

Log sin 47° =9·864127
log 180 =2·255273
log 215 =2·332438
Log sin 60° 52′=9·941258
Log sin 60° 53′=9·941328.

12. If A=55°, B=65°, and c=270, find a,

Given that—

Log sin 55°=9·9133645
log 3 =·4771213
log 2 =·3010300
log 2·5553=·4070508
log 2·5554=·4072209.

13. The sides a, b, and c of the triangle ABC are to each other as 4 : 5 : 6. Find B,

Given that—

Log cos 27° 53′=9·9464040
Log cos 27° 54′=9·9463371
log 2 =·3010300.

LI.

1. Illustrate the truth of proposition Euc. II. 5 by taking the whole straight line AB to be 12 feet, and the two parts AD and DB to be 9 feet and 3 feet respectively.

2. A point is taken within a circle and from that point two equal straight lines are drawn to the circumference. Prove that the straight line which bisects the angle between these two straight lines passes through the centre of the circle.

3. If the points of contact be joined prove that each angle of the triangle so formed together with half the angle opposite to it in the original triangle is equal to a right angle.

4. Prove that the area of the square inscribed in any circle is half that of the square described about the same circle.

5. Prove that the ratio of the diameter of the circle described about a triangle to any one of the sides is the same as the ratio of half the rectangle of the remaining two sides of the triangle to the area of the triangle.

6. Define the grade and the degree, and express the interior angle of a regular pentagon both in grades and degrees.

7. Define the circular measure of an angle, and find two angles such that their difference is one degree, and their sum is the unit of circular measure.

8. Define the principal goniometrical ratios, and find the sine of 30° and secant of 45°.

9. Prove that $\sin (A-B)=\sin A \cos B-\cos A \sin B$, stating what limitations your proof supposes in the values of A and B respectively.

10. Find sin 2 A and cos 3 A.

11. If $\sin A=\frac{2ab}{a^2+b^2}$ find $\tan \frac{A}{2}$.

12. Prove that the area of any triangle whose sides are a, b, and c is

$$\sqrt{s(s-a)(s-b)(s-c)}$$

s being the semiperimeter.

13. In the triangle ABC

A=60° 15′, B=54° 30′, and AB=100 yards, find AC having given

L sin 54° 30′=9·9106860
L sin 65° 15′=9·9581543
Log ·89646 =$\bar{1}$·9525317.

14. The sides and the included angle of a triangle are 327 feet, 256 feet, and 56° 28′ respectively, find the remaining angles; given

Log ·71 = $\bar{1}$·8512583
Log ·583 = $\bar{1}$·7656686
L tan 61° 46′=10·2700705
L tan 12° 46′= 9·3552267
L tan 12° 47′= 9·3558126.

15. ABC is a triangle and D the middle point of the base BC, prove that the sine of BAD is

$$\frac{b \sin A}{\sqrt{b^2+2bc\cos A+c^2}}.$$

LII.

1. If any number of equal straight lines be placed in a circle, find the locus of their points of bisection.

2. If from any point without a circle lines be drawn touching it, the angle contained by the tangents is double the angle contained by the line joining the points of contact and the diameter drawn through one of them.

3. Inscribe a circle in a given triangle. If the three

sides of a triangle equal a, b, and c respectively, find the lengths of the segments intercepted between the angles, and the points of contact of the inscribed circle.

4. If the angle of a triangle be divided into two equal angles, by a straight line which also cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

By means of this proposition prove that the three straight lines, which bisect the three angles of a triangle, meet in the same point.

5. Define the cosine, and versed sine of an angle, and trace the changes in magnitude and sign of the latter, as the angle increases from zero to 360°.

6. Prove that $\sin\left(\frac{\pi}{2}+A\right)=\cos A$, and that $\cos\left(\frac{\pi}{2}+A\right)=-\sin A$: and find an expression for all angles which have a given tangent.

7. Investigate a formula for expressing the cosine of half an angle in terms of the sine of the whole angle: and if the angle lies between 270° and 360°, show which signs of the roots must be taken.

8. Explain clearly the ambiguous case in the solution of triangles.

If AB=204, AC=145, and the $\angle B=35°$, find BC, and the angles BAC and C.

Given log 204=2·30963, log 145=2·16136, L sin 35 =9·75859, L cos 36° 12′=9·90686, L cos 1° 12′=9·99990, L sin 18° 48′=9·50821, log 252·744=2·40267, log 81·4687=1·91098.

9. Compare the areas of the circles inscribed in, and described about a given triangle.

10. Find all the values of θ, which satisfy the equation—

$$\cos\theta+\sin\theta=\sqrt{2}.$$

and prove that

$$\cos^{-1}\frac{1-m^2}{1+m^2}-\cos^{-1}\frac{1-n^2}{1+n^2}=\tan^{-1}\frac{2(m-n)(1+mn)}{(1+mn)^2-(m-n)^2}$$

also that

$$\frac{\text{vers A}}{\text{vers B}}=\frac{a(a+c-b)}{b(b+c-a)}.$$

LIII.

1. Divide a straight line into two parts, so that the rectangle contained by them may equal the square of the difference.

2. Describe a circle—

(1.) Passing through three given points, which are not in the same straight line.

(2.) Passing through two given points, and touching a given circle.

3. Explain the following terms:

(1.) Homologous sides.

(2.) Ex æquali.

(3.) Duplicate ratio.

(4.) Reciprocal figures

(5.) Alternando.

4. Show how a given straight line may be divided into any number of equal parts.

5. Define the tangent of an angle, find the value of tan 60°, and prove that tan (180−A)=−tan A and tan (90+A)=−cot A. Also trace the changes in its sign and magnitude, as A varies from 0° to 180°.

6. If A and B be each less than a right angle, and A>B, prove that cos (A−B)=cos A cos B+sin A sin B.

Draw the figure for a case (1) when A is greater than a right angle (2) when A is<B.

7. If the sine of an angle is given, show how to find the sine of half the angle, and explain why the general expres-

sion obtained ought to include four values.

Given $\sin 300° = \frac{-\sqrt{3}}{2}$, find $\sin 150°$.

8. Prove that $\sec A = \frac{1+\tan^2 \frac{A}{2}}{1-\tan^2 \frac{A}{2}}$

$$\operatorname{cosec} A = \frac{1}{2}\left(\tan \frac{A}{2} + \cot \frac{A}{2}\right)$$

and solve the equation—

$$\sin x \cos x + a \sin^2 x = b.$$

9. The angles in one regular polygon are twice as many as in another regular polygon; and an angle of the former is to an angle of the latter as 3 : 2. Find the number of sides.

10. In an oblique angled triangle, given $a=145$, $b=178$, $B=41° 10'$, find A

log 178=2·2511513
log 145=2·1613680
log sin 41° 10′=9·8183919
log sin 32° 21′ 54″=9·7286086,

and show whether there is any ambiguity in the result.

11. In any triangle given a, b, and A, find an equation for determining c, and discuss its values, according as (a) is equal to, less than, or greater than $b \sin A$.

CONTROL PAPERS.

LIV.

1. Reduce 4 tons 7 cwt. 1 qr. 13 lbs. 12 oz., to ounces.

2. If 13 lbs. of sugar cost 4*s.* $10\frac{1}{2}$*d.*, what will be the cost of 213 lbs.

3. Find (by Practice) the cost of 63 cwt. at £5 12*s.* 6*d.* a ton.

4. Find the simple interest on £106 13s. 4*d.* for 15 months at $4\frac{1}{2}$ per cent. per annum.

5. Add together $3\frac{1}{8}$, $\frac{2}{3}$, $1\frac{4}{15}$, $\frac{39}{40}$.

6. Subtract $5\frac{4}{9}$ from $6\frac{7}{12}$.

7. Multiply $12\frac{3}{8}$ by $1\frac{5}{11}$.

8. Divide $4\frac{2}{3}$ by $13\frac{1}{6}$.

9. Add together 84·006, ·002604, 845 and 70·0015603.

10. Subtract 73·4698 from 108·30125.

11. Multiply 302·468 by 400.

12. Divide 78 by 361·059 to 4 places.

13. Reduce 3·8375 ac. to roods and perches.

14. In 164723 pints how many quarters, bushels, pecks, &c.

15. If 15 men reap 20 ac. in 6 days, working 14 hours a day, how many men must be employed to assist 10 other men to reap 6 ac. in $1\frac{3}{4}$ days of 8 hours each?

16. Find (by Practice) the dividend on £721 13*s.* 6*d.* at 14*s.* 2*d.* in the pound.

17. Find the amount of £8900 in 3 years at $2\frac{1}{2}$ per cent. compound interest, neglecting fractions of a penny.

18. Add together $\frac{4}{17}$, $3\frac{1}{9}$, $1\frac{5}{51}$, $\frac{6}{27}$.

19. Subtract $5\frac{4}{13}$ from $6\frac{1}{9}$.

20. Multiply together $4\frac{1}{8}$, $\frac{16}{21}$, $1\frac{3}{11}$, $\frac{1}{2}$.

21. Divide $3\frac{7}{9}$ by $\frac{17}{63}$.

22. Add together 84 6912, ·001567, 1·0056, 549·2.

23. Subtract 23·69428 from 50·012.

24. Multiply 40·061 by ·0054.

25. Divide ·055757592 by ·009207.

26. Reduce 33 yds. to the decimal of a mile.

27. A person takes a railway return-ticket for a month, paying 25 per cent. for it more than he would have paid for a single ticket. At the end of the month he obtains an extension of the time for a week by paying 5 per cent. on the monthly ticket. The whole sum paid is £2 12*s.* 6*d.*; find the price of the single ticket.

28. Extract the cube root of 9555119848.

29. When the three per cents. are at 87½, and shares paying 5 per cent. are at 130¼, which is the more profitable investment? What sum does a person invest when the difference of the incomes arising from the two investments is £561?

30. Reduce ·2671875 to a vulgar fraction, and if the unit be £6 reduce the fraction to shillings, pence and decimals of a penny.

31. Find the prime factors of 111540, 42336, 67392: and thence write down the least number which they will all divide without a remainder.

32. If the weight of 1 cubic ft. of water is 62·35 lbs. avoirdupois, find the error in calculating the weight of 1000 cubic ft. on the approximate assumption that 1 cubic ft. weighs 1000 oz.

LV.

1. If the angles at the base of an isosceles triangle be bisected, the angle contained by the bisecting lines is equal to the exterior angle at the base.

2. Divide a circle into two segments such that the angle contained in one shall be five times that contained in the other.

3. Show that the straight lines joining the centres of the escribed circles of a triangle pass through its angular points.

4. If O is the centre of the circle inscribed in a triangle ABC, and P the centre of the circle which touches the side BC and the other two sides produced, show that the angles OBP, OCP are right angles.

5. (1.) Divide $x^6+4x^5-3x^4-16x^3+2x^2+x+3$ by x^3+4x^2+2x+1.

(2.) $a+\sqrt{ab}+b$ by $\sqrt{a}+\sqrt[4]{ab}+\sqrt{b}$.

6. (1.) Simplify $\dfrac{1}{(x-a)(x-2a)}-\dfrac{1}{(x-a)(x-3a)}+\dfrac{1}{(x-2a)(x-3a)}$.

(2.) $\dfrac{c\sqrt{ab}-ac}{bc-c\sqrt{ab}}$.

7. Find the L.C.M. of

$$x^2+5x+4,\ x^2+2x-8,\ x^2+7x+12.$$

8. Solve the equations:—

(1.) $(x+1)^2=(x+2)\sqrt{x^2+2}$.

(2.) $\left.\begin{matrix} x^2+xy=15 \\ xy-y^2=2 \end{matrix}\right\}$

(3.) $x(x+1)+3\sqrt{2x^2+6x+5}=25-2x$.

9. Sum the series:—

(1.) $2-2^2+2^3-2^4+\cdots$ to 32 terms.

(2.) $\frac{2}{3}+\frac{4}{9}+\frac{8}{27}+$ to infinity.

(3.) The series whose rth term is $2r-1$ to 10 terms.

10. Write down the first 4 terms and the middle term of the expansion of $(1+x)^{2n+1}$; show that the difference of the coefficients of x^{r+1} and x^r in this expansion is equal to the difference between the coefficients of x^{r+1} and x^{r-1} in the expansion of $(1+x)^{2n}$.

11. Define the cotangent of an angle and trace its change in sign and magnitude as the angle increases from 0° to 360°.

Find the value of cot 30°.

12. Prove the following relations:

(1.) $\text{Cos}\,(A-B)=\cos A\cos B+\sin A\sin B$.

(2) $$\text{Sin}\,2A=\frac{1-\tan^2\left(\frac{\pi}{4}-A\right)}{1+\tan^2\left(\frac{\pi}{4}-A\right)}$$

(3.) $$\tan\frac{A}{2}=\frac{-1+\sqrt{1+\tan^2 A}}{\tan A}$$ from (3) find tan 15° and tan 7°30′.

13. Calculate the value of $\sin\frac{A}{2}$ from a given value of sin A.

Ex. sin 210° = $-\frac{1}{2}$, find sin 105°.

14. A man wishing to find the distance between two signal staffs which are inaccessible, plants a staff on a spot from which one of the staves eclipses the other: he next measures 100 yds. in a line perpendicular to that which passes through the staves, and then observes that the angles between the staff which he has planted, and the two signals are 45° and 60° respectively. Find the distance required.

15. Given the three sides of a triangle; find the expressions for the radii of the inscribed and circumscribed circles.

16. Assuming $\cos A=\frac{b^2+c^2-a^2}{2bc}$, prove that

$\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$, where $s=\frac{a+b+c}{2}$ and thence find the greatest angle in the triangle whose sides are 5, 6, 7 ft. respectively. Having given

log 6 = ·7781514 L cos 39°14′ = 9·8890644 and the difference for 60″ = 1032.

LVI.

1. Reduce 5 tons 10 cwt. 3 qrs. 14 lbs. 7 oz. to ounces.

2. The proportion of the diameter of a circle to its circumference is very nearly 113 to 355. Find the circumference of the circle whose diameter is 132 ft.

3. Find (by Practice) the cost of 157 cwt. at £2 16*s.* 8*d.* per cwt.

4. Find the simple interest on £533 6*s.* 8*d.* for 146 days 3 per cent. per annum.

5. Add together $5\frac{3}{7}$, $4\frac{1}{28}$, $\frac{13}{21}$, $\frac{5}{98}$.

6. Subtract $6\frac{7}{12}$ from $10\frac{1}{3}$.

7. Multiply $15\frac{3}{4}$ by $\frac{15}{28}$.

8. Divide $13\frac{1}{8}$ by $4\frac{2}{3}$.

9. Add together 216·005, ·002604, 845, 6914·02.

10. Subtract 36·978 from 500·32105.

11. Multiply 216·79 by 750.

12. Divide 34·015 by 701·5 to 4 places.

13. Reduce 1·0165 ac. to square yards.

14. In 237164 ft., how many miles, furlongs, poles, &c?

15. How long will 40 men take to build a wall 10 ft. high, if 11 men require 17 days to build one of the same length, but only $7\frac{1}{2}$ ft. high?

16. Find (by Practice) the dividend on £1816 13*s.* 4*d.* at 15*s.* $3\frac{3}{4}$*d.* in the pound.

17. Find the compound interest on £4500 in 3 yrs. at 6 per cent. compound interest, neglecting fractions of a penny.

18. Add together $3\frac{5}{8}$, $\frac{3}{5}$, $17\frac{19}{20}$, $\frac{3}{4}$.

19. Subtract $12\frac{7}{8}$ from $19\frac{3}{28}$.

20. Multiply together $3\frac{5}{6}$, $\frac{15}{92}$, $1\frac{3}{4}$, $\frac{8}{21}$.

21. Divide $8\frac{5}{7}$ by $12\frac{1}{8}$.

22. Add together 38·9126, ·005169, 3·0012. 648·03.

23. Subtract 623·92842 from 1005· 6123.

24. Multiply 319·12 by ·0016.

25. Divide 485976·5 by 20·165.

26. Reduce 253 yds. to the decimal of 5 miles.

27. Find the square root of (1) 15661·521316; (2) $62\frac{133}{567}$.

28. The sum of £533 6*s.* 8*d.* is received for £560 due 2 yrs. hence; and what rate is the discount calculated?

29. Divide £2270 amongst A, B, C, D, and E, so that A's share : B's :: 1 : 2; B's : C's :: 5 : 4, D's : C's :: 6 : 5; E's : D :: 4 : 3.

30. A grocer buys coffee at £8 10*s.* per cwt. and chicory at £2 10*s.* per cwt., and mixes them in the ratio of 5 parts of chicory to 7 of coffee: at what rate per lb. must he sell the mixture to gain $16\frac{2}{3}$ per cent.?

31. Three sacks contain equal quantities of wheat, barley, and oats, and a mixture is formed by taking 50 per cent. from the first sack, 30 per cent. from the second, 70 per cent. from the third. Find what percentage of the whole quantity of corn is taken away.

A candidate being required to answer this question reasons as follows: '40 is the mean between 50 and 30, 55 is the mean between 40 and 70, therefore 55 is the mean percentage required.' Is this reasoning correct? If not show where the error lies.

32. Two clocks point to 8 o'clock A.M. at the same instant on New Year's Day: one loses 6″, the other gains 10″ in 24 hrs.: when will one be half an hour before the other, and what time will each clock then show?

LVII.

1. If in a triangle the square on the side subtending an angle be greater than the sum of the squares on the sides containing the angle, express the excess by a geometrical property applicable to all triangles in which this relation of the squares exists.

2. If two tangents, PA, PB, be drawn from a point P without the circle and a line be drawn from P cutting the

circle in C and D, D being nearer to P: show that the angle between the tangents at P is equal to half the angle subtended at the centre of the circle which stands on an arc equal to the difference of the arcs ADB, ACB.

3. Find the number of sides of the regular polygon each angle of which is to an angle of a regular quindecagon as 12 to 13.

4. Show that the perimeters of similar figures are as their homologous sides.

5. Prove that $(a+b+c+d)^2+(a+b-c-d)^2-(c+d-a-b)^2-(b+c-a-d)^2=4\ (a+d)\ (b+c)$, and divide $(x+y)^3+(a+b)^3$ by $(x+y)+(a+b)$.

6. Express in their simplest forms:—

$$(1)\ \frac{ab\ (x^2-y^2)+xy\ (b^2-a^2)}{ab\ (x^2+y^2)+xy\ (a^2+b^2)}.$$

$$(2)\ \frac{ab-2\ (a-b)\sqrt{c}-4c}{ab+2\ (a+c)\sqrt{c}+4c}.$$

7. Solve the following equations:—

(1) $(x-2)^2+(x-3)^2+(x-4)^2=3x^2-19\ x+36$.

(2) $(x+a)^2+bc=(b+c)\ (x+a)$.

(3) $3x^2+15x-2\sqrt{x^2+5x+1}=2$.

8. A broker bought 70 shares in a railway A, and 120 in a railway B for £5000. He bought 30 shares more in A for £1000 than he did in B for 600. Required the prices of the shares in A and B.

9. Sum the following series:—

(1) $a+nb+(2a+2nb)+(3a+3nb)+$ to n terms;

(2) $\cdot9+\cdot09+\cdot009+$ to infinity;

(3) $1+\frac{1}{20}+(\frac{1}{20})^2+(\frac{1}{20})^3+$ to 10 terms by the aid of logarithmic tables.

10. Assuming that the coefficients of a binomial follow throughout the expansion the law of the first four terms, write down the rth term of $(1+x)^n$.

In multiplying the terms of the expansions for $(1-x)^n$ and $(1-x)^{-n}$ together all the terms after the first vanish. Show this by actual multiplication as far as the term involving x^3.

11. Show that any number, however small or however great, positive or negative, may be the tangent of an angle. Find tan 30°. Give $\overline{3}=1{\cdot}7320508$; find without reference to tables the tabulated nos. for cos 30°, tan 30°.

12. Show that $\tan(A+B)=\dfrac{\tan A+\tan B}{1-\tan A\tan B}$.

(1) If A and B be the adjacent angles of a parallelogram what does the formula become ?

(2) Prove $\cot A+\tan A=\sec A\operatorname{cosec} A$.

(3) Find A if $\tan A+3\cot A=4$.

13. Given two sides and the included angle of a triangle, find formulæ for determining—

(1) The remaining angles of the triangle.

(2) The area of the triangle.

14. In the triangle in which each angle at the base is double the third angle, given the base 200 ft. find the area.

15. The angle of elevation of a tower 400 ft. high, when due north of an observer was 55° 14′; what will be the angle of elevation to the observer when he has walked to a point 720 ft. due west of his former position ?

LVIII.

1. In 1370695 oz. how many tons, cwts., qrs., and lbs. are there ?

2. How many persons in 14 days can do as much work as it takes 21 persons to do in 4 days ?

3. Find (by Practice) the dividend on £5146 15*s.* at 17*s.* 9*d.* in the £.

4. Find the simple interest on 200 guineas for 4 yrs. 7 mo. 25 days at $4\frac{1}{2}$ per cent. per annum.

5. Add together $1\frac{5}{9}$, $3\frac{7}{15}$, $\frac{13}{25}$.

6. Subtract $5\frac{2}{7}$ from $6\frac{4}{21}$.

7. Multiply $3\frac{6}{13}$ by $\frac{26}{115}$.

8. Divide $\frac{1}{7}$ by $3\frac{4}{21}$.

9. Add together ·602604, 845, 6914·02 and 71·935.

10. Subtract 43·78126 from 509·7.

11. Multiply 361·42 by ·00685.

12. Divide 61·0284 by ·3501 to 4 places.

13. Reduce 30 sq. yds. to the decimal of a rood.

14. Reduce 13 miles, 4 fur. 5 poles, to yards.

15. If 7 bushels of corn serve 3 persons 10 weeks, and it be sold at 65*s.* a quarter, what will be the cost of providing for 2 persons for 6 weeks?

16. Find (by Practice) the value of 5 qrs. 3 bush. 1 pk. of oats at £1 14*s.* 8*d.* per quarter.

17. Find the amount, at compound interest of £3546 in 3 years at 5 per cent. per annum, neglecting fractions of a penny.

18. Add together $4\frac{7}{12}$, $\frac{8}{9}$, $\frac{7}{16}$, $6\frac{5}{48}$.

19. Subtract $14\frac{4}{13}$ from $19\frac{3}{52}$.

20. Multiply together $\frac{7}{8}$, $5\frac{1}{9}$, $1\frac{7}{23}$, $\frac{24}{49}$.

21. Divide $13\frac{3}{4}$ by $5\frac{1}{2}$.

22. Add together 3·0012, 648·63, 9473·99, ·0006832.

23. Subtract 143·78065 from 8400.

24. Multiply 43·2568 by 720.

25. Divide ·003168 by ·000045.

26. Express ·45895 lbs. Troy in oz., dwts., grs., and the decimal of a grain.

27. Find (1) the square root of 100·861849, (2) the cube root of 12812904.

28. What is the present worth of a bill for £682 18*s.* $9\frac{1}{2}$*d.* due in 3 years 9 months, discount being allowed at the rate of $4\frac{1}{2}$ per cent per annum?

29. If 5 persons contribute 60 dollars each and employ

72 per cent. of their capital in the purchase of sugar at 18*s.* for 28 lbs.; then if dollars be exchanged at 4*s.* 2*d.* each, what part of a ton of sugar will they obtain?

30. A vessel is sent to the Arctic Ocean to catch whales; the captain, who is entitled to $\frac{1}{18}$ of the entire product of oil, receives, at the end of a 4 years' voyage, £730. Supposing the oil to be worth 60 guineas a tun (each whale on an average yielding 6 tuns), also the crew to have been $\frac{2}{3}$ of the entire period on the look out, what was the average number of days between the capture of two whales?

31. On what days of the week will the 10th of February fall in the years 1960 and 2060? Explain the calculations by which you obtain your results in this question.

LIX.

1. Show by a figure how the construction given in the 2nd Prop. Bk. I of Euclid must be applied to the 3rd. Prop. in order to draw from one extremity of the greater line a straight line equal to the less.

2. Show by a figure how to divide a triangle by a line so that its two parts made to coincide with a parallelogram on the same base as the triangle and half its altitude.

3. If the exterior angles of any quadrilateral be bisected by four straight lines, prove that the quadrilateral formed by these lines can be inscribed in a circle.

4. If the base BC of a triangle ABC be divided in D and produced to E so that DAE is a right angle, and BD : DC :: BE : CE, then AD and AE are the internal and external bisectors of the angle BAC.

5. Define the terms interest and discount, and show by general reasoning that the discount on a sum due at the expiration of a certain time is less than the interest on that sum for the same time by the interest on the discount.

The discount and interest on a certain sum for the same time are £22 and £24 respectively; find the sum.

6. Find the greatest common measure of the following quantities—

(1) x^3-3x^2+4x-2 and x^3-2x+4.

(2) x^m-x^{-m} and $x^m+x^{-m}+2$.

7. Solve the equations—

(1) $\frac{mx}{mx+a}+\frac{nx}{nx+a}=2\frac{px}{px+a}$.

(2) $x^{\frac{1}{n}}+x^{-\frac{1}{n}}=a-2$.

(3) $yz=bc$, $\frac{x}{a}+\frac{y}{b}=1$, $\frac{x}{a}+\frac{z}{c}=1$.

8. Give an algebraical solution of the proposition Euclid II. 11, and interpret the two answers.

9. If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, prove that each of these fractions is equal to

$$\frac{px+qy+rz}{pa+qb+rc}.$$

10. Show how to insert n harmonic means between two given quantities a and b.

An arithmetical progression and an harmonical progression have each the first term a and the last term l, and the same number of terms n; prove that the product of the $(r+1)^{\text{th}}$ term of one series and the $(n-r)^{\text{th}}$ of the other is independent of r.

11. Investigate the expression for $(1+x)^n$, n being a positive integer.

Find the sum of the squares of the coefficients.

12. Define the characteristic of a logarithm, and from your definition show that the mantissa must always be positive.

What is the characteristic of ·05 to the base 2? If $\log_{10}·02=\bar{2}·30103$ and $\log_{10}30=1·47712$, find $\log_{10}(·0020736)^{\frac{1}{4}}$.

13. Express sin A in terms of tan A, and tan A in terms of sin A.

Find the cosine, sine, and tangent of one-third of a right angle.

14. Prove geometrically that—

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

when $A+B<90°$, and also when A and B are each $>90°$, and $A+B<270°$.

15. If $\alpha+\beta=\omega$, then

$$\cos^2\alpha+\cos^2\beta-2\cos\alpha\cos\beta\cos\omega=\sin^2\omega.$$

16. In any triangle prove that $b\cos C+c\cos B=a$ and that $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{1}{2R}$, where R is the radius of the circumscribing circle.

17. Show how to solve a triangle, when two of the sides and the included angle are given. Ex.: $A=63°$, $b=13$ ft., $c=11$ ft.; find the number of degrees, minutes, and seconds in the angles B and C.

Given L cotan 31° 30′=10·21268. log 2=·30103
L tan 7° 44′= 9·13289. log 3=·47712
L tan 7° 45′= 9·13384.

18. Express the distances between the centres of the inscribed and escribed circles of a triangle in terms of the sides.

If a new triangle be formed by joining the centres of the three escribed circles of a triangle ABC, prove that the distances of the centres of its escribed circles from the centre of its inscribed circle are

$$8R\sin\frac{B+C}{4},\ 8R\sin\frac{C+A}{4},\ 8R\sin\frac{A+B}{4},$$

where R is the radius of the circle circumscribing the triangle ABC.

WOOLWICH PAPERS.

LX.

1. Add together $1\frac{1}{12}$, $2\frac{3}{20}$, and $2\frac{5}{18}$.
2. Subtract $9\frac{17}{18}$ from $10\frac{1}{3}$.
3. Multiply together $\frac{13}{66}$, $3\frac{1}{5}$, $2\frac{1}{2}$, and $1\frac{16}{39}$.
4. Divide $1\frac{27}{41}$ by $\frac{34}{451}$.
5. Add together 123·478, 1·03, 12, 1·0001, and ·437.
6. Subtract 83·0007 from 334.
7. Multiply 86·35 by 1·2203.
8. Divide 43555·952 by 324·56.
9. Express 12 oz. 8 drams as the decimal fraction of 2 cwt. 26 lbs.
10. Add together $\frac{3}{49}$ of $1\frac{3}{4}$, $\frac{2}{11}$ of $6\frac{2}{7}$, $2\frac{1}{6}$ of $2\frac{2}{5}$, and $\frac{\frac{1}{3}+\frac{1}{4}}{3\frac{1}{2}}$.
11. Subtract $\frac{\frac{7}{16}}{1\frac{3}{8}}$ from $\frac{5}{11}$ of $\frac{3}{4}$.
12. Multiply together $\frac{44}{81}$ of $9\frac{1}{10}$, $13\frac{1}{2}$, and $\frac{3}{91}$ of $\frac{9}{11}$.
13. Divide $4\frac{7}{8}$ by $1\frac{9}{56}$.
14. Add together 1·3 of a ton, ·001 of a cwt., ·93 of a quarter, and ·25 of a pound, and express the answer in pounds and the decimal fraction of a pound.
15. Multiply 101·037 by ·0173.
16. Divide ·00378 by 210.
17. Reduce 4 bushels 1 peck 1 gallon to the decimal fraction of 7 bushels.
18. Divide 26814 acres 1 rood 32 perches 0 yds. 1 ft. 3 in. by 72.
19. The construction of a railway 12 miles 6 furlongs 20 poles in length cost £328,000. What was the average expense per mile?

20. Find (by Practice) the cost of 12,480 things at £3 7*s.* 5¼*d.* each.

21. Find the simple interest on £3,121 6*s.* for 5 years at 3¾ per cent per annum.

22. If 45 men can excavate 2,790 tons of earth in 28 days of 10 hours, in how many days will 35 men working 12 hours per diem do twice that amount of work ?

23. Find the present value of £2,315 5*s.* at 5 per cent. (compound interest) due in 3 years.

24. Extract the cube root of 30·664297 ; and the square root of ·7 to four places of decimals.

25. Compute by means of the tables $\frac{\frac{3}{2}\sqrt{5{\cdot}2}}{5\sqrt{11{\cdot}31}} \times (\frac{3}{7})^{-\frac{1}{2}}$.

26. On the birth of an infant, 1,500 is invested so that it may accumulate at compound interest (3 per cent. per annum payable half-yearly) during the child's minority, calculate by logarithms the amount at the end of the 21 years.

LXI.

1. Find the number of inches in the length 1 mile 1 furlong 3 poles.

2. Find the value of 17 quarters 1 bushel of corn at the price of 4*s.* 8½*d.* the bushel.

3. What is the least weight which can be expressed either by a number of Troy pennyweights or by a number of Avoirdupois ounces ? Give the answer in Troy weight.

4. A vessel holds 2½ pints. How many times can it be filled from a cask of 56 gallons, and will there be any remainder ?

5. Find the value of $\frac{\cdot 06812}{\cdot 131}$ of £2 0*s.* 1¼*d.*

6. A school of boys and girls contains 453 children, and the boys are ·525252 .... of the girls. How many boys are there ?

7. If a metric system of area were adopted wherein 1 acre 1 rood 3 perches is represented by 5·12, express the unit of measurement in square yards and decimal parts of a square yard.

8. At what rate per cent. is the deduction made when 19*s.* 10½*d.* is taken from an account of £39 15*s.* in consideration of immediate payment?

9. Prove that if all the logarithms in a table were multiplied by the same quantity, the results would be the logarithms of the same numbers as before, but to a different base.

If, for instance, all the logarithms in the table supplied to you were doubled, what would the new base be to which they would continue to be logarithms of the same numbers? Give its value to four places of decimals.

Could we rely on the results of calculation made by a system of logarithms where the value of the base itself cannot be exactly determined?

10. Compute by logarithms the fraction $\sqrt[3]{\frac{\cdot0073288}{\cdot05482637}}$ to five places of decimals.

11. What is the compound interest to the nearest penny on £83 14*s.* 7*d.* in 7 years at the rate of 3 per cent. per annum, interest being due at the end of each year? How much does this compound interest exceed the simple interest in the same time on this principal at the same rate per cent.?

LXII.

1. Add together $1\frac{4}{5}$, $2\frac{5}{8}$, and $3\frac{44}{45}$.
2. Subtract $9\frac{14}{15}$ from $11\frac{3}{7}$.
3. Multiply together $1\frac{1}{2}$, $1\frac{1}{4}$, $2\frac{1}{2}$, and $3\frac{1}{5}$.
4. Divide $2\frac{15}{19}$ by $3\frac{19}{38}$.
5. Add together 11·234, ·007, 8, 9·0008, and ·7582.
6. Subtract 25·789 from 100·081.
7. Multiply 1·2345 by 9·807.

8. Divide 4·3046721 by ·0729.

9. Express 4 ounces 6 drams as the decimal fraction of a ton.

10. Add together $\frac{5}{11}$ of $1\frac{5}{17}$, $\frac{1}{10}$ of $\frac{37}{54}$, $5\frac{1}{2}$ of $\frac{3}{55}$, and $\frac{19\frac{6}{11}}{1\frac{1}{4}}$.

11. Subtract $\frac{1\frac{1}{16}}{3\frac{2}{5}}$ from $\frac{7}{8}$ of $1\frac{1}{2}$.

12. Multiply together $1\frac{1}{25}$ of $2\frac{2}{3}$, $1\frac{1}{64}$ and $\frac{3}{7}$ of $2\frac{2}{15}$.

13. Divide $2\frac{5}{13}$ by $2\frac{5}{6}\frac{1}{2}$.

14. Add together ·7 of a league, ·7803 of a mile, ·936 of a furlong, and ·864 of a pole.

15. Multiply 3·9876 by ·05004.

16. Divide 1 by 3·1415 to four places of decimals.

17. Reduce 6 bushels 2 pecks to the decimal fraction of 2 quarters 6 bushels.

18. Divide 18917 acres 2 roods 7 perches 7 yards 3 feet 108 inches by 54.

19. If 4 tons 2 cwt. 1 qr. cost £5 9*s.* 8*d.*, what is the cost of 1 ton 3 cwt. 2 qrs. ?

20. Find (by Practice) the cost of 12345 things at £2 18*s.* $11\frac{3}{4}$*d.* each.

21. Find the simple interest on £2345 5*s.* for 4 years at $4\frac{1}{4}$ per cent. per annum.

22. In a million inches find the number of miles, furlongs, poles, yards, feet.

23. If 9 men, 10 women, 11 boys do a piece of work in 64 days, in how many days will 10 men, 11 women, 12 boys do it, considering a man as equal to 3 boys, and a woman as equal to 2 boys?

24. Find the amount of £89 10*s.* in 3 years at 5 per cent., due annually, at compound interest.

25. Extract the square root of 64009 ; also of 2 and ·4 to four places of decimals.

26. If 5 horses eat 8 bushels $1\frac{2}{3}$ pecks of oats in 9 days, how much will last 18 horses 15 days?

27. Compute by means of the tables $\frac{7\cdot891}{\cdot0345} \times \sqrt[7]{89312\cdot7}$.

28. In what time will a sum of money treble itself at 3 per cent. compound interest paid annually, given log 3 $=\cdot4771213$, and log $103=2\cdot0128372$?

LXIII.

1. Add together $5\frac{1}{2}$, $4\frac{3}{4}$, and $3\frac{15}{16}$.

2. Subtract $11\frac{7}{19}$ from $21\frac{1}{5}$.

3. Multiply together $2\frac{10}{27}$, $1\frac{23}{35}$, $\frac{28}{87}$, and $2\frac{13}{16}$.

4. Divide $4\frac{12}{25}$ by $4\frac{31}{50}$.

5. Add together 189·47132, 3·0167, ·119, 97·625, and 5·43798.

6. Subtract 25·06744 from 40·055.

7. Multiply 3·0056 by 156·25.

8. Divide 126·01745 by 3·925.

9. Express £2 3*s.* $7\frac{1}{2}$*d.* as the decimal of £5.

10. Add together $3\frac{3}{11}$ of $4\frac{2}{5}$, $2\frac{2}{45}$ of $4\frac{3}{23}$, $\frac{5}{8}$ of 122, and $\frac{1\frac{1}{6}+\frac{1}{4}}{5}$.

11. Subtract $25\frac{15}{16}$ from $\frac{12\frac{1}{2}}{\frac{48}{121}}$.

12. Multiply together $7\frac{1}{5}$ of $2\frac{1}{2}$ of $4\frac{3}{4}$, $\frac{1}{6}$ of $\frac{9}{10}$ of $\frac{4}{17}$, and $3\frac{1}{3}$ of $33\frac{1}{3}$.

13. Divide $1\frac{38}{105}$ by $1\frac{63}{91}$.

14. Add together $21\frac{8}{15}$ lbs. $8\frac{5}{24}$ oz. and $18\frac{1}{2}$ dwts., and express the answer in lbs. and the decimal fraction of 1 lb.

15. Multiply together 48·1625, ·00075, and 8000.

16. Divide 352·95624 by ·000504.

17. Reduce 4 square poles 2 yards 6 feet 108 inches to the vulgar fraction and to the decimal fraction of 4 roods 20 poles.

18. Divide 489 tons 14 cwt. 1 qr. by 84.

19. If 63 gallons 3 quarts 1 pint of wine cost £94 4*s.* $3\frac{3}{4}$*d.*, what is the average price per gallon?

20. Find, by Practice, the cost of 59476 bales of goods at £15 9*s*. 10½*d*. per bale.

21. Find the simple interest on £257 18*s*. 4*d*. for 3¼ years at 5 per cent. per annum.

22. In how many days will the same amount of work be done by 75 men, working 8 hours a day, which 100 men will do in 9 days, working 10 hours a day?

23. In what time will the compound interest on £819 4*s*. amount to £224 16*s*. 3*d*. at 6¼ per cent. per annum?

24. Find the square root of 930677049, and the cube root of 2·9 to two places of decimals.

25. Compute, by means of the tables, the value of—

$$\sqrt[3]{24{\cdot}41} \times \frac{1}{\sqrt[3]{187}} \div {\cdot}00395.$$

26. A sum of one shilling, lent on condition of one penny interest being paid monthly, accumulates by compound interest at the same rate for twelve years: find, by logarithms, what the debt will amount to at the end of that time.

LXIV.

1. Add together $3\frac{6}{7}$, $1\frac{3}{16}$, and $4\frac{5}{28}$.
2. Subtract $8\frac{6}{19}$ from $12\frac{2}{17}$.
3. Multiply together $1\frac{19}{26}$, $7\frac{1}{11}$, $\frac{33}{41}$, $5\frac{3}{5}$, and $\frac{1}{4}$.
4. Divide $11\frac{8}{17}$ by $5\frac{25}{34}$.
5. Add together 74·36001, 2·9603, ·0013, 3·1010897, and ·0000037.
6. Subtract 48·978654 from 361·2041.
7. Multiply 6·0035 by ·107.
8. Divide ·008976744 by 712·44.
9. Reduce £1 14*s*. 6*d*. to the decimal of £3 7*s*. 6*d*.
10. Add together $\frac{5}{11}$ of $\frac{44}{75}$, $\frac{17}{23}$ of $7\frac{2}{3}$, and $2\frac{14}{19}$ of $\frac{19}{104}$.
11. Subtract $84\frac{5}{81}$ from $100\frac{1}{27}$.
12. Multiply together $4\frac{13}{18}$ of $2\frac{2}{117}$, $\frac{2}{15}$ of $\frac{7}{24}$, and $\frac{19}{21}$ of 12.
13. Divide $7\frac{1}{3}$ by $2\frac{7}{9}$.

14. Add together ·00465 of a ton and 2·745 of a hundredweight, and give the answer in pounds and the decimal fraction of a pound.

15. Subtract ·0175 of a mile from 8·65 of a furlong, and give the answer in poles and the decimal fraction of a pole.

16. Multiply 7·4361 by ·00542.

17. Divide 3·06541 by ·0328 to three places of decimals.

18. Reduce 6·42 minutes to the decimal of a day.

19. Reduce 7 lbs. 6 oz. 8 dwts. to grains.

20. If $13\frac{1}{2}$ cwt. of copper cost £40 10*s*., what will be the price of 18 lbs.?

21. Find, by Practice, the dividend on £6204 10*s*. at 12*s*. $9\frac{1}{2}$*d*. in the pound.

22. Find the simple interest on £1862 10*s*. for 7 years at $3\frac{3}{4}$ per cent. per annum.

23. In 3 acres 2 roods 10 square yards, how many square feet?

24. If either 5 oxen or 7 horses will eat up the grass of a field in 87 days, in what time will 4 oxen and 6 horses eat up the same?

25. Find, by Practice, the value of 3 tons and 21 lbs. at £12 10*s*. per cwt..

26. Find the amount of £6500 in 3 years at 5 per cent. compound interest (neglecting fractions of a penny).

27. Compute by means of the tables the value of $\frac{(2{\cdot}4806)^8}{(1{\cdot}2701)^{10}}$ to four places of decimals.

28. Given log 2=·3010300, log 9=·9542425, find *without using the tables*, log 5, log 6, log ·0216, and log $\sqrt[5]{{\cdot}375}$.

29. Extract the square root of 3915380329 and of $83\frac{19}{25}\frac{3}{5}$.

30. Find the present value of £2587 18*s*. 9*d*., due 5 years hence, at $3\frac{1}{2}$ per cent.

31. By selling goods for £817 19*s*. a person lost 9 per cent. on his outlay; find at what price he should have sold them in order to gain $10\frac{1}{2}$ per cent.

LXV.

1. Prove that $(x^2+ax+a^2)(x^2-ax+a^2)=x^4+a^2x^2+a^4$, and $(b-c)^3+(c-a)^3+(a-b)^3=3(b-c)(c-a)(a-b)$.

Find a simple expression for the continued product—

$(x^2+ax+a^2)(x^2-ax+a^2)(x^4-a^2x^2+a^4)(x^8-a^4x^4+a^8)$.

2. Divide—

$x^6+25x^4-13x^3+162x^2-129x+90$ by x^2+x+15.

Prove that $(x^2+x+1)^3+(x^2-x+1)^3$ is divisible by x^2+1.

3. Prove the rule for finding the Greatest Common Measure of two algebraical expressions.

Find the Greatest Common Measure of—

$x^3-x^2y-15xy^2+12y^3$ and $x^3-7x^2y+15xy^2-12y^3$.

4. If x be a real quantity, prove that $x+\frac{1}{x}$ is never less than 2; and find the least value of the expression $\frac{x^2+3x+3}{x+2}$.

Find the imaginary values of x which make $x+\frac{1}{x}$ equal to unity.

5. Solve the equations—

(α.) $(x-2)(x-3)+(x-4)(x-5)=2(x-6)(x-7)+62$.

(β.) $y+z=z+x=x+y=2a$.

(γ.) $x^2+3x-4\sqrt{x^2+3x+14}=82$.

(δ.) $\left.\begin{array}{l} x^2+xy+y^2=7a^2 \\ x^4+x^2y^2+y^4=21a^4 \end{array}\right\}$.

6. Find four consecutive numbers such the sum of their squares exceeds by 132 the product of the two middle numbers.

7. A man has six hours to spare for an excursion; how far can he ride out in a carriage which travels eight miles an hour so as to return home in time, walking back at the rate of four miles an hour?

8. Having given the first term and the common ratio, find the sum of n terms of a series in geometric progression.

If each term of the series be subtracted from the preceding, prove that the successive differences are in geometric progression.

9. Find the condition that the quadratic equation—

$$ax^2+bx+c=0$$

may have equal roots.

Determine the values of m which make the expression $3mx^2+(6m-12)x+8$ a complete square.

10. If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, prove that each of these fractions

$$=\frac{a+c+e}{b+d+f}=\sqrt{\frac{a^2+c^2+e^2}{b^2+d^2+f^2}}=\sqrt[3]{\frac{ace}{bdf}}.$$

11. Show how to express a given whole number in any proposed scale.

Express 4562 and 293 in the duodecimal scale, and multiply them together in that scale.

Find the base of the scale in which the number 531 is represented by 650.

12. Write down the expressions for—

(1) The number of permutations of n different things taken all together.

(2) The number of combinations of n different things taken r together.

(3) The number of permutations of $p+q$ things taken all together, p of which are alike and the rest all different.

A boat's crew, consisting of eight men and a coxswain, is to be selected from 15 men, of whom 11 can row and cannot steer, and the rest can steer but cannot row; in how many ways can this be done, and in how many of the different crews would one particular man be the coxswain ?

13. Find the sum of the coefficients in the expansion of $(1+x)^n$, n being a positive integer, and prove that the sum of the coefficients in the odd terms is equal to the sum of the coefficients in the even terms.

14. Find the coefficients of x^n in the expansions of—

$$(1-x)^{-4},\ \frac{1+x}{(1-x)^4},\ \text{and}\ \sqrt{\frac{1-x}{1+x}}.$$

LXVI.

1. Show how to find the centre of the sphere circumscribing a tetrahedron.

2. If two opposite edges of a tetrahedron be equal, prove that the section of the tetrahedron by a plane parallel to these edges is a parallelogram of constant perimeter.

3. Through the centre, O, of the circumscribing circle of a triangle ABC, AOD is drawn to meet BC in D ; prove that—

$$OD : BD : CD = \cos A : \sin 2C : \sin 2B.$$

4. Define a logarithm, and prove that—

$$\log_a b.\ \log_b a = 1.$$

Having given $\log_{10}2 = \cdot30103$ and $\log_{10}49 = 1\cdot690196$, find $\log_{10}(\cdot0056)$ and $\log_{100}1120$.

5. Enunciate and prove De Moivre's theorem.

Apply this theorem to find all the fifth roots of unity.

6. Obtain the exponential expressions for $\sin\theta$ and $\cos\theta$.

If $\tan\theta = n\tan\alpha$, prove that—

$$\theta = \alpha - \frac{1-n}{1+n}\sin 2\alpha + \frac{1}{2}\left(\frac{1-n}{1+n}\right)^2 \sin 4\alpha - \ldots\ldots$$

7. Prove that the infinite series—

$$\cos\theta - \tfrac{1}{2}\cos 2\theta + \tfrac{1}{3}\cos 3\theta - \tfrac{1}{4}\cos 4\theta + \ldots\ldots$$ is equal to $\log\left(2\cos\frac{\theta}{2}\right)$.

8. Define a parabola, and prove that if any chord be drawn through the focus, the circle of which that chord is a diameter touches the directrix.

9. Draw a pair of tangents to an ellipse from an external point, and prove that they subtend equal angles at either focus.

10. If the tangent PT at a point P of a hyperbola meet the transverse axis in T, and if PN be the ordinate, prove that, $CN.CT=AC^2$,

C being the centre and A one of the vertices.

If NQ parallel to AP meet CP in Q, prove that AQ is parallel to PT.

11. Explain what are meant by the co-ordinates of a point referred to a pair of rectangular axes, and find the equation of a straight line.

What is the locus of the equation—

$$ax^2+2bxy+cy^2=0,$$

in the several cases in which b^2 is greater than, equal to, or less than ac?

12. Find the equations of the straight lines which pass through a given point and make a given angle with a given straight line.

What are the equations of the straight lines which pass through the centre of the circle—

$$x^2+y^2=2a\,(x+y),$$

and are inclined at an angle of 45° to that diameter of the circle which passes through the origin?

13. Find the equations of the tangent and normal at the point (x, y) of the parabola, $y^2=4ax$.

Prove that the straight line, $x+y=1$, touches the parabola, $x^2=x-y$.

14. Trace the curve—

$$x^2+xy+y^2=a^2,$$

and find the lengths of its axes.

LXVII.

1. If ABC be a triangle right angled at A, and if circles be described upon the sides BA and AC as diameters, prove that the square on their common tangent is equal to the area of the triangle.

2. If one side of a triangle lie upon a given indefinite straight line, and if the bisectors of the exterior and interior angles between the two remaining sides meet that straight line in two given points, find the locus of the angle opposite to that line.

3. If x, y, and z be three unequal numbers, such that $2a-3y=\frac{(z-x)^2}{y}$ and $2a-3z=\frac{(y-x)^2}{z}$.

Prove that $x+y+z=a$.

4. Find the condition that the roots of the equation—

$$ax^2+bx+c=0$$

should be possible.

If x be real, prove that the fraction $\frac{2x^2+6x+3}{2x+1}$ cannot have any value between 1 and 3.

5. If x, y, and z be the third terms of an arithmetic, geometric, and harmonic series respectively whose first two terms are the same in each case, and positive, prove that x, y, and z are in ascending order of magnitude.

6. In how many ways may four eight-oars be manned with 32 rowers?

7. If x_n denote the number of permutations of x things n together, prove that—

$$(x+y)_n=x_n+nx_{n-1}\,y_1+\frac{n(n-1)}{1.2}\,x_{n-2}y_2+\&c.+y_n.$$

8. If $\frac{p}{q}$ and $\frac{p'}{q'}$, be successive convergents of any continued fraction, prove that $pq'-p'q=\pm 1$.

A straight line AB of length l is divided into a and b, equal parts. Show that if a be prime to b there will be two segments of the line, each equal to $\frac{l}{ab}$ in length, and that every other segment will be greater than this length.

9. At a concert the sum of £100 is taken in 5*s*. and 3*s*. tickets. Find in how many different ways the audience may be composed.

10. Find the sum of the squares of the first n, natural numbers.

Hence by observation of the expanded form of—

$$(1+2+3 \ldots +n)^2,$$

prove that the sum of the products of the first n numbers two and two together is $\frac{1}{24} n\,(n+1)\,(3n+2)\,(n-1)$.

11. Find the coefficient of x^n in the expansion of

$$\frac{1}{1-ax-bx^2}.$$

12. Find the present value of an annuity of £1 for n years, commencing one year hence, at compound interest.

A gentleman pays £10000 for a house. Show how, by means of a table of logarithms, to determine what sum he must lay by annually, beginning one year hence, in order that in 60 years he may have reimbursed himself.

13. A bag contains three half-crowns and two florins, and two people draw one coin in succession.

Prove that their expectations are of equal value.

14. Prove that $\cos x \cos (A-x)$ has its greatest value when $x=\frac{A}{2}$.

Hence prove that the product of the cosines of the three angles of a triangle cannot be greater than $\frac{1}{8}$.

15. Prove that if sin α and sin β be the two values of sin θ satisfying the equation—

$$a \cos 2\theta + b \sin 2\theta = c,$$

then $\cos^2\alpha - \sin^2\beta = \frac{ac}{a^2+b^2}$.

16. If R be the radius of the circle circumscribing the triangle ABC whose sides are a, b, and c, prove that—

$$\tan A = \frac{1}{R} \cdot \frac{abc}{b^2+c^2-a^2},$$

Hence show that if the squares of the sides be in arithmetical progression, the tangents of the angles will be in harmonical progression.

LXVIII.

1. Define a root of the equation $f(x)=0$, and prove that in an equation with real coefficients if $\alpha+\beta\sqrt{-1}$ be a root $\alpha-\beta\sqrt{-1}$ is also a root.

2. Enunciate Descartes' rule of signs, and prove that if any group consisting of an even number of consecutive terms is deficient in any equation there are at least as many imaginary roots.

3. Show that the curve $a^3y = x^4 - 2c^3x + 3c^4$ cannot be cut more than twice by any right line passing through the origin, and find the equation to the tangent through the origin.

4. When is one quantity said to be a function of another? If $x^2+y^2+z^2=r^2$ be the equation to a sphere, what condition must z satisfy in order that x may be a function of y only?

5. Show how to find the differential coefficient of a function of a function of x. If u and v be both functions of x and y, show that if u be expressible as a function of v

$$\frac{du}{dx}\frac{dv}{dy} = \frac{du}{dy}\frac{dv}{dx}.$$

6. Prove that if $f(x)$ and all its derived functions to the $(n+1)th$ inclusive are continuous for all values of x between a and $a+h$ then there must be some positive value of θ less than unity such that—

$$f(a+h)=f(a)+hf'(a)+\frac{h^2}{1\cdot 2}f''(a)$$
$$+\cdots+\frac{h^{n+1}}{\lfloor n+1}f^{n+1}(a+\theta h);$$

Why is it necessary to say that the functions must all be *continuous*?

Expand $(1+x)^3 \log (1+x)$ in a series ascending by powers of x.

7. Explain the method of finding the value of fractions which assume the form $\frac{0}{0}$. Can the same method always be applied to fractions of the form $\frac{\infty}{\infty}$, and if not, why not?

Evaluate $\frac{\sin x}{\text{versin } x}$ and $\frac{\sin^2 x}{\text{versin } x}$ when $x=0$.

8. Investigate a method of finding maxima and minima values of functions of one variable and of distinguishing between a maximum and a minimum.

If a and b be positive constants, find the maximum value of $ax+by$ subject to the condition that x^2+y^2 be constant.

9. If $\phi(x\,y)=a$ be the equation to a curve referred to rectangular co-ordinates find the equation to the normal at the point x, y.

Find the equation to the normal to the curve $y=ae^{\frac{x}{c}}$ at the point (ξ, η).

10. Prove that if the contact of a curve with its tangent be of an odd order, the curve in the neighbourhood of the point of contact lies wholly on one side of the tangent.

Find the points of inflexion of the curve $y=xe^{-\frac{x^2}{k^2}}$.

11. Find an expression for the radius of curvature in terms of the arc, and the angle between the normal and a fixed line.

12. Investigate an expression for the radius of curvature in terms of polar co-ordinates.

Find a curve in which the radius of curvature is proportional to the radius vector.

13. Explain the meaning of the expression $\int_b^a f(x)\,dx$; and find the value of $\int_0^{\pi} \cos^n x\,dx$ when n is a positive even integer.

Find the indefinite integrals of—

$$\cos 2x\,dx,\ x \log x\,dx,\ (a^2+x^2)^{-\frac{3}{2}}dx,$$

$$\text{and } \frac{dx}{(x-2)^2\,(x^2+6x+5)}.$$

14. How does the integral calculus enable us to determine the areas of plane curves? Find the area included between the axes and the curve $y=xe^{-\frac{x^2}{h^2}}$.

LXIX.

1. Define the unit of circular measure.

Show how to transform from degrees to circular measure. What would be the unit of circular measure if an angle of 120° expressed in it became $\frac{3}{4}$.

2. Define the principal trigonometrical ratios, and express each of them in terms of the tangent.

If the cosecant of an angle between 90° and 180° be $\frac{2}{\sqrt{3}}$, find the secant, and if the cosine of an angle between 540° and 630° be $-\frac{1}{2}$, find the cosecant.

3. Prove that—

$$\sin(A+B)=\sin A \cos B+\cos A \sin B,$$

and deduce the value of tan (A+B) in terms of the tangents of A and B.

4. Find sin 3 A and apply the formula to determine the values of sin 18° and cos 36°.

5. Prove that for all values of A—

$$(\sin 2\,A)^2=2\cos^2 A\,(1-\cos 2\,A),$$

and if A, B and C be the angles of a triangle prove that—

$$\sin A \sin B+\cos^2\left(A+\frac{C}{2}\right)=\cos^2\frac{C}{2}.$$

6. Eliminate θ between the equations—

$$x=2a\cos\theta\cos 2\theta-a\cos\theta.$$
$$y=2a\cos\theta\sin 2\theta-a\sin\theta.$$

7. What is the meaning of $\sin^{-1} m$?

Prove that—

$$\sin^{-1}\frac{2mn}{m^2+n^2}+\sin^{-1}\frac{m^2-n^2}{m^2+n^2}=\frac{\pi}{2}$$

for all values of m and n.

8. In any triangle prove that the sides are proportional to the sines of the opposite angles.

If α, β, γ, be the perpendiculars from the angles of the triangle ABC, upon the opposite sides a, b and c prove that—

$$a\sin A+b\sin B+c\sin C=2\,(\alpha\cos A+\beta\cos B+\gamma\cos C).$$

9. Prove that in any triangle ABC whose sides are a, b, and c—

$$\cos A=\frac{b^2+c^2-a^2}{2bc},$$

and hence show that in such a triangle—

$$\frac{\cos\frac{A}{2}}{\sqrt{a(s-a)}}=\frac{\sin\frac{B}{2}}{\sqrt{b(s-b)}}=\frac{\cos\frac{C}{2}}{\sqrt{c(s-c)}}.$$

10. Show how to solve a triangle when two sides and the included angle are given.

If two of the sides be $a \sin \alpha$ and $a \sin (120° - \alpha)$ and the included angle be 60°, solve the triangle.

11. The angles of elevation of a balloon are observed from two stations a mile apart, and from a point half way between them to be 60°, 30°, and 45° respectively. Prove that the height of the balloon in yards is $440\sqrt{6}$.

12. The adjacent sides AC and CB of the base of a rectangular building are 40 and 30 yards respectively, and the angles AOC and BOC subtended by them at a point O on the horizontal plane are each equal to $\tan^{-1}\frac{3}{4}$. Prove that the angle ACO is equal to $\pi - \tan^{-1}\frac{24}{25}$.

Hence, by the aid of a table of logarithms, find the distance OC.

LXX.

1. If a be a root of the equation $f(x)=0$ prove that $f(x)$ is divisible by $x-a$.

Find the ratio of b to a in order that the equations $ax^2+bx+a=0$ and $x^3-2x^2+2x-1=0$ may have either one or two roots in common.

2. If p be the numerically greatest of the negative coefficients in the equation—

$$x^n+p_1x^{n-1}+\&c.+p_n=0.$$

Prove that $p+1$ is a superior limit of the positive roots.

3. If $f(x)=0$ have equal roots, prove that the equations $f(x)=0$ and $f'(x)=0$ must have a root in common;

Hence prove that the equation—

$$x^n+nx^{n-1}+n(n-1)x^{n-2}+\&c.+n(n-1)\ 2.1=0$$

cannot have equal roots.

4. Define a differential co-efficient, and from your definition find the differential co-efficient of a quotient of one given function of x divided by another.

Find also, from first principles, the differential co-efficients of $\log_e x$ and $\tan x$.

5. Differentiate—

(1) $\log e^x \cdot \left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}$. (2) $\tan^{-1}\frac{1-\cos x}{1+\cos x}$

and find $\frac{dy}{dx}$ when $x^{y \log x}=y^{x \log y}$.

6. If $u=\sqrt{\sin^{-1} x}$ prove that—

$$\frac{d}{dx}\log\left(u\frac{du}{dx}\right)=\frac{x}{1-x^2}.$$

7. Prove that $f(a+h)-f(a)=hf'(a+\theta h)$, θ being some fraction less than unity, and $f(x)$ and $f'(x)$ being finite between the limits a and $a+h$ of x.

Assuming Maclaurin's theorem, prove that θ is very nearly equal to $\frac{1}{2}$ when h is very small.

8. If $e^y=e+xy$ find the first six terms of the expansion of y in powers of x.

9. If ABC be a triangle inscribed in a given circle, and the side BC be of constant length prove that any small variations db, and dc in the lengths b and c of the remaining sides are connected together by the equation

$$\frac{db}{\cos B}+\frac{dc}{\cos C}=0.$$

10. Find the value of—

(1) $\dfrac{x^2\cos x-\sin^2 x}{\frac{x^3}{2}-2\sin^2\frac{x}{2}}$ when $x=0$.

(2) $(x^2+3x-1)^{\frac{1}{x}}$ when $x=\infty$.

11. Find the conditions that $f(x)$ may be a maximum or a minimum when $x=a$.

If P and Q be two fixed points situated either, both without, or both within, the circumference of a circle whose centre is O, and if R be a point upon that circumference, prove that the sum of the distance PR and QR is the least possible when PR and QR make equal angles with the radius OR.

12. Find expressions—

(1) For the perpendicular from the pole upon the tangent,

(2) For the radius of curvature,

at any point of a curve referred to polar co-ordinates.

If S be the pole, Y the foot of the perpendicular (p) from S on the tangent PY at P, and ϕ the angle between SY and any fixed line, prove that $\frac{dp}{d\phi}=\text{PY}$.

13. Find the asymptotes to the curve—

$$(y-2x)(y^2-x^2)=a(y^2-9x^2),$$

and the form of the curve at the origin.

14. Integrate—

(1) $\frac{dx}{x^2-5x+6}$.

(2) $\sqrt{a^2-x^2}.dx$.

(3) $\frac{dx}{\cos x}$.

15. Find a formula of reduction for the integral—

$$\int x^m \sqrt{2ax-x^2}dx,$$

and find the value of the definite integral—

$$\int_0^{2a} x\sqrt{2ax-x^2}dx.$$

LXXI.

1. Prove that the sum of the angles of a spherical triangle is always greater than two right angles and less than six right angles.

2. State Napier's rules for the solution of a right-angled spherical triangle: and prove them in the two cases wherein the hypothenuse is the middle part.

3. Determine by the infinitesimal method the surface of a cone, and the content of a sphere.

4. Differentiate—

$$(1)\ y=\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}};$$

(2) $y=\log \tan^{-1}x$.

and find $\frac{dy}{dx}$ when $x^3-3ayx+y^3=0$.

5. Prove Maclaurin's Theorem, and by means of it expand, in terms of x, sec x to three terms.

6. Investigate a method by which a maximum or minimum value of a function of one variable may be determined.

Find the maximum and minimum value of

$$y=x^3-9ax^2+24a^2x+16a^3,$$

and give a figure explanatory of the result.

7. If the equation to a plane curve is $u=f(x, y)$, prove that the equation to the tangent is

$$-x)\left(\frac{du}{dx}\right)+(\eta-y)\left(\frac{du}{dy}\right)=0.$$

What simplification takes place when $f(x, y)$ is homogeneous?

8. Define curvature and radius of curvature. Find the value of the radius of curvature at any point of a plane

curve. At what points in an ellipse is the curvature a maximum and a minimum? Determine the values of the radii of curvature at these points.

9. If $r=f(\theta)$ is the equation to a curve in terms of polar co-ordinates, and $u=\frac{1}{r}$, and p is the perpendicular from the pole to the tangent, prove that

(1) The angle between radius vector and curve

$$= \tan^{-1}\frac{rd\theta}{dr}.$$

(2) $$\frac{1}{p^2}=u^2+\frac{du^2}{d\theta^2}.$$

10. Draw the curves whose equations are

(1) $x^3-3axy+y^3=0$. (2) $r=a\theta+b\sin\theta$.

11. Prove that the envelope of parabolas passing through a given point and having a common directrix is a parabola. What is the application of this theorem in gunnery?

12. Integrate the following :—

(1) $\frac{dx}{x^2-3ax+2a^2}$. (2) $\frac{dx}{(x^2+a^2)^{\frac{1}{2}}}$.

(3) $\sqrt{a^2-x^2}\,dx$. (4) $\tan x\,.\,dx$.

(5) $\frac{dx}{a+b\cos x}$.

13. Find the length of the cycloid

$$y=a\ \text{ver-sin}^{-1}\frac{x}{a}+\sqrt{2ax-x^2},$$

and the area contained between it and the base.

14. Prove that the volume of the solid formed by the revolution of a sector of a circle, the angle of which is α, about one of its bounding radii $=\frac{2\pi a^3}{3}(1-\cos\alpha)$.

15. Find the curve in which the perpendicular from the origin to the tangent is equal to the abscissa.

16. Solve the following differential equations :—

(1) $ydx=(x+y)\,dy$.

(2) $(1+e^x)\frac{dy}{dx}=1-ye^x$.

(3) $2z\left(\frac{du}{dx}\right)-x\left(\frac{du}{dy}\right)-(x+y)\left(\frac{du}{dz}\right)=0$.

LXXII.

1. Inscribe a circle in a given triangle. Also determine by geometrical constructions the centres of the escribed circles of a triangle, and prove that the circumscribing circle of a triangle is the nine-point circle of the triangle formed by the centres of its escribed circles.

2. Show that if the vertical angle of a triangle be constant, and the base given in magnitude and position, the bisecting line passes always through a fixed point.

3. Multiply $x^5+3x^3-x^2+1$ by x^2-8x+4, and divide x^6-2x^3+1 by x^2-2x+1 ; also resolve into factors the expressions—

$$(x^2+3x+1)^2-(x^2+x+1)^2 \text{ and } x^2-23x+132.$$

4. Find the greatest common measure, and also the least common multiple, of the expressions—

$$x^4-x^2-2x-1 \text{ and } x^4+2x^3+x^2-1.$$

If any numerical value be assigned to x, will the resulting numerical value of the algebraical greatest common measure be necessarily the greatest common measure of the numerical values of the expressions themselves ?

5. If n be a positive integer, prove that x^n-y^n is divisible by $x-y$, and that $(x+y+z)^{2n+1}-x^{2n+1}-y^{2n+1}-z^{2n+1}$ is divisible by $(y+z)\,(z+x)\,(x+y)$.

6. Solve the following equations:—

(1) $(x-1)(x-2)+2(x-2)(x-3)=3(x-3)(x-4)$.

(2) $4x^2-40x+99=0$.

(3) $(x-1)(x-2)+2(x-2)(x-3)=70$.

(4) $x^2+y^2=74$, $x+y=12$.

7. Having given the first term, and the common difference, find the sum of n terms of an arithmetic progression.

Also find the sums of n terms of a geometric progression of which the first two terms are given.

Find the sums of the series—

$1+3+5+\ldots$ to n terms,

$1^2+3^2+5^2+\ldots$ — — ,

and $1\times2+3\times4+5\times6+\ldots$ — — .

8. If α and β be the roots of the equation $ax^2+bx+c=0$, prove that $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$. Show that the equation of which the roots are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is $acx^2+(2ac-b^2)x+ac=0$.

9. Find an expression for the number of permutations of n things taken r together. In how many ways can a company of $2n$ soldiers be arranged, 1st, in one line, 2nd, two deep, a given half of the company being in the front rank and the other half in the rear rank?

10. Enunciate the binomial theorem, and write down the co-efficients of x^r in the expansions of—

$(1-x)^{-2}$, $(1-x)^{-n}$, and $(1+x+x^2+x^3+\ldots$ ad inf.$)^n$.

If n be a positive integer find the sum of the co-efficients in the expansion of $(1+x)^n$, and prove that the sum of the co-efficients of the odd terms is equal to the sum of the co-efficients of the even terms.

11. If from any angle of a triangle a straight line be drawn perpendicular to the base, the rectangle contained by the sides of the triangle is equal to the rectangle contained

by the perpendicular and the diameter of the circle circumscribing the triangle.

Having given n points on the circumference of a circle, show that $3.4.5 \ldots (n-1)$ polygonal figures of n sides can be formed, having the given points for vertices.

Show also that if, from any other given point on the circumference, perpendiculars be let fall upon all the sides of one of these poylgons, the product of the perpendiculars will be the same for all the polygons.

LXXIII.

1. If $x=4$, $y=3$, find the numerical value of

$x^4-4x^3y+6x^2y^2-4xy^3+y^4$, and show that for these values of x and y, $\sqrt[4]{x^3+x^2+1}=y$.

2. Reduce to their simplest forms the following expressions—

(1) $\dfrac{x^2-xy+y^2}{x-y}-\dfrac{x^2+xy+y^2}{x+y}$.

(2) $(a+b\sqrt{-1})^2\times(a-b\sqrt{-1})^2$.

3. Investigate a rule for finding what is called the least common multiple of two algebraical expressions. Find the least common multiple of $(x-a)$ and $x^{\frac{2}{3}}-2x^{\frac{1}{3}}a^{\frac{1}{3}}+a^{\frac{2}{3}}$.

Reduce to its lowest terms $\dfrac{acx^2-(ad+bc)x+bd}{a^2x^2-b^2}$.

4. Solve the following equations:

(1) $(x-1)^3+(x-2)^3+(x-3)^3=3(x-1)(x-2)(x-3)$

(2) $\left.\begin{aligned}\frac{12}{x}+\frac{12}{y}&=7.\\ \frac{8}{x}-\frac{9}{y}+\frac{11}{12}&=\frac{1}{x}-\frac{1}{y}.\end{aligned}\right\}$

(3) $(2x-1)(3x+1)-(x-1)(2x+1)=36.$

(4) $\left.\begin{aligned} x^2+y^2+z^2&=77. \\ xy+yz-xz&=26. \\ y+x&=11. \end{aligned}\right\}$

5. A person bought two farms, one of pasture the other of arable land, the latter exceeding the former by 20 acres, for the pasture land he paid £10 an acre more than he paid for the arable, and the whole cost of the pasture farm was £9,000 and of the arable £8,000. How many acres were there in each farm, and what price per acre was paid in each case?

6. Express algebraically the condition that one quantity shall vary directly as a second and inversely as a third. State any known geometrical example of such a law of variation.

Given that y is equal to the sum of two terms, one of which varies as $\frac{1}{x}$ and the other as x^2, and that when x is 1 or 2, y is equal to 7, find the equation between y and x.

7. A merchant transmits to a retailer a cask of spirits containing (a) gallons, the cask is consigned to three carriers in succession, the first when he receives it draws one fourth of a gallon from the cask and fills it up with water, the second when he receives it does likewise, and so does the third, find the ratio of the quantity of spirit in the cask to the quantity of water when the retailer receives it, and find the actual quantities of water and spirit remaining if $a=9$.

8. Without assuming the general expression for the sum of an arithmetic series, find the sum of (n) terms of the series—

$$\frac{a}{n^2}, \frac{2a}{n^2}, \frac{3a}{n^2}, \text{ \&c.}$$

There are x terms in arithmetical progression between 5 and 25, and the second term is $\frac{3}{7}$ths of the last term but one, express the terms of the series.

9. Find a geometrical mean between—

$$\left(1-\frac{2x}{3}+\frac{x^2}{9}\right) \text{ and } \left(1-\frac{4x}{3}+\frac{4x^2}{9}\right).$$

If a, b, c are in harmonical progression, prove that

$$a^3+c^3>2b^3.$$

10. Distinguish between permutations and combinations. Deduce from first principles the expression for the number of permutations of n things taken 4 at a time, and assuming the general form, show that the number of permutations of n things taken altogether when they all differ is double the number of permutations of n things taken together when 2 of them are alike.

11. Assuming the form of the expansion of a binomial, express the $(r+1)^{\text{th}}$ term of $(1+x)^{-n}$; show how to find the greatest term of $(1+x)^{-n}$ when x is a proper fraction and n an integer. Determine the place of the greatest term in the expansion of $(1+\frac{4}{7})^{-4}$.

LXXIV.

1. If a is a root of $f(x)=0$, show that the equation is divisible by $x-a$.

Resolve sin x into its factorials.

2. Solve the equations—

(1) $x^3+2x-12=0$.

(2) $x^4-6x^3+19x^2-14x+3=0$.

3. What is a determinant? Show that the product of two determinants is also a determinant.

4. Prove the following :—

(1) $\tan A = \tan (n\,180° + A)$.

(2) $\dfrac{1-\cos A}{1+\cos A} = \dfrac{2\sin A - \sin 2A}{2\sin A + \sin 2A} = \left(\tan \dfrac{A}{2}\right)^2$.

(3) $\tan 2A = \dfrac{2\tan A}{1-(\tan A)^2}$:

$$\tan 3A = \frac{3\tan A - (\tan A)^3}{1-3(\tan A)^2}.$$

5. From (3) in the preceding question deduce tan 18°.

6. Prove the following properties of a plane triangle, of which the angles are A, B, C, the sides are a, b, c, R is the radius of the circumscribed circle, and r, r_a, r_b, r_c are the radii of the inscribed and escribed circles, $2s = a+b+c$;

(1) $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = \dfrac{1}{2R}$.

(2) $\text{Area} = r\,s = r_a\,(s-a) = r_b\,(s-b) = r_c\,(s-c)$.

7. Given two sides and the included angle of a plane triangle, find the third side in a form adapted to logarithmic computation.

8. A measured line is drawn on a horizontal plane in a direction at right angles to the line joining one end to the base of a tower standing on the plane. The angles of elevation of the tower from the two ends of the measured line are 30° and 18° respectively. Find the height of the tower.

9. Prove that $2\cos\theta = \varepsilon^{\theta\sqrt{-1}} + \varepsilon^{-\theta\sqrt{-1}}$,

$2\sqrt{-1}\sin\theta = \varepsilon^{\sqrt{-1}} - \varepsilon^{-\theta\sqrt{-1}}$.

How is the former of these equivalents consistent with the fact that the sum of a number and its reciprocal is always greater than 2?

10. Find the equation of a straight line: and the length of the perpendicular distance from a given point on a given straight line.

Hence deduce the condition that

$$x \cos \alpha + y \sin \alpha - p = 0$$

should touch the circle $(x-a)^2+(y-b)^2=c^2$.

11. Find the condition that the lines whose equations are $Ax+By+C=0$, $A'x+B'y+C'=0$, $A''x+B''y+C''=0$ should meet in a point. Show that this condition is satisfied when $\frac{A}{B}=\frac{A'}{B'}=\frac{A''}{B''}$, and explain why.

12. Prove by means of the equation to a circle that the angle in a semicircle is a right angle, and that the angle in a segment greater than a semicircle is less than a right angle.

13. Find the equation to the tangent of an ellipse; prove that from a given point two tangents can be drawn to an ellipse; and that these tangents are at right angles to each other if the point is on a circle, concentric with the ellipse, of which the radius is $(a^2+b^2)^{\frac{1}{2}}$.

14. Find the equation to a normal of a parabola; and prove that the normal cuts the curve in a second point. Determine the co-ordinates of this second point, when the normal is drawn at the extremity of the latus rectum.

15. Define conjugate diameters of a conic; and prove geometrically that the sum of the squares of conjugate axes of an ellipse is constant.

16. Describe the conic whose equation is

$$A\alpha^2+B\beta^2+C\gamma^2=0.$$

LXXV.

1. Multiply $x^2+y^2+z^2-yz-zx-xy$ by $x+y+z$; and divide $x^5-124x^3+288x^2-81x-108$ by $x^2-11x+12$.

2. Prove that $(a^2-b^2)(c^2-d^2)=(ac\pm bd)^2-(ad\pm bc)^2$.

In how many ways can the product

$$(a^2-b^2)\,(c^2-d^2)\,(e^2-f^2)\,(g^2-h^2)$$

be written in the form of the difference of two squares?

Factorise the expressions, $(a^2+b^2)^2-4a^2b^2$ and x^6-y^6.

3. If $a=\frac{16}{9}$, $b=1$, $c=\frac{5}{3}$, prove that

$$(a+\sqrt{b})\,(\sqrt{a}-b)\,(\sqrt{a+b})=\frac{1}{3}\,\frac{c^3}{\sqrt{c^2-a}}.$$

4. Find the cube of $\frac{1}{2}(-1+\sqrt{-3})$, and the square root of $60+14\sqrt{11}$.

Prove that $\dfrac{3\sqrt{3}+2\sqrt{2}}{3\sqrt{3}-2\sqrt{2}}=\dfrac{35+12\sqrt{6}}{19}$.

5. Find the highest common factor, and the least common multiple of the expressions—

$$x^3-27x^2+196x-192 \text{ and } x^3-26x^2+180x-144.$$

6. If $\frac{x}{a}=\frac{y}{b}$, prove that each of these fractions $=\dfrac{x+y}{a+b}=\sqrt{\dfrac{xy}{ab}}$.

Prove also that $\left(\dfrac{x^2+y^2}{a^2+b^2}\right)^2=\dfrac{x^4+y^4}{a^4+b^4}$.

7. Solve the equations—

(α) $\dfrac{2x-\frac{5}{2}}{2}-\dfrac{2x-\frac{3}{2}}{5}=\dfrac{4x}{13}$.

(β) $\left.\begin{aligned}25x-11y&=126\\26x-13y&=117\end{aligned}\right\}$.

(γ) $132x^2-265x+132=0$.

(δ) $\left.\begin{aligned}x^2+\frac{16}{y^2}&=145\\x+\frac{4}{y}&=17\end{aligned}\right\}$.

8. A vessel is filled with a mixture of spirit and water in which there is 70 per cent. of spirit; 19 gallons are taken out, and the vessel is filled up again with water; the proportion of spirit is now found to be 56·7 per cent.; find how much the vessel contains.

9. Having given the first term (a), and the common difference (b), of an arithmetic series, find the sum of n terms.

Find the common difference of an arithmetic series, whose first term is unity, when the sum of n terms is equal to n^2.

Find the sum of 45 terms of the arithmetic series,

$$19+18\tfrac{1}{4}+17\tfrac{1}{2}+16\tfrac{3}{4}+ \ldots .$$

10. Prove that the sum of the roots of the quadratic equation,

$$ax^2+bx+c=0,$$

is equal to $-\frac{b}{a}$, and that the product of its roots is equal to $\frac{c}{a}$.

Prove also that the sum of the squares of the roots is equal to $\frac{b^2-2ac}{a^2}$.

11. Find an expression for the number of permutations of n things taken r together.

Determine n when the number of permutations of n things four together is fifteen times the number of combinations of $n+1$ things three together.

12. Write down the expressions for the coefficients of x^r in the expansions of $(1+x)^n$, $(1-x)^{-2}$, and $(1-x)^{-\frac{1}{2}}$.

Find the sums of the coefficients in the expansions of

$$(1+x)^6 \text{ and } (1+2x)^6.$$

LXXVI.

1. What in plane geometry is the limit of the magnitude of an angle? In what sense in analytical Trigonometry is the magnitude of an angle considered unlimited, and either positive or negative? Which of the ordinary trigonometrical functions are limited in magnitude and which unlimited?

If $\sin A=a$, express by a general formula all the angles which have a for their sine; if $\cos A=b$, prove $a^2+b^2=1$.

2. Find sin A and cos A in terms respectively of the sines and cosines of their supplements. Through what portions of the first four quadrants is the sine greater than the cosine in magnitude, and the tangent greater than the cotangent; when are they respectively equal and of the same sign?

Find sin 120°, tan 60°, cos 135°.

Prove $\sin A=\dfrac{\tan A}{\sqrt{1+(\tan A)^2}}$.

3. Prove $\cos(A+B)=\cos A\cos B-\sin A\sin B$, and deduce from the formula the corresponding expressions for sin (A+B), sin (A−B), and cos 2A.

4. Prove—

(1) $\dfrac{\sin A+\sin B}{\cos A+\cos B}=\tan\dfrac{A+B}{2}$.

(2) $\text{Cos } 72°=\dfrac{\sqrt{5}-1}{4}$.

(3) $\text{Sin A sin (A+2C)}+\sin B.\sin(B+2A)+\sin C.\sin(C+2B)=0$, if $A+B+C=180°$.

5. Explain the notation $\sin^{-1}s$, $\tan^{-1}t$.

Prove—

(1) $\text{Sin}^{-1}\dfrac{3}{5}+\sin^{-1}\dfrac{4}{5}=90°$.

(2) $\text{Tan}^{-1}\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}+\tan^{-1}\sqrt{\frac{(s-a)(s-c)}{s(s-b)}}$

$$+\tan^{-1}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=90^\circ$$

when $s=\frac{a+b+c}{2}$.

6. Show that $\frac{\text{arc}}{\text{radius}}$ may be properly taken for the measure of an angle; of what angle is 1·5708 the circular measure?

If θ be the circular measure of an angle less than a right angle,

Prove sin θ less than $\theta-\frac{1}{4}\theta^3$, and verify this when $\theta=\frac{\pi}{6}$.

7. In a plane triangle of which A, B, C are the angles, and a, b, c the sides subtending them,

Prove—

(1) $c=a\cos \text{B}+b\cos \text{A}$.

(2) $\frac{a}{b}=\frac{\sin \text{A}}{\sin \text{B}}$.

Deduce from these expressions the value $\cos \text{A}=\frac{b^2+c^2-a^2}{2bc}$.

Prove $a^2+b^2+c^2=2ab\cos \text{C}+2ac\cos \text{B}+2bc\cos \text{A}$.

8. Find the area of a triangle in terms of any two sides and their included angle, and also in terms of the three sides. Find the area of the greatest triangle that has for two of its sides 50 and 60 feet.

9. Given the difference between the angles at the base of a triangle 17° 48′, and the sides subtending those angles 105·25 feet and 76·75 feet: find the angle included by the given sides.

10. An observer in a balloon when it is one mile high observes the angle of depression of a conspicuous object on the horizontal ground to be 35° 20′; then, after ascending vertically and uniformly for 20 minutes, he observes the angle of depression of the same object to be 55° 40′: find the rate of ascent of the balloon in miles per hour.

11. A tower which stands on a horizontal plane is 200 feet high, and there is a small loophole in the tower at a certain height above the ground; an observer is at a horizontal distance from the tower of 300 feet, but stands on a mound so that his eye is 12 feet above the ground on which the tower stands, and in that position the angles subtended at his eye by the portions of the tower above and below the loophole are equal: find the height of the loophole from the ground.

12. In a circle which has a radius of 10 feet two chords AB, CD are drawn at right angles to each other, and intersecting in O, AO and CO are three and four feet respectively: find the sides and angles of the quadrilateral ABCD formed by joining the extremities of the chords.

LXXVII.

1. Prove that the three angles of every spherical triangle are greater than two and less than six right angles.

2. If a, b, and c be the sides of the spherical triangle ABC, opposite to the angles A, B, and C respectively, prove that

$$\cos a = \cos b \cos c + \sin b \sin c \cos A.$$

3. Enunciate Napier's rules for right-angled triangles. Prove them for the cases in which the side opposite to the right angle is the middle part.

4. Define the term *differential coefficient.*

Assuming that $lt_{x=\infty}\left(1+\frac{1}{x}\right)^x$ is e, prove that

$$lt_{y=0}\frac{e^y-1}{y} \text{ is } 1.$$

Deduce the differential coefficient of e^x.

5. Find $\frac{dy}{dx}$ in the following cases:—

(1) $y=\frac{(x^m-1)(x^n-1)}{x^{m+n}-1}$.

(2) $y=\tan^{-1}\frac{3a^2x-x^3}{a^3-3ax^2}$.

(3) $(x+a)^y=(y+b)^x$.

6. If $\sin^{-1} y=\log(1-x)$, prove that

$$(1-x)^2\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=0.$$

7. Write down Maclaurin's theorem for the expansion of functions of x in ascending powers of x.

Find the first five terms of the expansion of $\sin\log(1-x)$.

8. State fully the conditions that $f(x)$ shall be a maximum when $x=a$.

Prove that 1 is the least and $4\frac{13}{81}$ the greatest possible value of $3x^5-10x^4+10x^3-5x+3$.

If AB and AC be two fixed straight lines and PQ a line meeting AB and AC in P and Q, and moving subject to the condition that the area PAQ is constant, prove that when the length of PQ is a minimum AP and AQ will be equal.

9. Prove the following formulæ in the case of plane curves:—

$$\frac{1}{p^2}=\left(x\frac{dy}{ds}-y\frac{dx}{ds}\right)^{-2}=u^2+\left(\frac{du}{d\theta}\right)^2.$$

Prove that in the curve $r=\frac{a}{\theta}$ the polar subtangent is of constant length.

10. Show how to find the envelope of a series of curves of the form $f(x, y, \alpha)=0$ in which α is a variable parameter.

If x be the abscissa of a point on the envelope to the lines

$$x \cos 2\theta + y \sin 2\theta = 2a \cos \theta,$$

and if ϕ be the angle at which the normal to the envelope at that point is inclined to the axis of x, prove that $\cos \frac{\phi}{2}=\sqrt[3]{\frac{x}{2a}}$.

11. Integrate—

(1) $\sqrt{a^2-x^2}$. (2) $\frac{1}{a^2-x^2}$. (3) $\frac{1}{\text{versin } x}$.

12. Trace the curves—

(1) $a^2y^4=x^4(a^2-x^2)$.

(2) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

Find also the area of (1) and the length of (2).

13. Prove that the equations—

$$\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$$

represent a straight line through the origin, stating the geometrical meanings of l, m, and n.

Prove that the straight line $\frac{x}{5}=\frac{y}{7}=\frac{z}{6}$ bisects one of the angles between the straight lines—

$$\left.\begin{aligned}\frac{x}{2}=\frac{y}{3}=\frac{z}{4}\\ \frac{x}{3}=\frac{y}{4}=\frac{z}{2}\end{aligned}\right\}$$

14. Interpret the equation—

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=0.$$

Prove that the surface thus represented is met by the plane

$$x+4y+z=0$$

in the straight line

$$\frac{x}{2}=-y=\frac{z}{2}.$$

What is the geometrical interpretation of this?

15. Enunciate and prove Leibnitz's theorem for the differentiation of a product of functions.

Apply it to prove that if $y=e^{e^{mx}}$, then

$$\frac{d_y^{n+1}}{dx^{n+1}}=\left\{m^n+\frac{n}{1}.m^{n-1}\frac{dy}{dx}+\frac{n(n-1)}{1.2}m^{n-2}\frac{d^2y}{dx^2}+\&c.+\frac{d^ny}{dx^n}\right\}e^{mx}.$$

16. If p be the perpendicular from the origin upon the tangent at any point P of a plane curve and ϕ the inclination of this perpendicular to the fixed line in the curve, prove that the radius of curvature to the curve at P is $\frac{d^2p}{d\phi^2}+p$.

Hence prove that the radius of curvature at any point of the envelope of the lines in No. 10 is $\frac{3}{4}$ths of the perpendicular from the origin on the tangent to the envelope at that point.

17. Trace the curves :—

(1) $x^5-5ax^2y^2+y^5=0.$

(2) $\gamma=\frac{a}{\theta}+a\sin\theta.$

18. Prove that the plane

$$(b-c)^2\,x+(c-a)^2\,y+(a-b)^2\,z=0$$

touches the surface in No. 14 in the line

$$(b-c)\,x=(c-a)\,y=(a-b)\,z$$

for all values of a, b, and c.

Write down the equation of the tangent plane to the surface

$f(x, y, z)=0$ at the point x' y' z'.

LXXVIII.

1. In any spherical triangle prove that the sines of the angles are proportional to the sines of the opposite sides.

2. Enunciate Napier's rules for the solution of right-angled spherical triangles, and prove them for the cases in which one of the sides containing the right angle is the middle part.

3. If ϵ be the spherical excess of any spherical triangle, prove that $\cot\frac{\epsilon}{2}=\dfrac{\cot\frac{a}{2}\cot\frac{b}{2}+\cos C}{\sin C}$.

4. Find from definition the differential coefficients of x^n (n unrestricted) and of $\sin x$.

Differentiate

(1) $\left(\frac{a+x}{a-x}\right)^{\frac{3}{2}}$ (2) $(\log x)^x$ (3) $\cos(\tan^{-1} x)$.

5. Find the r^{th} differential coefficient of $x^n e^{mx}$, and prove that if

$$y^n-ny+(n-1)x=a, \text{ then } \frac{d^2y}{dx^2}-ny^{n-2}\left(\frac{dy}{dx}\right)^3=0.$$

6. *Write down* Taylor's expansion for $f(x)$, with the remainder after n terms.

Expand e^{e^x} to four terms, and find the first four terms of the expansion of y in terms of x in the equation

$$x^2+y^2-6x+4y+4=0.$$

7. *State* fully the conditions that $f(x)$ may be a maximum or minimum.

Apply to $3x^4+8x^3+6x^2+3$.

Upon the same circular base, and upon opposite sides of it, two right cones are constructed so that the volume of the whole figure thus formed may have a given value. Find the condition that the surface of the figure may be a minimum.

8. Find the equation of the tangent to the curve $y=f(x)$.

Apply to the curve

$$y=\frac{a}{2}\left\{e^{\frac{x}{a}}+\epsilon^{-\frac{x}{a}}\right\}$$

9. Find any expression for the radius of curvature at any point of a curve, and apply it to the curve in the last question.

10. Prove the formula $\frac{dA}{d\theta}=\frac{r^2}{2}$ where A is the sectorial area subtended by a curve at the origin.

If r be the distance of any point of an ellipse from one of the foci, and a be the semi-major axis, prove that $\int\sqrt{\frac{2a-r}{r}}ds$ along any arc is proportional to the sectorial area subtended by that arc at the other focus.

11. Integrate the expressions

(1) $\sqrt{a^2-x^2}$ (2) $\frac{1}{1+\tan x}$,

and find a formula of reduction for the integration of $(a^2+x^2)^{\frac{n}{2}}$.

12. Find the equation of a plane referred to three rectangular axes.

The planes $2x+3y+4z=a$

and $5x+4y+3z=b$

intersect the plane $x=c$.

Find the cosine of the angle between the lines of intersection.

13. Interpret the equations

$$(1)\ (x-a)^2+(y-b)^2+(z-c)^2=k^2\left\{(x-a)\,l+(y-b)\,m+(z-c)\,n\right\}^2$$

$$(2)\ (x-a)^2+(y-b)^2+(z-c)^2=\left\{x\,l+y\,m+z\,n\right\}^2$$

where $l^2+m^2+n^2=1$ in each case.

Prove that the surfaces thus represented intersect in plane curves, and find the equations to the planes of such intersection.

14. If the curve in question 8 have contact with a circle of radius r, whose centre is in the axis of y, and if the angle of inclination of the common tangent to the axis of z be 45°, prove that $a=\dfrac{r}{\sqrt{2}\log_e(1+\sqrt{2})}$.

Determine also whether the radius of curvature at the point of contact is greater or less than that of the circle.

15. How are solid angles measured?

Prove that the solid angle subtended by a square whose side is $2a$ at a point in the perpendicular to its plane through its centre and at a distance b from its plane is to the solid angle subtended by any closed surface surrounding the same point as $\sin^{-1}\dfrac{a^2}{a^2+b^2}$ is to π.

16. Find the asymptotes and any singular points of the curves

$$(1)\ y^2=\frac{ax^2(x-a)}{x^2-4a^2}$$

$$(2)\ r=a\frac{(3\theta-\pi)(5\theta-\pi)}{4\theta-\pi}$$

and trace the curves.

LXXIX.

1. Calculate, to four places of decimals, the value of the expression—

$$\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}.$$

2. Form the quadratic equation, the roots of which are $3\pm\sqrt{3}$, and the biquadratic equation, the roots of which are $\pm\sqrt{3}\pm\sqrt{5}$.

3. Define a continued fraction, and state the rule for the formation of its successive convergents.

Find the first four convergents of the fraction—

$$\frac{1}{2+}\ \frac{1}{3+}\ \frac{1}{4+}\ \frac{1}{5+}$$

Also express $\sqrt{17}$ in the form of a continued fraction.

4. Prove that a rational algebraic equation of an odd degree has at least one real root, and that an equation of an even degree with its last term negative has at least two real roots. What is the nature of the roots of the equation—

$$(x^2+2)(x^2-x+1)+2=0\ ?$$

Find the sum of the squares of the roots of this equation, and also the sums of the reciprocals and of the squares of the reciprocals of the roots.

5. Define the sine and cosine of an angle, and find the values of sin 30°, cos 30°, and cos 45°.

Find an expression for all the values of x which satisfy the equation, $\sin^2 3x=\frac{3}{4}$.

6. Prove the formulæ—

(1) $\sin \overline{A+B}=\sin A\cos B+\cos A\sin B$,

(2) $\sin A+\sin B=2\sin\frac{1}{2}\overline{A+B}\cos\frac{1}{2}\overline{A-B}$,

(3) $(\sin A+\cos A)(\tan A+\cot A)=\sec A+\operatorname{cosec} A$,

(4) $\sin 3A=3\sin A-4\sin^3 A$.

7. Find an expression for all the angles which have the same tangent as a given angle.

Prove that $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$,

and explain why it is that tan A has two values when tan 2A is given.

Find the value of tan 22° 30′.

8. Prove that in any triangle ABC, the sides of which are a, b, and c,

$$a = b \cos C + c \cos B.$$

Show also that

$$a^2 = b^2 + c^2 - 2bc \cos A,$$

and deduce expressions for $\cos \frac{A}{2}$, and $\sin \frac{A}{2}$, in terms of the sides.

9. Having given that, in a triangle ABC,

$$A = 30°,\ a = 3, \text{ and } c = 3\sqrt{2},$$

find the other parts of the triangle.

Explain the ambiguous case in the solution of triangles, and prove that the circumscribing circles of the two triangles are equal.

10. Find the area of a regular polygon of n sides, each side being equal to a.

Deduce the area of a circle of given radius, and find the areas of a sector of a circle, and of a segment of a circle.

11. State De Moivre's Theorem, and apply it to find the three cube roots of unity.

Expand sin θ in powers of θ, θ being in circular measure, and calculate approximately the value of sin 20″.

12. Write down the expansion of $\log_e(1+x)$ in powers of x, and prove that

$$\log 2 = \frac{1}{1\times 2}+\frac{1}{3\times 4}+\frac{1}{5\times 6}+\&c.$$

Calculate to five places of decimals the value of $\log_e(1{\cdot}02)$.

13. Explain what are meant by co-ordinates, x and y, of a point referred to rectangular axes, and define the locus of an equation connecting x and y.

What are the loci of the several equations—

(1) $x^2=a^2$, (2) $(x-a)^2+(y-b)^2=c^2$,

(3) $y^2=4ax$, (4) $(y^2-4ax)^2+(x-a)^2(x-2a)^2=0$?

14. Define the tangent to a curve, and find the equation to the tangent at the point (x, y) of the ellipse,

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

Prove either geometrically or analytically that the rectangle contained by the perpendiculars from the foci on any tangent to an ellipse is constant.

15. A straight line of given length slides with its ends on two fixed straight lines at right angles to each other; find the locus of any given point in the moving line.

LXXX.

1. In modern books on Trigonometry what ratios are expressed by the sine, tangent, and secant of an angle respectively? Formerly the Trigonometrical functions were defined as represented by straight lines; define the sine, tangent, secant, according to this method, and show how the two systems are made consistent with each other. Define the complement and supplement of an angle, and prove $\cos A=\sin(90°-A)=-\cos(180°-A)$.

What is the formula corresponding to

$$(\sin A)^2+(\cos A)^2=1$$

if the linear definitions of the sine and cosine be adopted?

2. Trace the sine and extreme values of the secant of an angle from 0 to 360°, find cos 30°, tan 45°, and secant 60°. What are the three least positive angles that each have $\sqrt{\frac{3}{2}}$ for their sine?

Given tan A= ·5, find sec A.

Given $\cos 36°=\frac{\sqrt{5}+1}{4}$ find sin 18°.

3. If A+B be an angle less than a right angle, prove by a geometrical construction

$$\cos(A+B)=\cos A\cos B-\sin A\sin B.$$

Exhibit also the construction when A and B are each less than a right angle, and (A+B) greater than a right angle.

Obtain the formula for sin (A−B) from the above value for cos (A+B).

4. Prove—

(1) $\tan 2A=\frac{2\tan A}{1-(\tan A)^2}$.

(2) $\frac{\sin A+\sin B}{\sin A-\sin B}=\frac{\tan\frac{A+B}{2}}{\tan\frac{A-B}{2}}$.

(3) $\tan 75°=2+\sqrt{3}$.

(4) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=45°$.

5. Prove generally—

$$\cos\frac{A}{2}+\sin\frac{A}{2}=\pm\sqrt{1+\sin A}$$

$$\cos\frac{A}{2}-\sin\frac{A}{2}=\pm\sqrt{1-\sin A}$$

and express with proper signs the values of

$$\cos\frac{A}{2},\ \sin\frac{A}{2},$$

derived from these formulas, when A lies between 270° and 360°.

From the above find sin 9° without reducing the surds.

6. In any plane triangle, if a, b, c are the sides subtending the angles A, B, C respectively,

prove (1) $\sin\frac{A}{2}=\frac{1}{2}\sqrt{\frac{(a+b-c)(a+c-b)}{bc}}$.

(2) $\tan B=\frac{b\sin C}{a-b\cos C}$.

Verify formula (1) if A be the greatest angle of a triangle whose sides are 30, 40, 50 feet.

7. Show that $\frac{\text{arc}}{\text{radius}}$ may be properly taken as the measure of an angle, find the angle which is the unit of measure when this mode of representing angles is referred to, and show also that the sine of a small angle is approximately equal to the circular measure of the angle itself.

If θ be the circular measure of an angle, show that the number of seconds it contains is equal to $\frac{\theta}{\sin 1''}$ nearly.

8. The sign of an angle defined as a ratio being less than unity, explain why the tabular logarithms of the sines of angles are expressed with whole numbers as characteristics. If the logarithm of the tangent of 18° be registered on the tables as 9·5117760, show what the tabular logarithm of cot 18° must be.

Find by the aid of the tables the value of A if $\cos A=\cot\alpha\tan\beta$ where $\alpha=75°\ 30'$, $\beta=28°\ 10'$.

9. In a triangle ABC, given A=35° 50′, BC=595 feet, AC=684 feet, show that the solution will be ambiguous, and determine the third side for the greater of the two triangles to which the data belong.

10. From the top of a cliff on the sea shore, 326 feet high, the angle of depression of a boat at sea was found to be 34° 30′; find the distance of the boat from the foot of the cliff.

11. An observer in a ship at sea sailing north observes two rocks, A and B, to bear 25° east of his course; he then sails in a direction north-west for four miles, and observes A to bear due east and B north-east of his new position. Find the distance from A to B.

12. If π be the ratio of the circumference of a circle to its diameter, point out any method by which the approximate value of π may be determined numerically, and if r be the radius of a given circle show that the area of the circle is equal to πr^2.

13. Find the radius of the circle circumscribing a given triangle in terms of its sides.

If ABCD be a quadrilateral that may be inscribed in a circle, AC a diagonal, show from the expression obtained above that the radius of the circle that would circumscribe the triangle ABC is equal to the radius of the circle that would circumscribe the triangle ACD, A and C being opposite angles of the quadrilateral.

LXXXI.

1. If a^x can be expanded in an infinite series of the form $1+Ax+Bx^2+$&c.; find A.

2. Deduce the expansion of $\log_e(1+x)$, and prove that

$$\log \frac{3}{2}=\frac{1}{2}-\left\{\frac{2}{1.3.2^3}+\frac{3}{2.5.2^5}+\text{\&c.}\right\}$$

writing down the r^{th} term of the series in brackets.

3. Prove De Moivre's Theorem for all forms of the index.

4. Find exponential expressions for sin θ and cos θ, and prove that

$$(\sqrt{-1})^{\sqrt{-1}} = e^{-\frac{\pi}{2}}.$$

5. Prove that in an equation with real coefficients imaginary roots occur in pairs.

The equation $x^4+4x^3+3x^2-2x-6=0$ has one root of the form $-1+\sqrt{-1}$; find all the roots.

6. Investigate the relation between the coefficients and roots of an equation.

If the roots of $x^3+px^2+qx+r=0$ be in arithmetical progression, prove that

$$2p^3-9pq+27r=0.$$

7. State and prove the use of the first derived function in determining the existence of equal roots.

If x and y be connected by the equation

$$x^5-5ax^2y^2+y^5=0,$$

prove that when $x=a^5\sqrt{108}$ there will be two equal values of y.

8. State and prove Descartes' rule of signs.

In the equation of the last question prove that—

(1) For every negative value of x there must be one positive value of y and four impossible values.

(2) For every positive value of x there must be one negative value of y and two impossible values.

9. In the parabola prove that the subnormal at any point is equal to half the latus rectum.

10. A circle APQ touches the parabola APR at its vertex A and cuts it in the point P.

At P a normal to the parabola is drawn, meeting the axis in G, and from G the straight line GQR is drawn perpendicular to the axis, and cutting the circle and parabola in the points Q and R respectively.

Prove that $RG^2=2.QG^2$.

11. Prove, either analytically or geometrically, that in any conic section the portion of the tangent intercepted between the point of contact and the directrix subtends a right angle at a focus.

12. In the ellipse the rectangle contained by the focal distances of any point is equal to the square on the conjugate semi-diameter.

13. Prove that every equation of the first degree represents a straight line.

Find the equations of the three diagonals of the complete quadrilateral formed by the straight lines whose equations are—

(1) $x=0$ (2) $y=0$ (3) $3x+2y=1$ (4) $2x+3y=1$.

14. Find the equation of the tangent at a given point of an ellipse.

If Y and Z be the feet of the perpendiculars from the focus of an ellipse, whose centre is C, upon the tangent and normal respectively at any point, prove that C, Z, Y are in one straight line.

LXXXII.

1. Find the centre of a circle cutting off three equal chords from the sides of a triangle.

2. From each angular point of a triangle a perpendicular is let fall on the opposite side; prove that the distance of the point of intersection of these perpendiculars from any one of the angular points is equal to twice the distance of the centre of the circumscribed circle from the side opposite to that angular point.

3. How much per cent. must a tradesman add on to the cost price of his goods that he may make 10 per cent. profit after allowing a customer a reduction of 5 per cent. on his bill?

4. Prove that the product of any four consecutive integers increased by unity is a perfect square.

5. Find the least number which, when expressed in the scale of which 3 is the base, has zero for its three right-hand digits; when in the scale of which 5 is the base, has two zero digits on the right; and when in the scale of which 7 is the base, has one zero digit on the right. Express this number in the duodecimal scale.

6. Two clocks start simultaneously from 12 o'clock; one gains and the other loses five minutes in an hour uniformly; after how long will they again show the same time?

7. Explain the ambiguous case in the solution of triangles.

Prove that in this case the circumscribing circles of the triangles are equal.

8. If $\cos^2\beta \tan(\alpha+\theta)=\sin^2\beta \cot(\alpha-\theta)$, prove that

$$\tan^2\theta=\tan(\alpha+\beta)\tan(\alpha-\beta).$$

9. Find the amount of a given sum in any time at compound interest.

Having given $\log 2=\cdot 301030$, and $\log 11=1\cdot 041392$, find the two consecutive years between which the amount will become double the principal, when the rate of interest is 10 per cent.

10. Investigate the law of formation of the successive convergents to a continued fraction.

Find the continued fraction equivalent to $7-\sqrt{7}$, and its first four convergents.

11. Find the sums of the series—

(α) $n^2+(n+1)^2+(n+2)^2+\ldots+(2n)^2$,

(β) $\frac{1}{2\times 5}+\frac{2}{5\times 8}+\frac{1}{8\times 11}+\ldots+\frac{1}{(3n-1)(3n+2)}$,

(γ) $1.2.3+2.3.4+3.4.5+\ldots+n(n+1)(n+2)$.

12. Having given one solution of the equation $ax+by=c$ in positive integers, find the general solution.

Find all the solutions, in positive integers, of the equation $7x+6y=151$.

13. Having given the probabilities of two independent events, find the probability that both will happen.

A man takes with one hand two coins from a bag containing five sovereigns and three shillings, and with the other hand he takes two coins from a bag containing four sovereigns and four shillings ; find the probability that he will take a sovereign and a shilling from each bag.

14. Resolve into partial fractions the expression—

$$\frac{2x-4}{(1-x^2)(1-2x)};$$

and find the coefficient of x, when it is expanded in a series of ascending powers of x.

LXXXIII.

1. Find the sums of the following series :—

(α) $1^2+2^2+3^2+ \ . \ . \ . \ +n^2$.
(β) $1\times2+2\times3+ \ . \ . \ . \ n\times(n+1)$.
(γ) $1.2.3+2.3.4+ \ . \ . \ . \ +n\,(n+1)\,(n+2)$.

2. Find the sum of n terms of the recurring series—

$$u_0+u_1x+u_2x^2+u_3x^3+ \ . \ . \ . \ ,$$

having given that, for every value of r greater than unity—

$$u_r-pu_{r-1}-qu_{r-2}=0.$$

Determine the scale of relation in the case of the series—

$$3+4x+13x^2+25x^3+ \ . \ . \ . \ ,$$

and find its general term.

3. Show that the convergents to a continued fraction, taken in order, are alternately less and greater than the continued fraction.

Find the continued fractions equivalent to

$$\frac{351}{251}, \ \sqrt{26}, \text{ and } 2+\sqrt{3}.$$

4. Resolve into partial fractions the expressions—

$$\frac{x+5}{(x^2-1)(x+4)}, \text{ and } \frac{x^2+5}{(x-1)^2(x^2+4)}.$$

5. Write down the expansions, in powers of x, of e^x and of

$$\log_e(1+x).$$

Prove that

$$\log_e m = 2\left\{\frac{m-1}{m+1}+\frac{1}{3}\left(\frac{m-1}{m+1}\right)^3+\frac{1}{5}\left(\frac{m-1}{m+1}\right)^5+\ldots\right\},$$

and employ this series to calculate $\log_e 2$ to three places of decimals.

6. A bag contains 4 white balls and 5 black balls, and 4 are taken out at random; find the chance that 2 will be white and 2 black. Find also the chance of taking 2 white of balls at least.

7. Prove the formula—

$$\tan \overline{A+B} = \frac{\tan A+\tan B}{1-\tan A \,.\, \tan B}.$$

Find $\tan \frac{A}{2}$ in terms of $\tan A$, and explain *à priori* why it is that there are two values.

Determine the value of $\tan 22° 30'$.

8. Prove that if R be the radius of the circle circumscribing the triangle ABC,

$$2R = \frac{a}{\sin A} = \frac{a+b+c}{\sin A+\sin B+\sin C}.$$

9. State and explain the ambiguous case in the solution of triangles.

Prove that in this case the circumscribing circles of the two triangles are equal.

10. If $a+\sqrt{b}$ be a root of a rational algebraic equation $f(x)=0$, prove that $a-\sqrt{b}$ is also a root.

Form the equation of the fourth degree, of which one of the roots is

$$\tfrac{1}{2}(\sqrt{3}+\sqrt{5}).$$

11. Show how to find the equal roots of an equation $f(x)=0$.

Solve the equation $x^3-6x^2+9x-4=0$, which has two equal roots.

12. Find the loci of the equations—

(α) $(x-a)(y-b)=0$.
(β) $(x-a)^2+(y-b)^2=0$.
(γ) $y^2-9xy+20x^2=0$.
(δ) $(x-a)(y-b)=c^2$.

13. Explain what is meant by the eccentric angle at any point of an ellipse.

Find the equation to the chord joining two points P, P′ of the ellipse $a^2y^2+b^2x^2=a^2b^2$, the eccentric angles of which are ϕ and ϕ'.

Deduce the equation to the tangent at P.

14. Find the locus of the point of intersection of tangents to an ellipse at right angles to each other.

15. Find the number of the divisors, and also the sum of the divisors, of any given number.

16. Find the sum of the series—

$\sin\theta+\frac{1}{2}\sin 2\theta+\frac{1}{3}\sin 3\theta+$ &c. to infinity.

17. Explain the method of divisors for determining the commensurable root of an equation, and apply it to the equation—

$$x^4-6x^3+47x-60=0.$$

18. Find the asymptotes of, and trace, the curve—

$$y^3-xy-2x^2-y+5x=6.$$

LXXXIV.

1. When m and n are whole numbers, express the result of the division of a^m by a^n; (1) when m is greater than n; (2) when m is less than n; and trace the steps in the theory of indices from which it is inferred that a^{-n} represents $\frac{1}{a^n}$ and $a^{\frac{1}{n}}$ represents $\sqrt[n]{a}$.

Express by an arithmetical fraction $(8^{\frac{2}{3}}+4^{\frac{3}{2}})\times 16^{-\frac{3}{4}}$.

2. Multiply $(x^{\frac{3}{2}}+2y^{\frac{3}{2}}+3z^{\frac{3}{2}})$ by $(x^{\frac{3}{2}}-2y^{\frac{3}{2}}+3z^{\frac{3}{2}})$.

3. Divide $(x+y)^2+(x+z)^2+(y+z)^2+2\ (x+y)\ (x+z)+2\ (x+y)\ (y+z)+2\ (x+z)$ by $(x+y+z)$.

4. Express $a^2\ (c-b)+b^2\ (a-c)+c^2\ (b-a)$ as the product of three simple binomial factors.

5. Reduce to its lowest terms

$$\frac{(a^2-b^2)\ (x^2-y^2)-4abxy}{(a^2-b^2)\ (x^2+y^2)+2\ (a^2+b^2)xy}.$$

6. Prove $\dfrac{a}{(a-b)\ (c-a)\ (x+a)}+\dfrac{b}{(a-b)\ (b-c)\ (x+b)}$

$$+\frac{c}{(b-c)\ (c-a)\ (x+c)}=\frac{x}{(x+a)\ (x+b)\ (x+c)}$$

7. Show how to find a factor which will rationalise the surd quantity $a^{\frac{1}{p}}+b^{\frac{1}{q}}$, where p and q are any whole numbers.

Express $\dfrac{5^{\frac{1}{2}}-7^{\frac{1}{4}}}{5^{\frac{1}{2}}+7^{\frac{1}{4}}}$ by an equivalent fraction with a rational denominator.

8. Solve the following equations—

(1) $(x-a)\ (x-b)=(x-c)\ (x-d)$;

(2) $\left.\begin{aligned} &\frac{3}{x}+\frac{5}{y}=\frac{8}{15} \\ &9y-22x=\frac{3xy}{25} \end{aligned}\right\}$

(3) $\frac{x^2}{6}-\frac{1}{7}=\frac{43x-8x^2}{42}$;

(4) $\left.\begin{aligned} &x^2(x+y+z)=108 \\ &y^2(x+y+z)=192 \\ &z^2(x+y+z)=300 \end{aligned}\right\}$

9. What are the roots of a quadratic equation? If α and β are the roots of the equation $x^2-px+q=0$, and α^4 and β^4 the roots of $x^2-Px+Q=0$; express P and Q in terms of p and q.

10. Two workmen, A and B, of unequal efficiency, working together are capable of making a drain, which they undertake to do, in 60 days. After working for 20 days A falls ill, and his place is supplied by another workman, C, whose efficiency is the same as that of B, and it is found that the whole time consumed in making the drain is 80 days. In how many days would A or B working alone make the drain?

11. A cubical tank contains 512 cubic feet of water. It was required to enlarge the tank, the depth remaining the same, so that it should contain seven times as much water as before, subject to the condition that the length added to one side of the base should be four times the length added to the other side. Find the sides of the new rectangular base.

12. Define the three progressions Arithmetical, Geometrical, Harmonical; and show that the three equations following severally determine for the three quantities, a, b, c, the conditions of each progression:

(1) $\frac{a-b}{b-c}=\frac{a}{a}$; (2) $\frac{a-b}{b-c}=\frac{a}{b}$;

(3) $\frac{a-b}{b-c}=\frac{a}{c}$;

Insert four harmonic means between $\frac{1}{3}$ and 2.

A watch, which is set right at noon, gains two minutes the first hour afterwards, three the second, four the third, and so on; after what interval will the watch be an hour and a half fast, and what time will it then indicate?

13. Assuming the expression for the number of permutations of n things taken r together, find the expression for the number of combinations of n things taken r together.

A dealer has for sale 8 bay, 7 grey, and 5 black horses. A purchaser requests that 12 horses, four of each colour, may be sent him; in how many different ways can the dealer execute the order?

14. Assuming the form of the expansion of a binomial when the index is a positive integer, and that $f(m)$ denotes the series $1+mx+\frac{m(m-1)}{1.2}x^2+$ &c. for any value of m; show how to obtain a proof of the theorem when the index is a negative integer.

Since $(1-x)^n(1-x)^{-n}=1$, it would follow that if the two expanded series for $(1-x)^n$ and $(1-x)^{-n}$ were multiplied together all the coefficients of the different powers of x should vanish separately; show that this is the case for the whole coefficient of x^4.

LXXXV.

1. If two straight lines be at right angles to the same plane, they shall be parallel to one another.

2. If two straight lines be cut by parallel planes, they shall be cut in the same ratio.

3. If two straight lines do not intersect, and along one of them AB, BC, CD, &c. be taken all equal to one another, and along the other *ab*, *bc*, *cd*, &c. all equal to one another, the straight lines A*a*, B*b*, C*c*, &c. shall be all parallel to a certain plane.

4. Define a logarithm, and prove that—

$$\log_a N = \frac{\log_b N}{\log_b a}.$$

Prove that if $\log_a b = m$ and $\log_b a = n$, then

$$\left(\frac{\log_a m}{\log_b n}\right)^2 = \frac{m}{n}.$$

5. Find the expansion of $\log_e (1+x)$ in ascending powers of x.

Prove that the coefficient of x^n in the expansion of $\log (1+x+x^2)$ is $\frac{-2}{n}$ or $+\frac{1}{n}$ according as n is or is not a multiple of 3.

6. Prove that

$$\cos \alpha + \sqrt{-1} \sin \alpha = e^{\alpha\sqrt{-1}}.$$

Hence prove that

$$\cos\left(\frac{\pi}{4} + \theta\sqrt{-1}\right) = \frac{1}{2\sqrt{2}}\left\{(e^{\theta}+\epsilon^{-\theta}) - \sqrt{-1}\,(e^{\theta}-\epsilon^{-\theta})\right\}.$$

7. If x be the circular measure of an angle, prove that

$$x = \tan x - \frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} - \&c.$$

Prove that

$$3^{\frac{1}{2}} - 3^{-\frac{1}{2}} - \frac{1}{3}\left(3^{\frac{3}{2}} - 3^{-\frac{3}{2}}\right) + \frac{1}{5}\left(3^{\frac{5}{2}} - 3^{-\frac{5}{2}}\right) - \&c. = \frac{\pi}{6}.$$

8. Find an expression for $\cos^n\theta$ in terms of cosines of descending multiples of θ when n is an even integer.

9. Prove that the foot of the perpendicular from the focus on any tangent to a parabola lies on the tangent at the vertex.

A long rectangular slip of paper is folded so that one of the corners always lies on the opposite side. Prove that the crease always touches a parabola of which the aforesaid side is the directrix.

10. In the ellipse prove that if CP and CD be conjugate semi-diameters

$$CP^2+CD^2=CA^2+CB^2.$$

11. In the hyperbola the foot of the perpendicular from the focus on the tangent lies on the circle whose diameter is the transverse axis.

What portion of this circle corresponds to either branch of the hyperbola?

12. Show how to pass from one set of rectangular axes to another without changing the origin.

Find the angle through which the axes must be turned in order that the equation

$$Ax+By+C=0,$$

may be reduced to the form $x=$constant, and find the value of this constant. What is the geometrical meaning of this result?

13. Find any form of the equation of the ellipse, and interpret the constants.

A number of ellipses are described upon the same major axis; prove that the locus of the extremities of their latera recta is a parabola.

14. Find the equation of the tangent at any point of an hyperbola, and deduce that of the normal.

If the hyperbola be equilateral, prove that the portion of the normal intercepted between the axes is bisected by the curve.

LXXXVI.

1. Prove that $(y+z-x)^2+(z+x-y)^2+(x+y-z)^2=4(x^2+y^2+z^2)-(x+y+z)^2,$

and that $(a^2+b^2)(c^2+d^2)=(ac\pm bd)^2+(ad\mp bc)^2.$

In how many ways can the expression

$$(a^2+b^2)(c^2+d^2)(e^2+f^2)$$

be written in the form of the sum of two squares?

2. Solve the equation—

$$6x^2-89x+323=0.$$

If α and β be the roots of the equation $ax^2+bx+c=0$, prove that $\alpha(\alpha+\beta)=-b$, and $\alpha\alpha\beta=c$.

Find, in terms of a, b, and c, the values of the expressions $\alpha^2+\beta^2$, and $\alpha^3+\beta^3$.

3. Sum the series—

$1+3+5+7+\ .\ .\ .\ .$ to n terms,
$1+3+9+27+\ .\ .\ .\ .$ to n terms,
and $1^2+2^2+3^3+4^2+\ .\ .\ .$ to n terms.

4. Write down the coefficients of x^3, and of x^r, in the expansions of $(1-x)^{-2}$, and $(1-x)^{-\frac{1}{2}}$.

If $\dfrac{n\,(n-1)\,(n-2)\ .\ .\ .\ (n-r+1)}{1.2.3.\ .\ .\ .\ .\ .\ r}={}_nC_r$, prove that ${}_{n+1}C_r={}_nC_r+{}_nC_{r-1}$.

5. Express $\dfrac{200}{119}$, and $\sqrt{10}$ in the form of continued fractions, and find the first four convergents to each of the continued fractions.

6. Define the sine, and the cosine, of an angle; and prove the formulæ—

$$\sin A=\sin(180^\circ-A),$$

$$\sin(A+B)=\sin A\cos B+\cos A\sin B.$$

Find the values of sin 15° and sin 75°.

7. Prove that in any triangle ABC, of which a, b, c are the sides—

$$b^2+c^2-a^2=2bc\cos A.$$

Find the angles of a triangle, the sides of which are 6 inches, 3 inches, and $3\sqrt{3}$ inches.

8. If R be the radius of the circumscribing circle of a triangle ABC, prove that—

$$2R=\frac{a}{\sin A}.$$

State the *ambiguous case* in the solution of triangles, and prove that, in that case, the circumscribing circles of the two triangles are equal.

9. Define the logarithm of a number to a given base, and find the logarithms of 64 to the base 2, and of 128 to the base $\frac{1}{2}$.

Prove that $\log (m \times n) = \log m + \log n$.

10. Explain how probability is measured. A bag contains five sovereigns and ten shillings; if four coins be taken out at random, find the chance that two sovereigns and two shillings will be taken out.

11. Prove that an equation of an odd degree has at least one real root; and that an equation of an even degree with its last term negative has at least two real roots of contrary signs.

Can the equation—

$$x^6 + 3x^4 + 4x^2 + 5 = 0$$

have any real roots?

12. Find the equation to a straight line referred to rectangular axes.

Determine the equation to the straight line which passes through the point (1, 2) and is perpendicular to the line $y = x + 1$.

13. Define a parabola, and find its equation in the form $y^2 = 4ax$.

Show that the equation to the tangent at the point (x', y') is—

$$y - y' = \frac{2a}{y'}(x - x');$$

and that if p be the length of the perpendicular upon it from the vertex—

$$p^2(a + x') = ax'^2.$$

14. Prove that the equations—

$$\frac{x}{a^2}+\frac{y^2}{b^2}=1, \text{ and } \frac{x^2}{a^2+c^2}+\frac{y^2}{b^2+c^2}=1,$$

represent ellipses having the same foci.

Show that the ellipse, $x^2+4y^2=4$, and the hyperbola $x^2-2y^2=2$, have the same foci; find the co-ordinates of their point of intersection, and draw figures of the two curves.

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15. If a, b, and c be positive integers, prove that one solution of the equation $ax-by=c$ can always be found such that x and y are positive integers, and hence find the general solution in positive integers. Prove also that the successive values of x are in an arithmetical progression of which b is the common difference.

Find the least solution in positive integers of the equation—

$$15x-19y=10.$$

16. Find all the solutions, in positive integers, of the equation—

$$xy-2x-y=8.$$

17. Find the sums of the series—

(α) $1\cdot n+2\cdot(n-1)+3\cdot(n-2)+\ \ldots\ +n\cdot 1.$

(β) $1+\frac{2}{2}+\frac{3}{2^2}+\frac{4}{2^3}+\frac{5}{2^4}+\ \ldots$ to infinity.

18. Find the number of cannon balls in a pyramidal pile: 1st. When the base is an equilateral triangle having n balls in a side; 2nd. When the base is a square having n balls in a side. Hence show that a square pile is equivalent to two triangular piles.

19. If the fraction $\frac{x^2-7x+13}{x^2-7x+12}$ be expanded in ascending powers of x, find the coefficient of x^2 and of x^r.

20. A sum of money is distributed amongst a certain number of persons. The second receives one shilling more

than the first, the third two shillings more than the second, the fourth three shillings more than the third, and so on. If the first person receives one shilling and the last £3 7*s.*, what is the number of persons, and the sum distributed?

21. If an event may happen in different independent ways, prove that the probability of its happening is the sum of the probabilities of its happening in the different ways.

Find the chance of throwing 6 with two dice. Find also the odds against throwing 6 twice at least in three throws with two dice.

LXXXVII.

1. Resolve each of the expressions—

$$x^4-5a^2x^2+4a^4 \text{ and } x^3-2ax^2-a^2x+2a^3$$

into their simplest factors, and hence find their highest common measure.

Does the highest common measure of two algebraical quantities always give the greatest common measure of the numbers which result when particular numbers are given to the algebraical symbols?

2. Solve the equations—

(1) $\frac{6x}{7}-\frac{7-3x}{8}=25+\frac{x-21}{5}$.

(2) $\left.\begin{aligned}\frac{xy}{x+4}&=y-1\\ \frac{xy}{x-3}&=y+1\end{aligned}\right\}$.

(3) $\frac{x-3a}{3x-a}+\frac{3x-a}{x-3a}=3$.

3. Prove that x^4-12x has the same value whether x is $1-\sqrt{2}$ or $-1+\sqrt{-4}$.

4. If a, b, and c are real quantities, prove that no real values of x and y can satisfy the equation

$$ay-bx=c\sqrt{(x-a)^2+(y-b)^2}$$

unless c^2 is less than a^2+b^2.

5. If $x^2-\frac{1}{x^2}=y$, express $\frac{1+x^4}{1-x^4}$ in terms of y.

6. A given sum of money, $£a$, is to be distributed in equal shares among a number of persons. If there had been n more persons to receive, each would have had b shillings less for his share. How many receivers were there?

7. Find the first term and the common difference of the arithmetical progression wherein the seventh, eighth, and ninth terms together make 15, and the third and fifth terms together make 2.

8. A bag contains two sovereigns along with shillings and pence, and the value of its contents is two guineas and a half. The value of the shillings is a geometrical mean (proportional) between the values of the sovereigns and the pence. How many pence are there in the bag?

9. Expand $(a^{\frac{1}{3}}-x)^{\frac{3}{2}}$ in ascending powers of x to four terms by the Binomial Theorem, and prove that no coefficient in the whole expansion can be greater than $\frac{3}{2}$.

10. To inscribe a square in a given circle.

Prove that a square is the largest figure of four sides which can be inscribed in a given circle.

11. If a straight line parallel to the side BC of the triangle ABC cuts the sides BA, CA produced through A in the points D,E respectively, prove that BD : DA :: CE : EA.

12. Define compound ratio.

Equiangular parallelograms have to one another the ratio which is compounded of the ratios of their sides.

13. Prove that every equilateral polygon inscribed in a circle must be equiangular. Is the converse also true?

LXXXVIII.

1. A dealer bought $3\frac{1}{2}$ hundredweight of sugar for seven guineas; what did the sugar cost per pound? At what price per pound must it be sold, so that when all is disposed of the profit may be £2 9*s.*?

2. If the weight of a sovereign be 5 dwts. $3\frac{1}{2}$ grains, what is the value of 3 oz. 1 dwt. 15 grains of standard gold?

3. If $\frac{13}{15}$ of a ship's cargo be worth £170 6*s.*, what is the whole cargo worth? What fraction of another cargo which is worth £235 16*s.* will have the same value as the whole of the first cargo?

4. The net rental of a freehold estate is £365 15*s.*; what must a purchaser pay for the estate so as to obtain $3\frac{1}{2}$ per cent. per annum on his purchase money?

5. On the first of July 1871 the price of a ton of coal was 20*s.*, on the 1st of July 1872 the price of a ton of the same coal was 33*s.*; at what rate per cent. has the price increased in the year? If the annual consumption of a householder had been 66 tons at the lower price, to what should his consumption be reduced at the higher price that his annual expenditure on coal may be unchanged?

6. Reduce to their simplest forms—

(1) $\dfrac{\frac{99}{198}+\frac{198}{297}}{\frac{55}{66}-\frac{13}{39}}$.

(2) $\dfrac{1\cdot3+\cdot39}{\cdot001+\cdot0003}$.

7. Find $\left(\dfrac{\cdot064}{\cdot08}\right)^2$. Extract the square root of 25·70591401.

8. If $\log_{10}2=\cdot3010300$, write down the logarithms of 20 and ·02.

Find by the aid of the tables a mean proportional to $(\cdot0456)^3$ and $(4\cdot72)^{\frac{2}{3}}$.

9. If a French mètre contain 3·28089 English feet, find by the aid of the tables the length in English yards of the radius of a circle the area of which is 78·54 square mètres, the ratio of the circumference of a circle to its diameter being 3·1416 to 1.

10. If the straight line joining the vertices of two triangles, described on the same base, bisect the base at right angles, each of the triangles will be isosceles.

11. All the angles made by any number of straight lines meeting in a point are together equal to four right angles.

Enunciate the proposition in Euclid's First Book, of which the above is a corollary, and *prove* that all the interior angles of any rectilineal figure together with four right angles are equal to twice as many right angles as the figure has sides.

Each angle of a regular polygon contains $\frac{6}{5}$ of a right angle; what is the number of sides of the polygon?

12. Show that a square is the greatest rectangle that can be inscribed in a given circle.

LXXXIX.

1. Prove that every plane section of a sphere is a circle. Assuming the earth to be a sphere, find the distance between two places which have the same latitude and different longitudes, (1) along the great circle which joins them, (2) along their parallel of latitude.

2. Prove that in a spherical triangle—

$$(1)\ \frac{\sin A}{\sin a}=\frac{\sin B}{\sin b}=\frac{\sin C}{\sin c}.$$

$$(2)\ \tan\frac{a+b}{2}\cos\frac{A+B}{2}=\cos\frac{A-B}{2}\tan\frac{c}{2}.$$

3. If the arcs forming the sides of a spherical triangle are small relatively to the radius of the sphere, prove that

each of the angles exceeds the corresponding angle of the plane triangle whose sides are of the same length as the arcs forming the sides by one third of the spherical excess.

4. What do you mean by finding the differential of a quantity or of a function? and show how the Differential and Integral Calculus enable you (1) to draw a tangent to a curve; (2) to find the area of any surface plane or curved.

Find the differentials of

$$(1)\ \frac{f(x)}{\phi(x)}; \qquad (2)\ (a^2-x^2)^{\frac{1}{2}}; \qquad (3)\ a^x.$$

5. Find $\frac{d^2y}{dx^2}$ in the following cases; when

$$(1)\ y=\log\frac{x+\sqrt{x^2-a^2}}{x-\sqrt{x^2-a^2}}; \qquad (2)\ y=(\sin x)^x.$$

6. Prove Maclaurin's Theorem; and deduce Taylor's Theorem from it.

Expand y in ascending powers of x when

$$(1)\ y=a\sin\left(x+\frac{\pi}{4}\right)+b\cos\left(x+\frac{\pi}{4}\right);$$

$$(2)\ y-b-xa^y=0.$$

In what cases is Maclaurin's Theorem said to fail?

7. What forms of expression are indeterminate? How are the true values determined? Evaluate the following—

$$(1)\ \frac{x^4-6ax^3+8a^2x^2-2a^3x-a^4}{(x-a)^2}, \text{ when } x=a.$$

$$(2)\ (\tan^{-1}x)^{\tan x} \text{ when } x=0.$$

8. Find the equation to the tangent of the curve $y=f(x)$; and the length of the perpendicular from the origin on the tangent when the curve is referred to (1) rectangular co-ordinates, (2) polar co-ordinates.

9. Find the position of that normal of an ellipse the perpendicular on which from the centre is a maximum.

10. Explain a general method of finding asymptotes to plane curves. Trace the following curves—

(1) $y^3=2ax^2-x^3$. (2) $y^2=x^2\dfrac{x-a}{x+a}$.

11. How are points of inflexion determined in curves referred to polar co-ordinates? Trace the curve $r\theta^{\frac{1}{2}}=a$, showing the position of its rectilinear asymptote and of its point of inflexion.

12. Find the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at the point $x=y$; and also prove the evolute to be a semi-cubical parabola.

13. Find the integrals of

(1) $\sec 2x\,dx$. (2) $x^2\sin^{-1}x\,dx$.

(3) $\dfrac{x^n dx}{\sqrt{a^2-x^2}}$. (4) $\dfrac{(x^2-a^2)dx}{x^3-5ax^2+6a^2x}$.

14. What is a definite integral? Find the length and the included area of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

15. Prove that the sum of the projections of the sides of a closed figure on any straight line is zero.

16. Find the equation to a plane; and the equations to the line of intersection of the two planes—

$$x\cos\alpha+y\cos\beta+z\cos\gamma=p.$$
$$x\cos\alpha'+y\cos\beta'+z\cos\gamma'=p'.$$

XC.

1. If $\dfrac{a}{\alpha}=\dfrac{b}{\beta}=\dfrac{c}{\gamma}=\ldots$, prove that each of these fractions is equal to $\dfrac{la-mb+nc-\ldots}{l\alpha-m\beta+n\gamma-\ldots}$; and also equal to

$$\frac{\{a^2+b^2+c^2+\ldots-2ab-2ac-2bc-\ldots\}^{\frac{1}{2}}}{\{\alpha^2+\beta^2+\gamma^2+\ldots-2\alpha\beta-2\alpha\gamma-2\beta\gamma-\ldots\}^{\frac{1}{2}}}.$$

2. Explain the method of indeterminate coefficients, and apply it (1) to the expansion in ascending powers of x, to five terms, of $\frac{1+2x}{1-3x+5x^2}$; and (2) to the summation of the series

(1) $1.2+2.3+3.4+\ldots+n(n+1)$;

(2) $1^3+2^3+3^3+\ldots+n^3$.

3. Define chance odds in favour of an event. In throwing with two dice what is the most likely number to be thrown? What is the chance of throwing that number? and what are the odds in its favour as against 4?

4. Give some general method for solving a biquadratic equation; and solve by it $x^4-6x^3+8x^2-2x-1=0$.

Also determine the condition that two roots of an equation should be equal to one another. Apply it to determine whether two roots of the preceding equation are equal.

5. Define sine, tangent, secant, versine, of an angle; and show that all angles of the form $n360°\pm\alpha$ have the same secant, n being an integer.

6. Prove the following equivalents:—

(1) $\frac{\cos B-\cos A}{\cos B+\cos A}=\tan\frac{A+B}{2}\tan\frac{A-B}{2}$;

(2) $(\sin A)^2-(\sin B)^2=\sin(A+B)\sin(A-B)$;

(3) $\sin(60°+A)=\sin A+\sin(60°-A)$;

(4) $\sin 3A\sin^3 A+\cos 3A\cos^3 A=\cos^3 2A$.

7. Prove that in a plane triangle ABC, of which a, b, c are the sides, and R is the radius of the circumscribed circle,

(1) $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=\frac{1}{2R}$;

(2) $\frac{\sin B+\sin C-\sin A}{\sin B+\sin C+\sin A}=\tan\frac{B}{2}\tan\frac{C}{2}$;

(3) $(b-c)\cot\frac{A}{2}+(c-a)\cot\frac{B}{2}+(a-b)\cot\frac{C}{2}=0$.

8. A tower seventy feet high stands on the top of a hill, and at a point a quarter of a mile down the hill subtends an angle equal to that of the slope of the hill; determine the angle of the slope, and the vertical depth descended in the quarter of a mile.

9. Prove the following :—

$$(1)\ e^x=1+\frac{x}{1}+\frac{x^2}{1.2}+\frac{x^3}{1.2.3}+;$$

$$(2)\ \log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-;$$

and explain the principle and the formation of the tables of proportional parts.

10. Determine the distances of the centres of the escribed circles of a triangle from the centre of the inscribed circle.

11. If θ is the circular measure of an angle, prove that $\sin\theta=\theta=\tan\theta$, when θ is infinitesimal. Hence prove that in all cases—

$$\sin\theta=\theta-\frac{\theta^3}{1.2.3}+\frac{\theta^5}{1.2.3.4\cdot5}-.$$

Apply this series to the approximate determination of the value of π.

12. Prove that an equation of the form $Ax+By+C=0$ represents a straight line. What geometrical quantities are represented by A, B, C ?

If A and B are constant, and C varies, what property is common to all the lines ?

13. Find the equation to the tangent of the circle $y^2=2ax-x^2$, and show that the perpendicular from the origin on the tangent is equal to the abscissa.

14. Prove either analytically or geometrically that the subnormal of a parabola is equal to half the latus rectum, and that the subtangent is equal to twice the abscissa.

15. Define conjugate diameters of an ellipse; and having given the co-ordinates of the extremity of one diameter, find those of the extremity of its conjugate diameter.

Hence prove that the sum of the squares of a pair of conjugate diameters is constant.

16. Prove that by change of origin and of co-ordinate axes the equation of the second degree may always be reduced to the form

$$y^2 = 4mx + nx^2.$$

XCI.

1. Find the value of a ton of sugar at fourpence farthing the pound.

2. There are six silver spoons, of which each weighs 7 oz. 4 dwt. 4 grains. What is the excess of their weight together over the weight of another set of spoons which is 3 lbs. 6 dwt. ?

3. In a large institution where there is always the same number of inmates to be fed, the contract price of meat rises 20 per cent., and the daily allowance of each person is at the same time reduced from 9 oz. to 8 oz. If the yearly charge for meat is thenceforward £597, what was it before the changes were made?

4. A string 67 feet long is cut up into lengths of $6\frac{1}{4}$ inches each. How many of these lengths are formed, and how many inches of string remain over?

5. Multiply £2 7*s.* $4\frac{3}{4}$*d.* by 14·6.

6. After an income tax of threepence in the pound has been deducted, an income becomes £448 6*s.* 6*d.* What was the income before it was reduced by the taxation?

7. Find the square root of ·00367236.

8. Compute by logarithms the value of $\frac{\cdot 010378}{(\cdot 103)^7}$ to the nearest integer.

9. Employ logarithms to find to the nearest yard the length of a side of a square whose area is 1,275 acres.

10. If two triangles have two sides of the one equal to *two* sides of the other, each to each, but the angle contained

by the two sides of one of them greater than the angle contained by the two sides equal to them of the other, the base of that which has the greater angle shall be greater than the base of the other.

11. In a triangle in which one angle is obtuse, the square on a side subtending an acute angle is less than the squares on the sides containing that angle, by twice the rectangle contained by either of those sides and the straight line intercepted between the perpendicular let fall on it from the opposite angle and the acute angle.

12. When does a straight line touch a circle? If a straight line touch a circle, the straight line drawn from the centre to the point of contact shall be perpendicular to the line touching the circle.

13. A, B are the centres of two circles which touch each other externally in C. A straight line touches the circles at points D and E respectively. If EB and DC be joined, these straight lines when produced will meet in the circumference of the circle of which B is the centre.

XCII.

1. If the square inscribed in one circle be equal to the square described about another circle, compare the radii of the circles.

2. Multiply $(x^2-5ax+6a^2)$ by $(x^2+5ax+6a^2)$.

Reduce

$$\left\{(x^2+2y^2)^2-(x^2-2y^2)^2\right\}\times\left\{(2x^2+3y^2)^2-(2x^2-3y^2)^2\right\}.$$

Divide $x^3-(a-b+c)x^2-(ab-ac+bc)x+abc$ by $x^2-(a+c)x+ac$.

3. If $x^2-px+q=0$ be a quadratic equation, whose roots are a and b, find p and q in terms of a and b, find also a and b in terms of p and q.

Solve the equations—

(1) $$\left.\begin{aligned} \frac{5y-3x}{9}-\frac{2x-3y}{5}&=2+\frac{x}{30} \\ \frac{x}{4}-\frac{y}{5}&=2 \end{aligned}\right\}.$$

(2) $(x-2)(x-3)+(x-3)(x-4)=32.$

4. If A, B, C, be three angles of the hexagon taken in order, and the angles A and C be joined by the straight line AC, prove the square of AC equal to three times the square of the radius of the circle.

5. If an algebraical expression X is a divisor of each of the two algebraical expressions A and B, prove that X is a divisor of A−B.

Reduce to its simplest form—

$$\frac{x^2-px-2p^2}{x^2-(2p+q)x+2pq}-\frac{x^2+px-2p^2}{x^2+(2p+q)x+2pq}.$$

6. State and prove the property in proportion known as 'ex æquali.' Give a geometrical example of its application in any proposition of Euclid.

If four magnitudes of the same kind are proportionals, show that the greatest and least together are greater than the other two.

7. Solve the following equations :—

$$\left.\begin{aligned} x-2y+3z&=9 \\ x^2+4y^2+9z^2&=189 \\ 3xz&=4y^2 \end{aligned}\right\}.$$

There are two chests of mixed tea, in one the green is mixed with the black in the ratio of 3 to 5, in the other in the ratio of 5 to 11. What number of pounds must be taken from each chest so as to form another mixture which shall contain exactly 19 pounds of green tea and 37 pounds of black ?

8. There are two similar rectilineal figures, the area of one being four times the area of the other. In what ratio are their homologous sides?

9. There are seven terms of an arithmetic series, of which -4 is the first, and 26 the last term. Find the intermediate terms of the series.

Show that a decimal which recurs from the decimal point represents a geometrical series continued ad infinitum, and that the vulgar fraction which is equivalent to the decimal is the limit of the sum of that series.

Find the harmonic mean between $\frac{2a}{a+b}$ and $\frac{2a}{a-b}$.

10. If a regiment, all Scotch, contain n men, and another regiment, all Irish, contain n_1 men, find the number of combinations that can be formed out of the two regiments, each combination containing r Scotchmen and r_1 Irishmen.

11. Assuming that the product of the five factors $(x+a)$, $(x+b)$, $(x+c)$, $(x+d)$, $(x+e)$ is of the form $x^5+Px^4+Qx^3+Rx^2+Sx+T$, where P, Q, R, &c., represent the combinations of a, b, c, d, e, taken one and one, two and two, three and three, &c., together, show what those coefficients become when $a=b=c=d=e$.

Write down the 12th term of $\frac{1}{(1-x)^3}$ and express the middle term of $(a^2-x^2)^{10}$ without giving the preceding terms of the expansion.

12. Given the area of a right-angled triangle inscribed in a circle, and the rectangle contained by the segments of the base of the triangle, find the radius of the circle.

XCIII.

1. Prove that if m and n be positive integers,

$$a^m.a^n=a^{m+n}.$$

Assuming that this equation is true for all values of m and n, deduce interpretations of a^0, a^{-1}, and $a^{\frac{2}{3}}$.

2. Divide $x^5-10a^2x^3+7a^3x^2+14a^4x-24a^5$ by $x^2-ax-4a^2$, and $x^3+y^3+z^3-3xyz$ by $x+y+z$.

Resolve $x^4+a^2x^2+a^4$ into two quadratic factors.

3. Prove that—

$$n(n+1)(n+2)(n+3)+1=(n^2+3n+1)^2,$$

and simplify the expressions—

(1) $\left(bc+ca+ab\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-abc\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)$.

(2) $\frac{y+z-x}{(z-x)(x-y)}+\frac{z+x-y}{(x-y)(y-z)}+\frac{x+y-z}{(y-z)(z-x)}$.

4. Define the least common multiple of two algebraical expressions; and investigate a rule for finding it.

Find the highest common factor, and the least common multiple of the expressions—

$x^4+3x^3-5x^2+6x-14$, and $x^4+7x^3-x^2+14x-6$.

5. Solve the equations—

(α) $\frac{1}{x+a}+\frac{1}{x-c}=\frac{1}{x-a}+\frac{1}{x+c}$.

(β) $(x-1)(x-2)(x-3)+(x-3)(x-4)(x-5)$
$=2(x-1)(x-4)^2$.

(γ) $5x^2-144x+891=0$.

(δ) $\left.\begin{array}{l}5x^2+14y=150\\14x^2+5y=249\end{array}\right\}$.

6. If $a:b::c:d$, prove that

$$ma+nb:ma-nb::mc+nd:mc-nd$$

and that $a^2+c^2:b^2+d^2::ac:bd$.

7. Having given the first term, a and the common difference, b of an Arithmetic Series, find the sum of n terms.

If S_n represent this sum, prove that the sum of the series, $S_1+\frac{1}{2}S_2+\frac{1}{3}S_3+\ldots+\frac{1}{n}S_n$, is equal to the sum of n terms of a series, of which a is the first term, and $\frac{b}{2}$ the common difference.

8. Show how to express a given whole number in any proposed scale.

Transform the number 648972 into the scale of which 7 is the radix.

Find in what scale the number 729 is represented by 10404.

9. If ${}_nC_r$ represent the number of combinations of n things taken r at a time, prove that

$$ {}_nC_r = {}_nC_{n-r} $$

and that

$$ {}_{n+1}C_r = {}_nC_r + {}_nC_{r-1}. $$

Out of nine consonants and four vowels, how many words can be formed, each containing two consonants and one vowel?

10. Prove that a quadratic equation cannot have more than two roots.

Form the quadratic equation the roots of which are—

$$ \tfrac{1}{2}(11 \pm \sqrt{-5}). $$

11. What is the price of eggs per dozen when four more in a half-crown's worth lowers the price one penny per dozen?

12. Write down the expressions for the coefficients of x^r in the expansions of $(1+x)^n$, and $(1-x)^{-n}$.

Find the 10th term in the expansion of $(1-x)^{-\frac{1}{2}}$, and the middle term in the expansion of $\left(x+\frac{1}{x}\right)^{12}$.

XCIV.

1. Prove that the sum of the squares on the three straight lines, drawn from the angles of a triangle to the points of bisection of the opposite sides, is equal to three-fourths of the sum of the squares of the sides of the triangle.

2. Given one side of a triangle, the angle opposite to that side and the point at which the inscribed circle touches that side; construct the triangle.

3. Semicircles BDA and CEA are described upon the sides BA and AC of the right-angled triangle BAC, and another semicircle BFAGC is described upon the hypotenuse BC;

Prove that the sum of the areas of the two curvilinear figures BDAF and CEAG is equal to the area of the triangle ABC.

4. If $a=xy^{p-1}$, $b=xy^{q-1}$, $c=xy^{r-1}$,
prove that $a^{q-r}\,b^{r-p}\,c^{p-q}=1$;
and if $2s=a+b+c$,
prove that $(s-a)^3+(s-b)^3+(s-c)^3+3abc=s^3$.

5. If x be a possible quantity, prove that

$$\frac{2x^2+6x+3}{2x+1}$$

must be either less than 1 or greater than 3.

6. If the m^{th} term of a harmonic progression be equal to n, and the n^{th} term be equal to m, prove that the $(m+n)^{\text{th}}$ term is equal to $\frac{mn}{m+n}$.

7. Expand $\frac{1}{(1-x)^2}$ in ascending powers of x, and prove that if x be equal to $\frac{n+1}{n}$, the expression will be equal to *the sum* of the first n terms of the expansion.

8. If n be an even number, prove that the sum of the series—

$$1+2^2+3^2+4^2+\&c.$$

to n terms is—

$$\frac{n(n+1)(2n+1)}{6},$$

and find the sum of n terms of the series—

$$\frac{1}{1\cdot 6}+\frac{1}{6\cdot 11}+\frac{1}{11\cdot 16}+\&c.$$

9. Find all the solutions in positive integers of the equation—

$$xy+x+y=35.$$

10. Express $\sqrt{11}$ in the form of a continued fraction, and find the values of—

$$\frac{1}{2-\frac{1}{2-\frac{1}{2}}},\ \&c.,\ \text{ad infinitum}.$$

11. Three men toss a guinea in succession on the condition that he who first tosses heads keeps the coin;

Prove that the value of the expectation of the man who begins to toss is 12*s*.

12. A debt is to be paid by equal annual instalments extending over 20 years.

If the rate of interest be 5 per cent., prove that each annual payment is just a very little more than 8 per cent. of the debt.

13. If α and β be two values of θ satisfying the equation—

$$\frac{\cos\theta}{a}+\frac{\sin\theta}{b}=\frac{1}{c},$$

prove that $a\cos\frac{\alpha+\beta}{2}=b\sin\frac{\alpha+\beta}{2}=c\cos\frac{\alpha-\beta}{2}$.

14. Equilateral triangles DBC and EBC are described

upon opposite sides of BC, one of the sides of the triangle ABC.

Prove that $AD^2+AE^2=AB^2+AC^2+BC^2$.

15. If in the triangle ABC the angle A be three times the angle B, prove that

$$\sin B=\frac{1}{2}\sqrt{\frac{3b-a}{b}}.$$

16. Prove that—

$$\log (x+y\sqrt{-1})=\frac{1}{2}\log (x^2+y^2)+\tan^{-1}\frac{y}{x}\sqrt{-1}.$$

XCV.

1. If a straight line CD be parallel to a plane, prove that the intersection of any plane passing through CD with the given plane will be a straight line parallel to CD.

2. The base of a right pyramid is a regular hexagon inscribed in a circle whose radius is 10 inches; find the ratio of the contents of the pyramid and of a cone having the circle for its base, the altitudes both of the cone and pyramid being 12 inches.

3. In any plane triangle of which A, B, C, are the angles, and (a), (b), (c), the sides subtending them;

Prove—

(1) $c^2=(a+b)^2\sin^2\frac{C}{2}+(a-b)^2\cos^2\frac{C}{2}$.

(2) $\sqrt{bc\sin B\sin C}=\frac{b^2\sin C+c^2\sin B}{b+c}$

4. If $\left(x+\frac{1}{x}\right)=2\cos\theta$, and $y+\left(\frac{1}{y}\right)=2\cos\phi$, prove—

(1) $x^m+\frac{1}{x^m}=2\cos m\theta$;

(2) $x^my^n+\frac{1}{x^my^n}=2\cos(m\theta+n\phi)$.

5. Investigate the expression—

$$\sin\theta=\theta\left(1-\frac{\theta^2}{\pi^2}\right)\left(1-\frac{\theta^2}{2^2\pi^2}\right)\left(1-\frac{\theta^2}{3^2\pi^2}\right),\ \&c.$$

and hence prove $\pi=2\cdot\frac{2^2}{1\cdot3}\cdot\frac{4^2}{3\cdot5}\cdot\frac{6^2}{5\cdot7}\cdot\frac{8^2}{7\cdot9},\ \&c.$

6. Find the sum of either of the two following series—

(1) $\sin\theta+\sin2\theta+\sin3\theta+,\ \&c.,+\sin n\theta,$

(2) $\cos\theta+\cos2\theta+\cos3\theta+,\ \&c.,+\cos n\theta.$

7. If A be the vertex of a parabola, S the focus, P any point on the curve, T the point where the tangent at P meets the axis produced, PQ perpendicular to the directrix; Prove—

(1) PT bisects the angle SPQ;

(2) the quadrilateral SPQT is a rhombus.

8. Prove that the perpendiculars drawn from the foci of an ellipse on a tangent at any point intersect the tangent in the circumference of the circle described on the major axis.

If a tangent be drawn at the extremity of the latus rectum of the ellipse, find the lengths of the perpendiculars upon this tangent from the foci.

9. If a right cone be cut by a plane which meets the cone on both sides of the vertex, the section is an hyperbola.

10. Given the equations to two straight lines referred to rectangular co-ordinates, find the angle at which they intersect.

Prove that the equation—

$$3y^2-8xy-3x^2-29x+3y-18=0$$

represents two straight lines at right angles to each other.

11. Prove that the equation—

$$5y^2+5x^2+2xy-12x-12y=0$$

represents an ellipse; find its axes and determine its area.

12. When is an hyperbola said to be equilateral? Find the eccentricity of such an hyperbola. In a rectangular hyperbola, the distance of any point of the curve from the centre is a mean proportional between the distances of the same point from the foci.

XCVI.

1. Add together $3\frac{3}{4}$, $4\frac{5}{8}$, and $5\frac{7}{8}$.
2. Subtract $14\frac{5}{6}$ from $21\frac{1}{3}$.
3. Multiply together $21\frac{2}{5}$, $4\frac{5}{28}$, $\frac{4}{45}$, and $1\frac{5}{107}$.
4. Divide $1\frac{29}{35}$ by $1\frac{19}{21}$.
5. Add together 15·7, 4·092, ·0075, and 3·6505.
6. Subtract 72·0975 from 73·332.
7. Multiply 22·48 by 1·125.
8. Divide 917·3245 by 65·29.
9. Express 1 oz. 5 grs. as a decimal fraction of 3 oz. 4 dwts. 16 grs.
10. Add together $13\frac{1}{3}$ of $\frac{41\frac{1}{2}}{2}$, $11\frac{1}{5}$ of $5\frac{1}{3}$, $1\frac{1}{14}$ of $9\frac{1}{10}$, and $\frac{3\frac{1}{2}-2\frac{3}{4}}{\frac{5}{7}}$.
11. Subtract $\frac{9}{80}$ of $3\frac{3}{4}$ from $\frac{5\frac{5}{24}}{4\frac{4}{9}}$.
12. Multiply together $5\frac{4}{55}$, $1\frac{1}{24}$ of $1\frac{89}{115}$, and $2\frac{13}{17}$ of $\frac{55}{63}$.
13. Divide $2\frac{7}{69}$ by $\frac{29}{161}$.
14. Add together 1·1375 of a fathom, ·875 of a yard, 2·965 of a foot, and 9·75 of an inch, expressing the answer in feet and a decimal fraction of a foot.
15. Multiply together 2·56, 1·75, and ·000125.
16. Divide ·273 by ·0028.
17. Reduce 15 cwts. 2 qrs. 21 lbs. to the decimal of 2 tons 10 cwts.
18. Divide 10 miles 1 furlong 6 poles by 22.
19. An estate of 625 acres 1 rood 10 poles is bought for £7,878 18*s.* 9*d.* What is the average price per acre?

20. Find by practice the cost of 3,257 tons of coals, at £1 3*s.* 7½*d.* per ton.

21. In what time will the simple interest on £2,987 10*s.* 6*d.* amount to £248 19*s.* 2½*d.*, at 3½ per cent. per annum?

22. If 7 horses draw 5½ tons along 56 yards on a road in 10 minutes, how many horses will be required to draw 11 tons along 50 yards in 5 minutes?

23. Find the difference between the simple and compound interest on £933 6*s.* 8*d.* in 2½ years at 2½ per cent. per annum.

24. Find the cube root of 10·218313.

If the diagonal of a square is 3 ft. long, find the length of each side, correct to a thousandth of an inch.

25. Compute by means of the tables the value of

$$\frac{25}{27}\sqrt[3]{34\cdot3}\div\sqrt{14\cdot4}.$$

26. If a sum of £1,000 is lent on the condition that it shall bear compound interest at the rate of 5 per cent. per annum for the first 5 years, 10 per cent. per annum for the next 5 years, and afterwards 15 per cent. per annum; what will the debt amount to in 20 years?

XCVII.

1. 'The product of two algebraical quantities having like signs is positive, and having unlike signs negative.' State briefly the steps by which this rule is obtained, and show its arithmetical correctness when in the product obtained for $(a-b)\times(c-d)$, $a=12$, $b=9$, $c=10$, $d=4$.

If x and y differ in value, explain why

$$(x-y)^2=(y-x)^2.$$

Show how the rule of signs stated above leads to the introduction of impossible or imaginary quantities in algebra.

2. Multiply—

$$(1+x+2x^2)^2-(1-x-2x^2)^2$$

by $(1+x-2x^2)^2-(1-x+2x^2)^2$.

Find the product of

$$x^2-(a+b)\,x+ab \text{ by } x^2+(a-b)\,x-ab,$$

and examine what the product becomes if in it either a or b be substituted for x.

3. Divide $x^4-2bx^3-(a^2-b^2)\,x^2+2a^2bx-a^2b^2$ by $x^2-(a+b)\,x+ab$.

Reduce to its simplest form—

$$\frac{(x-y)(y-z)+(x-y)(z-x)+(y-z)(z-x)}{x(z-x)+y(x-y)+z(y-z)}.$$

4. If m be a whole number, prove that $x+y$ is divisible by $x^{\frac{1}{m}}+y^{\frac{1}{m}}$, when m is odd.

Write down the three last terms of the quotient of $x+y$ divided by $x^{\frac{1}{2n+1}}+y^{\frac{1}{2n+1}}$, and examine how many terms the quotient will contain.

5. Find the fraction in its lowest terms which is the square root of—

$$\frac{1+4x^{\frac{1}{2}}-2x-12x^{\frac{3}{2}}+9x^2}{1-4x^{\frac{1}{2}}+6x-4x^{\frac{3}{2}}+x^2}.$$

6. Solve the following equations:

(1) $(x-4)(x-8)(x-10)=(x-6)(x-7)(x-9)$.

(2) $5x^2-14x=3$.

(3) $2cx^2-abx+2abd=4cdx$.

(4)
$$\left.\begin{aligned}(x+y)^2-z^2&=65\\ x^2-(y+z)^2&=13\\ x+z-y&=9\end{aligned}\right\}.$$

7. Form a quadratic equation whose roots are $-5\pm6\sqrt{-1}$.

If $x^2-px+q=0$ and $x^2-p_1x+q_1=0$ have a common root, determine it, and in that case show that—

$$(q-q_1)^2=(p-p_1)(p_1q-pq_1).$$

8. A bill of £63 5*s.* was paid in sovereigns and half-crowns, and the number of coins used in the payment was 100, how many sovereigns were paid, and how many half-crowns ?

9. A cask A is filled with 50 gallons of water and a cask B with 40 gallons of brandy ; C gallons are drawn from each cask, mixed, and replaced. The same operation is repeated. Find C when there are $8\frac{7}{8}$ gallons of brandy in A after the second replacement.

10. Find the sum of n terms of the series

$$(a-b)+(a-3b)+(a-5b), \&c.,$$

without assuming a formula for the sum of an arithmetic series, and apply the result to find the sum of 10 terms of $76+70+64+$, &c.

11. The sum of the two first terms of a geometric series is 12, and of the three first terms is 39, find the series, and determine if the condition is satisfied by more than one series.

Find the sum of the infinite series—

$$1+r+(1+b)r^2+(1+b+b^2)r^3+, \&c.,$$

r and b being proper fractions.

12. Show how to transfer a whole number from one scale of notation to another, and prove that if r be the radix of the scale, the number itself will be divisible by $r+1$ if the difference between the sum of the digits in the odd places and the sum of the digits in the even places be divisible by $r+1$.

13. Find the number of different permutations of n letters taken altogether when one letter occurs p times and another q times in each permutation.

If the n letters contain only a and b, and n be even, show that the number of permutations will be greatest when $p=q=\frac{n}{2}$.

14. Assuming the form of the binomial theorem, find the greatest term of $(1+x)^n$ when n is a positive integer and x is also positive. Apply the result to the determination of the greatest term of $(1+\frac{5}{2})^6$, and express the value of the greatest term.

Show by actual multiplication that the whole coefficient of x^4 in the product of the expansions of $(1+x)^n$ and $(1-x)^n$ is equal to the coefficient of x^4 in the expansion of $(1-x^2)^n$.

XCVIII.

1. Define the degree and the unit of circular measure, and show how to pass from one of these units to the other.

If the radius of a circle be 25 feet, find to four decimal places the length of the arc subtending an angle of 3°, taking π to be 3·1416.

2. Define the principal trigonometrical ratios, and trace the changes in sign and magnitude of $\frac{\sin 3\theta}{\cos 2\theta}$ as θ varies from 0 to $\frac{\pi}{2}$.

3. Prove that $\sin (A+B)=\sin A \cos B+\cos A \sin B$ where A and B are positive angles each less than 90°.

Assuming that the formula holds for all values of the angles, deduce that for $\cos (A-B)$.

4. Find $\cos 3A$ and $\tan 3A$ in terms of $\cos A$ and $\tan A$ respectively.

5. Prove that for all values of A—

(1) $2 \operatorname{cosec} 4A+2 \cot 4A=\cot A-\tan A$;

(2) $\dfrac{\cos A+\cos 3A}{\cos 3A+\cos 5A}=\dfrac{1}{2 \cos 2A-\sec 2A}$.

6. Prove that—

$$\tan^{-1}m+\tan^{-1}n=\cos^{-1}\frac{1-mn}{\sqrt{(1+m^2)(1+n^2)}}.$$

7. Find sin 18°.

If cos A=tan B and cos B=tan A;
Prove that cos A=2 sin 18°.

8. Prove the formula—

$$a^2=b^2+c^2-2bc \cos A,$$

and apply it to prove that if the straight line which bisects the vertical angle of a triangle also bisects the base, then the triangle must be isosceles.

9. Find the area of a triangle in terms of the sides.

10. Find the radius of the circle which touches one side of a triangle and the two other sides produced.

If ABC and DEF be two triangles with the same perimeter, prove that the radius of the escribed circle touching the side BC of the first is to the radius of the described circle touching the side EF of the second as $\tan \frac{A}{2}$ is to $\tan \frac{D}{2}$.

11. Given two sides b and c, and the included angle A of a triangle, find a formula for determining the side a, without the previous determination of the angles B and C, adapted to logarithmic computation.

Apply to the case of b=354 yds., c=426 yds., A=49° 16′.

12. A building on a square base ABCD has two of its sides AB and CD parallel to the bank of a river.

An observer standing on the river's bank in the same straight line with DA finds that the side AB subtends, at his eye, an angle of 45°.

Having walked a yards along the bank he finds that the side DA subtends an angle whose sine is $\frac{1}{3}$.

Prove that the length of each side of the building is $\frac{a}{\sqrt{2}}$ yards.

XCIX.

1. Construct a triangle of which the sides shall be equal to three given straight lines, but any two whatever of these must be greater than the third.

How would the process fail if the last condition were not fulfilled ?

2. Parallelograms on the same base and between the same parallels are equal to each other.

Show that if two triangles have two sides of the one equal to two sides of the other, each to each, and the sum of the two included angles equal to two right angles, the triangles are equal.

3. In a right-angled triangle the square on the side subtending the right angle is equal to the sum of the squares on the sides containing the right angle.

Show how to construct a straight line, the square on which shall be any given multiple of a given square.

4. If a straight line be bisected and produced to any point, the rectangle contained by the whole line thus produced, and the part produced, together with the square on half the line bisected, is equal to the square on the straight line which is made up of the half and the part produced.

5. The angle at the centre of a circle is double of the angle at the circumference on the same base that is on the same arc.

6. If two straight lines cut one another within a circle, the rectangle contained by the segments of the one shall be equal to the rectangle contained by the segments of the other.

In a given straight line AB, find a point O such that the rectangle contained by the segments AO and OB shall be equal to a given rectangle not greater than the square on half of AB.

7. Describe a circle about a given triangle.

Lines are drawn parallel to one side of a triangle

cutting the other sides or the other sides produced. Show that if circles be described about the new triangles so formed, their centres all lie in a straight line, and find the angles which this line makes with the two sides of the original triangle.

8. If the vertical angle of a triangle be bisected by a straight line which also cuts the base, the segments of the base shall have the same ratio which the other sides of the triangle have to one another.

If ABC be the triangle, A the vertical angle, BE and CE the segments of the base, and if circles be described about the triangles ABE, ACE, the diameters of these circles are to each other in the same ratio as the segments of the base.

9. If four straight lines be proportionals, the rectangle contained by the extremes shall be equal to the rectangle contained by the means.

On a given straight line describe an isosceles triangle equal to a given triangle.

10. Similar triangles are to each other in the duplicate ratio of their homologous sides.

11. Given a point O in the line AB, find two other points C, D, such that a line OP given in direction shall bisect the angle CPD, and the segments CO, OD, shall bear a given ratio to each other.

C.

ARITHMETIC.

1. What would be the cost of painting the four walls of a room whose length is 24 ft. $8\frac{1}{2}$ in., breadth 16 ft. $3\frac{1}{2}$ in., and height 11 ft. 6 in., at 4*s.* a square foot?

2. Reduce to a simple fraction—

$$\frac{2\frac{1}{2}+1\frac{1}{4}-2\frac{1}{8}}{3\frac{3}{4}+2\frac{1}{3}-4\frac{1}{4}} \times \frac{\frac{9}{49}-\frac{4}{25}}{\frac{75}{121}-\frac{16}{27}} \div \frac{\frac{3}{7}+\frac{2}{5}}{\frac{5}{11}+\frac{4}{9}}.$$

3. By selling a horse for £82 5*s.* a person lost $6\frac{1}{2}$ per cent.; how much per cent. would he have lost or gained if he had sold the horse for £90 10*s.*?

4. Find within an inch the diagonal of a square field containing two acres.

5. Multiply, by duodecimals, 10 ft. 5 in. 3 pts. by 9 ft. 7 in. 10 pts. What does the product become when expressed in square inches and a fraction of a square inch?

6. A person invests £2500 in the 9 per cent. Japanese Loan, which was issued at 98 per cent. at $1\frac{1}{2}$ discount; how much stock will he have, and what rate of interest will the investment give?

7. Find the difference between the interest and the discount on £2598 15*s.* for 3 years at 5 per cent. simple interest.

8. Express 4 lbs. 3 oz. 12 dwts. 6 grs. troy weight as a decimal of a pound avoirdupois.

9. Reduce ·2671875 to a vulgar fraction, and if the unit be £3 reduce the fraction to shillings, pence, and decimals of a penny.

10. A wall 5 times as high as it is broad, and 8 times as long as it is high, contains 18225 cubic feet. Find the breadth of the wall.

11. A person had £6900 stock paying 3 per cent. in s

company, and was offered the choice of being paid off at par, or of receiving £110 of a new 2½ per cent. stock for every £100. He chose the former alternative, and invested his money in the 3 per cent. Consols at 92. Find the amount of his stock in Consols, and the excess of his income above what it would have been if he had agreed to the proposed conversion.

12. Find the prime factors of 111540, 296352, 404352; and thence write down all the numbers which will divide them all without remainder, and the smallest number which they will all divide without remainder.

13. If 56 cubic feet 1044 cubic inches of timber are required to floor a room 29 ft. 3 in. by 25 ft. 4 in., what is the thickness of the boards ?

14. A tradesman starts with a capital of £960, and after 3 years takes another into partnership with £2100. After 4 years more the whole profits amount to £2304. How ought this to be divided between them ?

15. Extract the square root of 2854·7649.

16. Extract the cube root of 1194389981.

17. Multiply, by the method of duodecimals, 6 ft. 7 in. 5 pts. by 8 ft. 3 in. 10 pts.

18. Express the result obtained in the last question in square feet, square inches, and a fraction of a square inch.

19. A tradesman's annual losses during 5 years average 1¼ per cent. on the capital with which he began, and at the end of the 5 years his effects are worth £2531 5*s*. What capital did he begin with ?

20. A person sells out of the 3 per cent. Consols at 99, and invests in Exchequer Bills, bearing interest at the rate of 2½*d*. a day per cent., when the bills are at a premium of 7*s*. 6*d*. What effect has this on his income ?

21. In the month of December last the number of paupers in a certain union was 336, the number of women being double that of the men, and the children being as

many as the men and women together. If a man cost $\frac{1}{3}$ more than a woman, and 3 children as much as a man and a woman together, and the whole cost for the month be £83 6*s*., how much is the daily cost of each man, woman, and child?

22. In 1858 the value of £100 tithe rent-charge, reckoned on the average price of corn in the 7 years immediately preceding, was £105; in 1859, reckoned in the like way, it is 3 per cent. more. If it were reckoned on the price in 1851 only, it would be but £69. What would it be if reckoned on the price in 1858 only?

23. Show that $\sqrt{2}$ lies between $\frac{17}{12}$ and $\frac{41}{29}$.

24. Extract the cube root of 669·921875 cubic feet, and reduce the result to inches.

25. What is the present worth of £1842 15*s*., payable a quarter of a year hence, at 5 per cent.?

26. What length of paper $22\frac{1}{2}$ inches wide would be required to paper the walls of a room 18 ft. 9 in. long, 13 ft. 3 in. broad, and 14 ft. 6 in. high?

27. Express 4 acres 2 roods 16 perches as the decimal of a square mile.

28. Multiply, by duodecimals, 7 ft. 3 in. 5 pts. by 5 ft. 7 in. 4 pts., and the product by 4 ft. 2 in. What does the product become when expressed in cubic feet and inches?

29. A man purchases £700 stock in the 3 per cent. Consols at $94\frac{1}{2}$, and also invests £585 in the purchase of Russian 5 per cent. stock at $97\frac{1}{2}$. How much stock has he standing in his name? If he sells out of the 3 per cents. at 95, and out of the 5 per cents. at $96\frac{1}{4}$, does he gain or lose by the transaction, and how much?

30. Divide 91·8$\dot{6}\dot{3}$ by 87·5$\dot{6}$.

31. A tradesman's stock in trade is valued on January 1st, 1868, at £8000, he has also £350 in cash and owes £1870; during the year his personal expenses, £300, are paid out of the proceeds of his business, and on January *1st, 1869,* his stock is valued at £7950, he has £570 in

cash and owes £1510. What is the whole profit on the year's transactions after deducting 5 per cent. interest on the capital with which he began the year?

32. A grocer buys coffee at the rate of £8 10*s.* per cwt. and chicory at £2 10*s.* per cwt., and mixes them in the proportion of 5 parts of chicory to 7 of coffee. At what rate must he sell the mixture so as to gain £16$\frac{2}{3}$ per cent. on his outlay?

33. Two clocks point to 2 o'clock at the same instant on the afternoon of Christmas-day; one loses 8 seconds, and the other gains 9 seconds, in 24 hours. When will one be half an hour before the other, and what time will each clock then show?

34. Three sacks contain *equal* quantities of wheat, barley, and oats, and a mixture is formed by taking 60 per cent. from the 1st sack, 40 per cent. from the 2nd, and 70 per cent. from the 3rd. Find what percentage of the whole quantity of corn is taken away. A candidate being required to answer this question reasons as follows: '50 is the mean between 60 and 40, and 60 is the mean between 50 and 70, therefore 60 is the mean percentage required.' Is this reasoning erroneous? If so, show clearly where the error lies.

35. If the light received by an object at different distances from a candle be inversely as the squares of the distances, and a mirror reflect $\frac{4}{5}$ of the light which falls on it, at what distance from the candle must an object be placed, in order to receive the same amount of light directly, which it would receive by reflection from the mirror so placed that the whole distance traversed by the light is 5 feet?

36. A contractor has stores worth £300; he sells $\frac{1}{3}$ of them so as to lose 10 per cent. What must he sell the remainder for so as to gain 20 per cent. on the whole?

37. (1) How many cubic feet of lead $\frac{1}{16}$ inch thick will be required to cover the sides and bottom of a cistern 10 ft.

long, 6 ft. 6 in. wide, and 7 ft. deep? (2) What weight of water will the cistern hold, a cubic foot of water weighing 1000 ounces?

38. If 2 cubic inches of iron weigh as much as 15 of water, and a cubic foot of water weighs 1000 oz., find the weight of a beam of iron 5 yds. long, 2 ft. wide, and 1 in. thick.

39. Two workmen contract to do a piece of work for £25, which they reckon will take them 75 days to execute. After working together 45 days, one of them falls ill, and his mate completes the job in 50 days more. How much is each entitled to receive?

40. A contractor engaged to finish a work of 3150 cubic yds. in 50 days, and employed at once 60 men upon it, but at the end of 35 days he finds only 1800 cubic yds. completed. How many extra men must he put on to complete the work in the given time?

41. If a person purchase £4000 stock at $93\frac{5}{8}$, what loss will he sustain by selling out at $92\frac{7}{8}$?

42. How many square feet of board will be required to make a cubical box which shall contain 81·370 cubic feet?

43. If a new unit of weight were instituted containing 1 lb. 1 oz. 3·326 drs. avoirdupois, how many of such units of weight would make 537 lbs. 11 oz. 15 drs.?

44. If a company, when its capital was £2000000, paid 7 per cent. to the shareholders, what is its capital now when it can pay no more than 3 per cent., although the profit to be divided is six times as much as it was in the former case?

45. If gold be beaten out so thin that 1 oz. will form a leaf of 20 square yards, how many of these leaves will make ·01 in. thick, if 1 cubic foot of gold weighs 1095 lbs.?

46. The stuff out of a lead mine contains at first 15·9 per cent. of lead; after washing, by which process the amount of lead ore is not diminished, the stuff contains 87·45 per cent. of lead; how much rock was washed away out of *216 tons 5* cwts. of the original stuff?

47. A field is 300 yds. long, 200 yds. broad; if a belt of trees 30 yds. wide be planted round it, find the area of the interior space. Find also the distance from corner to corner.

48. A fraudulent tradesman sells a piece of cloth for $31\frac{1}{2}$ yds., at 19*s.* 6*d.* a yard; what is his profit from having used a yard measure 1 in. too short?

49. An artisan going to his work every week-day is obliged to pay $\frac{1}{2}d.$ toll, the toll being converted into one of 2 mils. How much will he gain in a year by the change of currency?

50. A sovereign, standard gold, weighs 5·136 dwts.; a shilling, standard silver, weighs $\frac{1}{6\,6}$ lb. troy. What weight of standard silver is equal in value to 4 oz. of standard gold?

51. If 9 tons $7\frac{1}{2}$ cwts. of iron be sold for £245, and the gain upon it be 20 per cent., what was the cost of 1 cwt.?

ALGEBRA.

CI.

MULTIPLICATION AND DIVISION.

1. When m and n $(m>n)$ are whole numbers, show that $a^m \times a^n = a^{m+n}$; $a^m \div a^n = a^{m-n}$; $a^0 = 1$; and that $\frac{1}{a^n} = a^{-n}$.

2. What is the meaning of the term *factor*?
$x-1$ is a factor of x^2+x-2, what is the other factor?

3. Divide—

(1) $x^5-3x^4+4x^3+26x^2-92x+55$ by $x^2-3x+11$.

(2) $x^5-\frac{23}{30}x^4+\frac{31}{10}x^3-\frac{7}{3}x^2-\frac{181}{18}x+\frac{5}{3}$ by $x^2-\frac{x}{6}+5$.

(3) $21a^4-16a^3b+16a^2b^2-5ab^3+2b^4$
by $3a^2-ab+b^2$.

(4) $x-4y$ by $x^{\frac{1}{2}}-2y^{\frac{1}{2}}$.
(5) $(x^3-12x+16)\times(x^3-12x-16)$ by x^3-16.
(6) x^8+x^4+1 by x^2-x+1.

4. If $xz+yz-xy=2c^2$,
$xz+xy-yz=2a^2$,
$xy+yz-xz=2b^2$,
show that $x^2=\frac{(a^2+c^2)(a^2+b^2)}{b^2+c^2}$.

5. Resolve into factors—
$25x^2-16$; x^3-8 ; x^3+27y^3 ; $a^3x^3-64b^3$.

6. Divide—
(1) $hkx^4+2(h-k)x^3-(h^2+4-k^2)x^2+2(h+k)x-$ by kx^2-h+2x.
(2) $mn(x^5+1)+(m^2+n^2)(x^4+x)+(n^2+2mn)(x^3+$ by nx^2+mx+n.
(3) $(b-c)a^3+(c-a)b^3+(a-b)c^3$ by $(a-c)(a-$

7. Multiply $\frac{a^{\frac{1}{2}}}{4}+\frac{a^{\frac{1}{4}}b^{\frac{1}{4}}}{2}+\frac{3}{8b^{\frac{1}{2}}}$ by $\frac{a^{\frac{1}{2}}}{4}-\frac{a^{\frac{1}{4}}b^{\frac{1}{4}}}{2}+\frac{3}{8b^{\frac{1}{2}}}$.

8. If $a^3-pa^2+qa-r=0$, then x^3-px^2+qx-r is divisible by $x-a$.

9. Divide—
(1) $(x^2+xy+y^2)\times(x^3+y^3)$ by $x^4+x^2y^2+y^4$.
(2) $x^4+x^{-4}-x^{-2}-x^2$ by $x-x^{-1}$.
(3) $a^2-b^2+c^2-d^2-2(ac-bd)$ by $a-c+b-d$.
(4) Divide $1+2x$ by $1-3x$ to 4 terms.

10. (1) Prove that—
$\{(a-b)^2+(b-c)^2+(c-a)^2\}^2$
$=2\{(a-b)^4+(b-c)^4+(c-a)^4\}$.
(2) Divide $(x^2-y^2)^3-z^6$ by $x^2-y^2-z^2$.

(3) Simplify $(a+b+c)(b+c-a)+(a+b-c)(a+c-b)$.

(4) Divide $x^3+(a+b+c)x^2+(bc+ac+ab)x+abc$ by $x^2+ax+bx+ab$.

11. Resolve into factors—

(1) $(3a+2b-c-d)^2+(3a-b-c)^2-(c+d-3a-2b)^2-(a-3b-c)^2$.

(2) $(x-2y)^3-(y-2x)^3$.

(3) $2x^2-(a-2)x-a^2+a$.

(4) $x^2-p^2q^2+2pq-1$.

(5) $2x^2-ax-3a^2$.

(6) $3x^2+10ax+3a^2$.

(7) $pqx^2+(q^2-p^2)xy-pqy^2$.

(8) $(ab+cd)(c^2+d^2)+cd(a^2+b^2-c^2-d^2)$.

(9) $6x^3-7x^2-16x+12$.

(10) $(a+b+c)(ab+bc+ac)-abc$.

(11) $x^{16}+x^8y^8+y^{16}$.

(12) $x^2-2ax-(1-a^2)$.

CII.

GREATEST COMMON MEASURE AND LEAST COMMON MULTIPLE.

1. Find the G.C.M. of—

(1) $6x^2+x-35$ and $10x^2+21x-10$.

(2) $2x^4-12x^3+19x^2-6x+9$ and $4x^3-18x^2+19x-3$.

(3) $a^2-12a+32$ and $a^2-10a+16$.

(4) $2x^3-3x^2y-4xy^2+6y^3$ and $2x^3-x^2y+xy^2-6y^3$.

(5) $21x^3-15x^2-14x+10$ and $21x^3+27x^2-14x-18$

(6) x^5-4x^3+3x and $2x^4-5x^2-3$.

(7) $3x^3-8x-18$ and $2x^2+2x-12$.

2. Find the L.C.M. of—

(1) $-3x^3y^2z$, $-12x^2yz^3$, and $42x^3yz^2$.

(2) $x^2-3x-70$ and $x^3-39x+70$.

(3) $3x^3-2x^2-x$ and $6x^2-x-1$.

(4) $a^3-4a^2b+9ab^2-10b^3$,
and $a^3+2a^2b-3ab^2+20b^3$

(5) $x^3+2x^2y-xy^2-2y^3$ and $x^3-2x^2y-xy^2+2y^3$.

(6) $2x-4$, $4x^2-1$, and $4x^2+1$.

(7) $x-1$, $x-2$, x^2-4, and $x+1$.

(8) a^3-x^3, a^2-x^2, and a^3+x^3.

(9) x^3-3x^2+3x-1 and x^3-x^2-x+1.

(10) x^4-2x^3+2x-1 and $x^4-2x^3+2x^2-2x+1$.

(11) $x^2-(a-3)x-3a$, $x^2-(a+2)x+2a$,
and $x^2+(a-1)x-a$.

(12) $2x^2+xy-3y^2$, $2x^2-xy-3y^2$, x^2-1,
and $4x^2-9y^2$.

CIII.

FRACTIONS.

1. Reduce to their lowest terms—

(1) $\dfrac{15x^2+xy-2y^2}{9x^2+3xy-2y^2}$.

(2) $\dfrac{x^4+ax^3-9a^2x^2+11a^3x-4a^4}{x^4-ax^3-3a^2x^2+5a^3x-2a^4}$.

(3) $\dfrac{3x^3-2x^2-8x}{9x^3-12x^2-36x+48}$. (4) $\dfrac{a^2-b^2-2bc-c^2}{a^2+2ab+b^2-c^2}$.

(5) $\dfrac{x^4+5x^3-2x^2-15x-3}{x^5+5x^4+x^3-2x^2-10x-2}$.

(6) $\dfrac{\dfrac{x^2}{y^2}-2+\dfrac{y^2}{x^2}}{2\left(\dfrac{x^2}{y^2}-\dfrac{y^2}{x^2}\right)}$.

(7) $\dfrac{a^2-ab}{a^2b-abc-ab^2+b^2c}$.

(8) $\dfrac{x^5-2x^4+3x^3-3x^2+6x-9}{x^5+2x^4-3x^3-3x^2-6x+9}$.

(9) $\dfrac{x^4+2x^2+9}{x^4+4x^3+10x^2+12x+9}$.

(10) $\dfrac{a^3+ab^2-a^2b-b^3}{4a^4-2a^2b^2-4a^3b+2ab^3}$.

(11) $\dfrac{ab\,(x^2-y^2)+xy\,(a^2-b^2)}{ab\,(x^2+y^2)+xy\,(a^2+b^2)}$.

(12) $\dfrac{x^4-4x+3}{3x^5-20x^2+15x+2}$.

(13) $\dfrac{2x^3+x^2y-6xy^2-3y^3}{6x^3+3x^2y+2xy^2+y^3}$.

(14) $\dfrac{(x^2-y^2)\,(x^3+y^3)}{(x^2-xy+y^2)\,(x-y)}$.

(15) $\dfrac{12x^4+x^2y^2-6y^4}{6x^3+9x^2y-4xy^2-6y^3}$.

(16) $\dfrac{a^2+(2ac-b^2)x^2+c^2x^4}{a^2+2abx+(2ac+b^2)x^2+2bcx^3+c^2x^4} \div \dfrac{a^2+(ac-b^2)x^2+bcx^3}{a^2+(ac-b^2)x^2-bcx^3}$.

(17) $\dfrac{y^3-(2a+b)y^2+(2ab+a^2)y-a^2b}{3y^2-(4a+2b)y+2ab+a^2}$.

2. Simplify—

(1) $\dfrac{\dfrac{a}{a-b}-\dfrac{a}{a+b}}{\dfrac{b}{a-b}-\dfrac{b}{a+b}}$.

(2) $\dfrac{x^2+a^2}{x^2-a^2}-\dfrac{x^2-a^2}{x^2+a^2}$. (3) $\dfrac{x^2-(a+b)x+ab}{x^2+(a-b)x-ab}$.

(4) $\dfrac{1}{abx}+\dfrac{1}{a(a-b)\,(x-a)}-\dfrac{1}{b\,(a-b)\,(x-b)}$.

(5) $\dfrac{x}{a^2-ax+x^2}+\dfrac{1}{a-x}+\dfrac{a^2}{a^3+x^3}$.

(6) $\dfrac{x^4+x^2y^2+y^4}{x^3+y^3}\times\dfrac{x^2-y^2}{x^3-y^3-2xy\,(x-y)}$.

(7) $\dfrac{a}{x+\dfrac{m}{y+\dfrac{n}{z}}}$

(8) $\dfrac{1}{x+4}+\dfrac{x+1}{x^2-4x+16}-\dfrac{x^2+x-124}{x^3+64}$.

(9) $\dfrac{\dfrac{1+a}{1-a}-a}{1+\dfrac{a(1+a)}{1-a}}$.

(10) $\dfrac{1+x^2+x^4}{1-x^4}\div\dfrac{1+x+x^2}{1+x^2}$.

(11) $\dfrac{\dfrac{a+b}{a-b}+\dfrac{a-b}{a+b}}{\dfrac{a-b}{a+b}-\dfrac{a+b}{a-b}}\times\dfrac{ab^3-a^3b}{a^2+b^2}$.

(12) $\dfrac{1+4x}{1-4x}\cdot\dfrac{1-16x^2}{1+16x^2}\cdot\dfrac{1-256x^4}{1+8x+16x^2}$.

(13) $\dfrac{x\,(x+1)\,(x+2)}{1.2.3}+\dfrac{x\,(x-1)\,(x-2)}{1.2.3}$
$+\dfrac{2\,(x-1)\,(x+1)}{3}$.

(14) $$x\left(\frac{x-2y}{x+y}\right)^3+y\left(\frac{2x-y}{x+y}\right)^3.$$

(15) $$\frac{\frac{m^2+n^2}{n}-m}{n^{-1}-m^{-1}}\times\frac{m^2-n^2}{m^3+n^3}.$$

(16) $$\frac{a(a+2b)+b(b+2c)+c(c+2a)}{a^2-b^2-c^2-2bc}.$$

(17) $$\frac{(1-x^2)(1-x^3)}{x(1+x)(1-x)^2}-\frac{x^3+\frac{1}{x^3}}{x^2+\frac{1}{x^2}-1}.$$

(18) $$\frac{a(b+c)}{(a-b)(c-a)}+\frac{b(a+c)}{(a-b)(b-c)}+\frac{c(a+b)}{(c-a)(b-c)}.$$

(19) $$\frac{(1-xy)(1+xy)-(x-y)(x+y)}{(1+xy)^2+(x-y)^2}.$$

(20) $$\frac{a^2-bc}{(a+b)(a+c)}+\frac{b^2-ca}{(b+a)(b+c)}+\frac{c^2-ab}{(c+a)(c+b)}.$$

(21) $$\frac{x^2+\left(\frac{a}{b}+\frac{b}{a}\right)xy+y^2}{x^2+\left(\frac{a}{b}-\frac{b}{a}\right)xy-y^2}.$$

(22) $$\frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c+a}{(a-b)(b-c)}.$$

(23) $$\frac{a^2}{(a-b)(a-c)}+\frac{b^2}{(b-a)(b-c)}+\frac{c^2}{(c-a)(c-b)}.$$

(24) $$\frac{1}{a(a-b)(a-c)}+\frac{1}{b(b-a)(b-c)}+\frac{1}{c(c-a)(c-b)}.$$

(25) $$\frac{a^2-b^2}{1-\frac{a}{b}+\frac{a^2}{b^2}}\times\frac{1-\frac{b}{a}+\frac{b^2}{a^2}}{1-\frac{b^2}{a^2}}.$$

(26) $\left(\frac{b}{c}+\frac{c}{b}\right)^2+\left(\frac{a}{c}+\frac{c}{a}\right)^2+\left(\frac{a}{b}+\frac{b}{a}\right)^2$
$-\left(\frac{b}{c}+\frac{c}{b}\right)\left(\frac{a}{c}+\frac{c}{a}\right)\left(\frac{a}{b}+\frac{b}{a}\right)$.

(27) $\frac{pr+(pq+qr)x+q^2x^2}{p-qx}\times\frac{ps+(pt-qs)x-qtx^2}{p+qx}$.

CIV.

SURDS.

1. Reduce to their simplest forms—
   (1) $(\sqrt[3]{a^2}+\sqrt[3]{ab}+\sqrt[3]{b^2})(\sqrt[3]{a}-\sqrt[3]{b})$.
   (2) $(2\sqrt{32}-3\sqrt{2}+4)\div 8$.

2. (1) Square $(a+b\sqrt{-1})^{\frac{1}{2}}+(a-b\sqrt{-1})^{\frac{1}{2}}$.
   (2) Extract the square root of $31-12\sqrt{-5}$.

3. Simplify—
   (1) $\sqrt[3]{\frac{195}{125}}$. (2) $\sqrt[n]{\frac{1}{a^{n+1}}\cdot\frac{1}{a^{n-1}}}$.
   (3) $7\sqrt{45}+2\sqrt{180}+11\sqrt{245}$.
   (4) $2\sqrt[3]{4}\times 3\sqrt[3]{54}$.
   (5) $(x^{\frac{3}{2}}-xy^{\frac{1}{2}}+x^{\frac{1}{2}}y-y^{\frac{3}{2}})\div(x^{\frac{1}{2}}-y^{\frac{1}{2}})$.

4. (1) Show that if $x+\sqrt{y}=a+\sqrt{b}$, $x=a$, $y=b$, and apply the principle to determine the square root of $12-4\sqrt{5}$.
   (2) Express $\frac{12-3\sqrt{3}}{12-4\sqrt{3}}$ as an equivalent fraction with a rational denominator.

5. Express
$$\frac{4}{\sqrt[3]{5}-\sqrt[3]{4}},\quad \frac{x}{\sqrt{x^2+x}-x},\quad \sqrt{61-28\sqrt{3}}$$
in simpler forms.

6. Express

$$\frac{6}{\sqrt[3]{6}+\sqrt[3]{5}}, \frac{\sqrt{a+b}+\sqrt{a}}{\sqrt{a+b}-\sqrt{a}}, \sqrt{5-12\sqrt{-1}}$$

in simpler forms.

7. Simplify $\sqrt[3]{a^2bc}.\sqrt[4]{abc^2}+a^{\frac{11}{12}}b^{\frac{7}{12}}c^{\frac{2}{3}}$.

8. Simplify—

(1) $\left\{\frac{a^{-2}}{b^{-2}}\sqrt{\frac{a}{b}}\right\}^{\frac{1}{2}}$.

(2) $(\sqrt{5}-\sqrt{2})(\sqrt{2}+1)(\sqrt{5}+\sqrt{2})(\sqrt{2}-1)$.

9. Simplify—

(1) $\sqrt{75a^5b^3}$. (2) $\sqrt[3]{135}$.

(3) $\left\{a^{\frac{q}{p-q}}.a^{\frac{p}{q-p}}\right\}^{\frac{1}{p^2-q^2}}$. (4) $\frac{5}{\sqrt{7}-\sqrt{2}}$.

(5) $\sqrt{10-4\sqrt{-6}}$.

10. Square $3\sqrt{2}-2\sqrt{7}$, and extract the square root of $57-12\sqrt{15}$.

11. Simplify—

(1) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$. (2) $\sqrt{18}+\sqrt{200}-\sqrt{338}$.

(3) $\sqrt{2}+\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{8}}$. (4) $\sqrt{27+7\sqrt{5}}$.

(5) $\frac{1+\sqrt{2}}{1+\sqrt{2}+\sqrt{3}}$. (6) $\sqrt{75-12\sqrt{21}}$

(7) $\sqrt{128}-\sqrt{32}+\sqrt{8}-\sqrt{16}$.

(8) $\sqrt{5}.\sqrt[3]{2}.\sqrt[4]{4}$.

12. (1) Prove that

$$\left\{\sqrt{3+4\sqrt{-1}}+\sqrt{3-4\sqrt{-1}}\right\}^2=16.$$

(2) Simplify $\sqrt{7+4\sqrt{3}}+\sqrt{7-4\sqrt{3}}$.

(3) If $a=\frac{-1+\sqrt{-3}}{2}$, show that $\frac{1}{a}=\frac{-1-\sqrt{-3}}{2}$, and that $a^3+a^{-3}=2$.

(4) Extract the square root of

$$x^2y^{-2}+x^{-2}y^2-\frac{xy^{-1}-x^{-1}y}{\sqrt{-1}}-\frac{9}{4}.$$

(5) Rationalise the denominator of $\frac{1}{a-\sqrt{b}+\sqrt{c}}$, and show that the difference of $\frac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$ and $\frac{2\sqrt{2}+\sqrt{3}-1}{\sqrt{3}+1}$ exceeds their product by 1.

(6) Divide $x+y+z-3\sqrt[3]{xyz}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

13. (1) Simplify

$$\left\{\frac{x^2+\sqrt{x^4-a^4}}{x^2-\sqrt{x^4-a^4}}-\frac{x^2-\sqrt{x^4-a^4}}{x^2+\sqrt{x^4-a^4}}\right\}\div 4\sqrt{\frac{x^2-a^2}{x^2+a^2}}.$$

(2) Show that $\frac{a+b\sqrt{-1}}{(a-b\sqrt{-1})^2}+\frac{a-b\sqrt{-1}}{(a+b\sqrt{-1})^2}$ is a possible quantity.

14. Show that $\frac{2(x+2+\sqrt{x^2-4})}{x+2-\sqrt{x^2-4}}=x+\sqrt{x^2-4}$.

15. If $\sqrt{ax}+\sqrt{by}+\sqrt{cz}=0$, show that

$$a^2x^2+b^2y^2+c^2z^2=2(abxy+acxz+bcyz).$$

16. Show that $\frac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}=\sqrt{2}$.

17. Simplify $\frac{7+\sqrt{-3}}{2-\sqrt{-3}}+\frac{8+3\sqrt{-3}}{2+\sqrt{-3}}-\frac{4(2-\sqrt{-3})}{1-\sqrt{-3}}$.

18. Reduce to its lowest terms

$$\frac{x+3x^{\frac{3}{4}}-2x^{\frac{1}{2}}-7x^{\frac{1}{4}}+2x^{-\frac{1}{4}}}{x^{\frac{1}{4}}-2x^{-\frac{1}{4}}}.$$

19. Extract the square root of—

(1) $x^{\frac{3}{2}}+xy^{-\frac{1}{2}}-2x^{\frac{5}{4}}y^{-\frac{1}{4}}-2x^{\frac{1}{2}}y^{\frac{1}{4}}+2x^{\frac{3}{4}}y^{\frac{1}{2}}+y.$

(2) $\cdot 03+\cdot 04\sqrt{-1}.$

20. Show that $\sqrt[4]{-1}=\frac{1+\sqrt{-1}}{\sqrt{2}}.$

EQUATIONS.

CV.

Solve the equations:—

1. $\frac{a+b}{x+b}=\frac{a}{x-a}+\frac{b}{x-b}.$

2. $\frac{6x+7}{15}-\frac{2x-2}{7x-6}=\frac{2x+1}{5}.$

3. $\frac{x-a}{x-b}=\left(\frac{2x-a}{2x-b}\right)^2.$

4. $\frac{x+6}{2x+5}=\frac{x}{2x-5}$

5. $\frac{4x+3}{9}=\frac{8x+19}{18}-\frac{7x-29}{5x-12}.$

6. $\frac{x}{4}-\frac{2x-1}{2\frac{1}{2}}-\frac{6x-4}{15}=\frac{x}{6}.$

7. $cx-q=bx+r.$

8. $\frac{x-5}{x-4}-\frac{x-6}{x-5}=\frac{x-2}{x-3}-\frac{x-3}{x-4}.$

9. $\sqrt{12+x}=\sqrt{x}+6.$

10. $\sqrt{x}-\sqrt{\frac{a}{x}}=\sqrt{a+x}.$

11. $\frac{a-mx}{abx}=\frac{1}{b^2}-\frac{n}{ab}$.

12. $\sqrt{x-1}+\sqrt{x+1}=2$.

13. $\frac{a-2x}{abx}=\frac{1}{b^2}-\frac{1}{ab}$.

14. $\frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}-\sqrt{3}}=\frac{2}{3}$.

15. $\sqrt[3]{a+x}+\sqrt[3]{a-x}=\sqrt[3]{b}$.

16. $\frac{x-1}{x-2}-\frac{x-2}{x-3}=\frac{x-5}{x-6}-\frac{x-6}{x-7}$.

17. $\frac{\sqrt{3x+1}+\sqrt{3x}}{\sqrt{3x+1}-\sqrt{3x}}=4$.

18. $\frac{x^2-(a+b)x-bc}{x-b}=\frac{x^2-(a+c)x-bc}{x-c}$.

CVI.

Solve the equations :—

1. $\frac{x}{2y}=\frac{2}{5}$, and $\frac{y-x}{3}+\frac{2x+y}{13}=\frac{y+1}{4}$.

2. $\frac{x}{5}-\frac{y}{9}=16$, and $\frac{x+5}{13}=2+\frac{y-1}{10}$.

3. $\left.\begin{array}{l} ax+by=c \\ bx-ay=d \end{array}\right\}$

4. $\frac{x}{a}-\frac{y}{b}=1$, and $\frac{x}{c}-\frac{y}{d}=1$.

5. $\left.\begin{array}{l} \frac{a}{x}+\frac{b}{y}=p \\ \frac{b}{x}+\frac{a}{y}=q \end{array}\right\}$

6. $\left.\begin{array}{l} \frac{2}{x}+\frac{3}{y}=\frac{5}{6} \\ \frac{4}{x}+\frac{9}{y}=2 \end{array}\right\}$

7. $\frac{xy}{x+y}=1$; $\frac{xz}{x+z}=2$; and $\frac{yz}{y+z}=3$.

8. $9xy=20(x+y)$; $10xz=24(x+z)$; $11yz=30(y+z)$.

9. $3x=2y$ and $x^3=y^2$.

10. $\left.\begin{array}{l} \sqrt{x^2+ay-b^2}=c+x \\ \sqrt{y^2+mx}=x+y \end{array}\right\}$

11. $\dfrac{11y-5x}{22}=\dfrac{x+3y}{32}$, and $8y-5x=1$.

12. $\dfrac{3x+2y-3}{4x-5y+16}=\dfrac{9}{4}$, and $3x=5y$.

13. $\left.\begin{aligned} xy&=3(x+y)\\ xz&=8(x+z)\\ 7yz&=24(y+z)\end{aligned}\right\}$

14. $\left.\begin{aligned} 3x+2y+z&=23\\ 5x+2y+4z&=46\\ 10x+5y+4z&=75\end{aligned}\right\}$

15. $\left.\begin{aligned} x+\tfrac{1}{2}y+\tfrac{1}{3}z&=32\\ \tfrac{1}{3}x+\tfrac{1}{4}y+\tfrac{1}{5}z&=15\\ \tfrac{1}{4}x+\tfrac{1}{5}y+\tfrac{1}{6}z&=12\end{aligned}\right\}$

16. $\left.\begin{aligned} 3x+5y&=171\\ 7x+2z&=209\\ 7y+2z&=90\end{aligned}\right\}$

17. $\left.\begin{aligned} x^{-1}+y^{-1}+z^{-1}&=13\\ y^{-1}+x^{-1}&=7\\ x^{-1}y^{-1}-2z^{-1}&=0\end{aligned}\right\}$

CVII.

Solve the equations :—

1. $x^{-1}+x^{-\frac{1}{2}}=6$.

2. $\sqrt{x-3}+\sqrt{x}=\dfrac{3}{\sqrt{x-3}}$.

3. $\dfrac{3}{1-2x}-\dfrac{3}{1+2x}=4$.

4. $\sqrt{7x+1}=5+\sqrt{x}$.

5. $x^{10}+3x^5=1120$.

6. $5\{1+\sqrt{1+x^2}\}=5\sqrt{1-x^2}+2\sqrt{1-x^4}$.

7. $\dfrac{x-4}{x-8}=\sqrt{x}-2$.

8. $\sqrt{x^2+\tfrac{1}{2}\sqrt{x^2+96}}=x+1$.

9. $(x-m)\sqrt{an}-(a-n)\sqrt{mx}=0$.

10. $\dfrac{x-1}{\sqrt{x}+1}=4+\dfrac{\sqrt{x}-1}{2}$.

11. $\dfrac{15-4x^2}{\sqrt{15-4x}}=2.$

12. $\dfrac{x-1}{x+3}+\dfrac{x-3}{x+1}=2.$

13. $7x^2-13x=2.$

14. $x^6-7x^3=8.$

15. $\dfrac{\sqrt{2}}{x}=\dfrac{3}{\sqrt{x+x^2}}-\dfrac{\sqrt{2}}{1+x}.$

16. $x^2-16x=80.$

17. $\dfrac{7x+1}{2x-1}+\dfrac{3x}{5}=7.$

18. $\dfrac{x-2}{2}+\dfrac{3}{x-1}=3.$

19. $\dfrac{2x+\sqrt{4x^2-1}}{2x-\sqrt{4x^2-1}}=9.$

20. $(a+x)(b+x)-a(b+c)=\dfrac{a^2c}{b}+x^2.$

21. $\left(\dfrac{a+x}{a-x}\right)^2=1+\dfrac{cx}{ba}.$

22. $\dfrac{1}{18+x}=\dfrac{1}{7}+\dfrac{1}{11}+\dfrac{1}{x}.$

23. $\dfrac{9}{1+x+x^2}=7-x-x^2.$

24. $\dfrac{1}{x-12}=\dfrac{1}{x}-\dfrac{1}{5}-\dfrac{1}{7}.$

25. $\dfrac{x}{7-x}=\dfrac{7-x}{x}-\dfrac{5}{6}.$

26. $\dfrac{2x+3}{x+1}=\dfrac{4x+5}{4x+4}+\dfrac{3x+3}{3x+1}.$

27. $\dfrac{2x-3}{3x-4}=\dfrac{4x-6}{6x-7}.$

28. $\sqrt{x+13}=2+\sqrt{x-}$

29. $x(x+1)+3\sqrt{2x^2+6x+5}=25-2x.$

30. $\dfrac{5x}{x-2}-\dfrac{8}{x+1}=5+\dfrac{3(2x+1)}{x^2-4}.$

31. $x^4-\dfrac{5x^3}{6}-\dfrac{19x^2}{3}-\dfrac{5x}{6}+1=0.$

32. $(x+a)^5-(x-a)^5=242a^5$.

33. $x^2-4x+5=\sqrt{61-52x+11x^2}$.

34. $\frac{1}{24+x}=\frac{1}{13}+\frac{1}{11}+\frac{1}{x}$

35. $(7+4\sqrt{3})x^2+(2+\sqrt{3})x=2$.

36. $\frac{x+a}{x+b}=\left(\frac{2x+a+c}{2x+b+c}\right)^2$.

37. $x^4+x^3-4x^2+x+1=0$.

38. $x(x-2)(x^2-4)=2$.

39. $x-\frac{4}{\sqrt{x-2}}=4$.

40. $ab(x^2+1)=x(a^2+b^2)$.

41. $4\sqrt{\sqrt{x}+6}+3(\sqrt{x}+3)=x$.

42. $x^4+8x=4(x^2+1)$.

43. $\frac{\sqrt{2a^2-x^2}+b\sqrt{2a-x}}{\sqrt{2a^2-x^2}-b\sqrt{2a-x}}=\frac{\sqrt{a}+b}{\sqrt{a}-b}$

44. $7x+\sqrt{x^2-17x+4}=2x^2-27x+5$.

45. $\frac{x}{a+x}+\frac{a}{\sqrt{a+x}}=\frac{b}{x}$.

CVIII.

Solve the equations:—

1. $\left.\begin{aligned} y(x^2+y^2)&=4(x+y)^2 \\ xy&=4(x+y) \end{aligned}\right\}$

2. $x^4+y^4=337$, and $x+y=7$.

3. $xy+xy^2=12$, and $x+xy^2=18$.

4. $x^3+y^3=9$, and $xy=2$.

5. $x+y+3\,(x+y)^2=310$, and $xy=21$.

6. $\frac{a}{\sqrt{x}}+\frac{b}{y^2}=m$, $\frac{b}{\sqrt{x}}+\frac{a}{y^2}=n$.

7. $x^2+xy=a$, and $y^2+xy=b$.

8. $x+y+z=a+2b$;
$b(x+y)+az=b(x+z)+ay=2ab+b^2$.

9. $x^2+y^2=\frac{5}{2}$, and $x^2-xy+y^2=\frac{7}{4}$.

10. $\left.\begin{aligned} x^2-xy&=10\\ 2x-3y&=1\end{aligned}\right\}$

11. $\left.\begin{aligned} x+y&=3\\ x^2+y^2&=\frac{13}{2}\end{aligned}\right\}$

12. $\left.\begin{aligned} \frac{xy}{z}&=4\\ \frac{xz}{y}&=9\\ \frac{yz}{x}&=16\end{aligned}\right\}$

13. $\left.\begin{aligned} x^2+y^2 &: xy :: 5 : 2\\ x+y &: 5 :: 3 : y-4\end{aligned}\right\}$

14. $x^2+xy=15$, and $xy-y^2=2$.

15. $x^2-xy+y^2=7$, and $x^4+x^2y^2+y^4=133$.

16. $\sqrt{\frac{5x}{x+y}}+\sqrt{\frac{x+y}{5x}}=2$, and $xy-x-y=75$.

17. $\left.\begin{aligned} xy+\frac{x}{y}&=\frac{5}{3}\\ \frac{1}{xy}+\frac{y}{x}&=\frac{20}{3}\end{aligned}\right\}$

18. $\left.\begin{aligned} \frac{5xy}{6}&=x+y\\ \frac{3xz}{10}&=x+z\\ \frac{2yz}{15}&=y+z\end{aligned}\right\}$

19. $\frac{ax}{a+x}+\frac{by}{b+y}=\frac{(a+b)c}{a+b+c}$, and $x+y=c$.

20. $\left.\begin{aligned} x-y&=3\\ x^4-y^4&=609\end{aligned}\right\}$

21. $\left.\begin{aligned} \frac{y}{x}-\frac{x}{y}&=\frac{x-y}{x^2+y^2}\\ \frac{x^2}{y^2}-\frac{y^2}{x^2}&=\frac{x+y}{y^2}\end{aligned}\right\}$

22. $\left.\begin{aligned} (x-y)(x^2-y^2)&=a\\ (x+y)(x^2+y^2)&=b\end{aligned}\right\}$

23. $x^{\frac{1}{m}}y^{\frac{1}{n}}=a$, and $x^{\frac{1}{p}}y^{\frac{1}{q}}=b$.

24. $\frac{a^2}{x}+\frac{b^2}{y}=\frac{(a+b)^2}{c}$, and $x+y=c$.

25. $\left.\begin{array}{l} x^2+xy+y^2=49 \\ x^4+x^2y^2+y^4=931 \end{array}\right\}$

26. $\begin{array}{l} 4(x+y)=3xy \\ x+y+x^2+y^2=26 \end{array}$

27. $\left.\begin{array}{r} x\sqrt{\frac{x}{y}}+y\sqrt{\frac{y}{x}}=34 \\ x-y=12 \end{array}\right\}$

28. $\left.\begin{array}{l} ay^2+bxy-b=0 \\ bx^2+axy-a=0 \end{array}\right\}$

29. $\left.\begin{array}{l} 13y\sqrt{\frac{x^2}{y}+3}=6x^2+20y \\ 24x^2+y^2=2x(5y+4x) \end{array}\right\}$

30. $\left.\begin{array}{l} x^{\frac{3}{5}}+y^{\frac{1}{2}}=35 \\ x^{\frac{1}{5}}+y^{\frac{1}{6}}=5 \end{array}\right\}$

31. $\left.\begin{array}{r} x^2+y^2+z^2=50 \\ x=y+z-6 \\ x(y+z)=27 \end{array}\right\}$

32. $\frac{x^2}{y^2}+\frac{y^2}{x^2}+\frac{x}{y}-\frac{y}{x}=\frac{106}{9}$, and $xy=3$.

33. $x+2\sqrt{xy}+y=143+2\sqrt{x}+2\sqrt{y}$,
and $3\sqrt[4]{xy}-\sqrt{y}=9$.

34. $\frac{(x+y)^2}{a^2}+\frac{(x-y)^2}{b^2}=8$, and $x^2+y^2=2(a^2+b^2)$.

35. $\left.\begin{array}{l} 4x^2+4y^2=17xy \\ \sqrt{x}+\sqrt{y}=3 \end{array}\right\}$

36. $\left.\begin{array}{l} x^2-4y^2=36 \\ xy+2y^2=12 \end{array}\right\}$

37. $\left.\begin{array}{l} x+y+z=a+b+c \\ bx-cy+az=ay+bz-cx=ab-(a+b)c \end{array}\right\}$

38. $\sqrt{x+y}+\sqrt{x-y}=a$,
and $\sqrt{x^2+y^2}+\sqrt{x^2-y^2}=b^2$.

39. $xy+\frac{x}{y}=20$, and $\frac{1}{xy}+\frac{y}{x}=\frac{5}{16}$.

40. $x+y=a+b$; $\frac{a}{x}+\frac{b}{y}=2$.

41.
$$\left.\begin{aligned} &\frac{x}{2}+\frac{y}{3}-\frac{z}{4}=1 \\ &4x+y-\frac{3z}{2}=1 \\ &\frac{3z}{4}-2x-\frac{y}{3}=1 \end{aligned}\right\}$$

42.
$$\left.\begin{aligned} &2x-y=2 \\ &8x^3-y^3=98 \end{aligned}\right\}$$

43.
$$\left.\begin{aligned} &x^4+y^2=49+2x^2y \\ &x^4+y^4-x^2=20+(2x^2-1)y^2 \end{aligned}\right\}$$

44. $(x+y)^3+z^3=1125$; $x+y+z=15$; $xy=24$.

45.
$$\left.\begin{aligned} &3xy-y^2=8 \\ &4x^2-xy=33 \end{aligned}\right\}$$

CIX.

THEORY OF EQUATIONS.

1. Prove that a quadratic equation has two roots, and only two.

2. If α and β are the roots of the equation $ax^2+bx+c=0$, show that

(1) $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$;

(2) that $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ are the roots of
$acx^2+(2ac-b^2)x+ac=0$;

(3) $\frac{\alpha^3}{\beta^3}$ and $\frac{\beta^3}{\alpha^3}$ are the roots of
$a^3c^3x^2+(5a^2bc^2-5ab^3c+b^5)x+a^3c^3=0$;

(4) find the values of $\alpha^2+\alpha\beta+\beta^2$, $\alpha^3+\beta^3$, $\alpha^4+\alpha^2\beta^2+\beta^4$, and $\alpha^{-2}+\beta^{-2}$.

Show that the roots of the equation

x^2+mx+n are real and different, real and equal, or impossible, according as

$$m^2>, =, \text{or} < 4n.$$

If $\frac{1}{3}$ of the sum of the squares of the roots of the equa- $ax^2+bx+c=0$ is equal to their product, show that $5ac$.

If α and β are the roots of $ax^2+x+b=0$, show that

$$\left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)=ab.$$

If the equation $x^2+px+q=0$ have equal roots, show $ax^2+p(a+b)x+q(a+2b)=0$ has one of them, and the other.

Form the equation whose roots are 2, $\frac{1}{2}$, $-\frac{1}{2}$.

When will the roots of $x^2+2px+q=0$ be (1) both tive, (2) both negative?

If α, β are the roots of $ax^2+bx+c=0$, find in terms , b, and c, values of—

(1) $\frac{1}{\alpha^2}+\frac{1}{\beta^2}$; (2) $\alpha^3+\beta^3$.

). Show that impossible roots enter equations in pairs. at assumption is here made respecting the coefficients of equations? In what case will irrational roots enter in s?

1. Solve the equations—

(1) $x^3-7x-6=0$,

(2) $3x^3+4x^2-35x-12=0$,

ch have a common root.

2. Form the rational equation two of whose roots are $+\sqrt{3}$, $-1-\sqrt{3}$.

3. Reduce by 2 the roots of the equation

$$x^3-6x^2+8x+1=0.$$

4. Find all the roots of the equation

$$x^4-2(x^3+x^2-x)+1=0.$$

15. Given $x^3-24x-72=0$, to find x by Cardan's method.

16. State and prove the relations between the coefficients and roots of the equation

$$P_n x^n+P_{n-1}x^{n-1}+P_{n-2}x^{n-2}+\ldots\ldots P_1 x+P_0=0.$$

17. Prove that the roots of the equation $x^3+prx-r^3=0$ are the products of every pair of the roots of the equation $x^3+px^2+r=0$.

18. Show that the sum of the cubes of the roots of the equation

$x^n+p_1x^{n-1}+p_2x^{n-2}+p_3x^{n-3}+\ldots.=0$ is $3p_1p_2-3p_3-p_1^3$.

19. Transform the equation $x^3+px^2+qx+r=0$, whose roots are a, b, c, into one whose roots are ab, ac, bc.

20. Find all the roots of the equation

$$x^4-6x^2-16x+21=0.$$

21. Find the other roots of the equation

$x^4-2x^3-6x^2+8x+8=0$, one root being $1+\sqrt{3}$.

22. $3+\sqrt{-1}$ is a root of the equation

$$x^4-6x^3+13x^2-18x+30=0.$$

Find all the other roots.

23. Find the sum of the fourth powers of the roots of the equation $x^4+x^3-7x^2-x+6=0$.

24. Prove that one real root of the equation $x^3-2x-5=0$ lies between 2 and 2·1; also approximate to the value of the roots to 4 places of decimals.

25. Extract the cube root of 20, and find a root of the equation $x^3+2x=30$ by Horner's method.

26. Solve the equation $x^3-12x=16$, one root being 4.

27. Given $x^3-12x^2+44x-48=0$, and $y=x-2$; eliminate x and solve the resulting equation.

28. Show that $x^3-px^2+qx-r=0$ is the equation that results from the elimination of y and z from the equations $x+y+z=p$, $xy+xz+yz=q$, and $xyz=r$.

29. Solve the equation $x^4+4x^3-18x^2+20x-7=0$, which *has equal* roots.

30. If the same transformation banishes both the second and third terms in the cubic equation $x^3-px^2+qx-r=0$, what relation must subsist between the coefficients ?

31. Prove that x^n-a^n is divisible by $x-a$; and show that if a be a root of the equation $X=0$, X is divisible by $x-a$.

32. Solve the equation $x^3=2x+2$, by Cardan's method.

CX.

PROBLEMS.

1. An uncle is older than his nephew by 10 years; and 15 years ago the uncle was twice as old as his nephew. What are their respective ages ?

2. Two shepherds, A and B, owning a flock of sheep, agree to divide it. A takes 144 sheep, and B takes 184 sheep, paying £70 to A. Required the value of a sheep.

3. A general, after losing a battle, found that he had only $\frac{2}{3}$ of his army fit for action; $\frac{1}{9}$ of the army were wounded, and the remainder, 2,000 men, were either killed or missing. Of how many men did his army consist at first ?

4. There is a fraction which becomes equal to $\frac{1}{3}$ if 1 be added to its numerator, and becomes equal to $\frac{1}{4}$ if 1 be added to its denominator. Determine the fraction.

5. A workman was employed for 60 days, on condition that for every day he worked he should receive 2*s.* 6*d.*, and for every day he was absent he should forfeit 10*d.*: at the end of the time he had to receive £2. How many days did he work ?

6. A luggage train leaves a station, and travels at the rate of 10 miles an hour; after 4 hours another train follows from the same station, travelling $16\frac{2}{3}$ miles an hour. How far must the second train travel before it comes up with the first ?

7. Find the fraction to the numerator of which, if 16 be added, the fraction becomes equal to 4, and if 11 be added to the denominator, the fraction becomes $\frac{1}{4}$.

8. A man engages to perform, at a uniform rate, a certain distance on horseback in 10 hours, but when he is half way he increases his speed by 2 miles an hour, and thus arrives at his journey's end $1\frac{1}{4}$ hour before the time stipulated. Required the distance he performed and the rate at which he started.

9. At the election of a member of parliament the successful candidate was returned by a majority of 88; but if 1 out of every 7 of his supporters had voted for his opponent, he would have been in a minority of 18. How many votes were recorded for each candidate?

10. A body of infantry is marching in regular column with 10 men more in depth than in front; on the enemy coming in sight the front is increased by 720 men, and then the infantry is drawn up in 9 lines. Find the number of men on the march.

11. Two casks, A and B, are filled with 2 kinds of sherry, mixed in A in the ratio of 2 : 7, and in B in the ratio of 1 : 5. What quantity must be taken from each in order to have 11 gallons mixed in the ratio of 2 : 9?

12. A has £3, and B has £2 8*s*. How much shall A give to B, that B may then have 3 times as much as A?

13. In a school of 600 children there are 18 more boys than girls, and the numbers of boys and girls are together double the number of infants. How many boys are there?

14. Two merchants, A and B, start with the same capital in trade; at the end of a year A has gained £3,000 and B has lost £1,000; it is then found that A's capital is double the capital of B. What did they each start with?

15. In a regiment 1,200 strong, the number of English and Scotch together was double the number of the Irish, and the number of English and Irish together was 5 times

the number of Scotch. How many were there of each nation in the regiment?

16. A wine merchant has 2 sorts of wine, one sort worth 12*s.* a gallon, the other sort 9*s.* a gallon; he wants to make a mixture of 36 gallons, to be sold at 10*s.* a gallon. How many gallons of each sort must he take?

17. Two persons, A and B, have the same annual income; A lays by $\frac{1}{5}$ of his income, and B, spending £800 a year more than A, at the end of 4 years finds himself £2,200 in debt. Find the annual income and expenditure of each.

18. A man buys a case of oranges at 6*d.* a dozen; he finds 50 spoiled, and selling the remainder 3 for 2*d.* has 10*s.* 9*d.* for profit. How many oranges were there in the case?

19. Find a fraction whose denominator exceeds its numerator by 3, and which is equal to $\frac{1}{2}$ when 2 is added to the denominator.

20. A draper buys a piece of cloth for £12, and adds as his profit on each yard as many pence as there are yards in the piece; when he has sold 20 yards he finds that he has cleared the cost price of the whole. How many yards are there in the piece?

21. A can do a piece of work in 9 days, B in twice that time, C can only do $\frac{3}{4}$ as much work as A in a day. How long would A, B, and C, working together, require to do the same piece of work?

22. A and B are two workmen; A alone takes 4 hours more to perform a certain work than B alone takes; A and B can do it together in $3\frac{3}{4}$ hours. How long would each man take to do the whole?

23. A railway train running from L to X encounters an accident on the way, in consequence of which its rate is reduced to $\frac{3}{5}$ of what it was at first, and it is 2 hours 5 minutes late; if the accident had happened 18 miles nearer X,

it would have been only 1 hour 50 minutes late. Find the distance from X to the place of the accident.

24. There is a looking-glass whose plate is 2 feet greater in height than in width; the frame cost 5*s*. per linear foot, inside measure, and the plate 10*s*. per square foot; the whole cost was £11 10*s*. What is the width?

25. One side of a room is 5 feet longer than the other, and 1,000 square feet of paper are required to cover its walls. Now, if it were 3 feet higher, the same quantity of paper would be required for 3 of its walls, that remaining unpapered being one of the longer sides. What are the dimensions of the room?

26. A number of persons undertake to erect a public building, contributing equally to the cost, which is £5002. Whilst it is in progress, 10 of the number become bankrupts, and there are 10 others who refuse to contribute to the deficiency thus occasioned. This deficiency, being equally borne by the remainder, increases the contribution of each by £40. How many subscribers were there in the first instance?

27. Two gunners keep up a fire on a battery from 2 different guns; the first one, who has spent 24 shots before the second opens fire, discharges 8 shots to 7 of those of his comrade, but uses in 4 rounds only the same quantity of powder as the other in 3. How many shots must the second fire before he has consumed as much powder as the first?

28. Two paintings are to be framed and glazed, one being 6 inches longer and 4 inches wider than the other; the framing and glazing of the larger costs 28*s*. 8*d*., and that of the other 20*s*., the glass being charged at the rate of 4*s*. per square foot, and the frames at 2*s*. per lineal foot, inside measure. What are the dimensions of the paintings, the part of each concealed by the frame being neglected?

29. A garrison is provisioned for 180 days; at the end of 30 days it is found that $\frac{1}{4}$ of the stores then remaining is unfit for use; at the end of 30 days more it is reinforced by

2000 men, and then the stores are made to last 110 days longer by putting the men on half rations. Find the number of the original garrison.

30. A book is published at a cost of 5*s*. a copy, and sold at 10*s*. a copy. Now, if it had been sold at 6*s*. 8*d*. a copy, the annual sale would have been 200 copies more, the edition would have been sold off 2 years sooner, and it would have begun to pay profit 1 year later. How many copies were published, interest of money to be neglected?

31. A government contractor imported a number of mules which he bought for delivery at 10 guineas per head under the contract price, at a total cost of £350, and although 7 of them died on the passage he realised a profit of £14 on the transaction. How many mules did he sell to the government?

32. A wine merchant mixes 2 kinds of wine, and sells the mixture so as to gain 8 per cent.; had he sold each kind of wine at the same price per gallon as he sells the mixture, he would have gained 10 per cent. and 6 per cent. respectively on the cost price of each. In what ratio were the 2 kinds mixed?

33. A farmer has 1400 acres; for every 10 sheep that he keeps he requires to plough an acre of land for turnips, and for every 4 sheep he requires an acre of land for pasture. How many sheep can he keep altogether?

34. An engineer had a certain number of telegraph posts to extend the wires a certain distance; he found that if he set them 15 yards apart he should have too many by 150, but if he set them 5 yards apart he should have too few by 70. How many posts had he?

35. A merchant bought for £40 a number of articles at the same price; reserving 5 of them, he sold the rest for £36, thus gaining 8*s*. on each. How many articles were there?

36. A contractor begins a work with 21 bricklayers and 8 carpenters, and their weekly wages are £25 1*s*. As the

work proceeds he takes off 6 bricklayers and adds 4 carpenters, thus lessening the weekly wages by £1 10*s*. Find the weekly wages of a bricklayer.

37. A gentleman is in the habit of walking from his house, to take a certain train, in 20 minutes. One day he is detained beyond his usual time of starting, and although he quickens his pace to $\frac{19}{17}$ of his accustomed speed, he is too late at the station by 2 minutes. How long was he detained?

38. Two men start together from the same point and run round a ring with different uniform speed. If they run in opposite directions they meet at 36 yards from the starting-point. If they run in the same direction, they are at the starting-point together after one has been 7 times round, and the other 6 times. Find the length of the ring.

39. A railway train, after travelling 1 hour, meets with an accident which delays it 1 hour, after which it proceeds at $\frac{3}{5}$ of its former rate, and arrives 5 hours late; if the accident had occurred 50 miles further on, the train would have been 3 hours 20 minutes late. Find the length of the line.

40. The fore-wheel of a carriage makes 6 revolutions more than the hind-wheel in going 120 yards; if the circumference of the fore-wheel be increased by $\frac{1}{4}$ of its present length, and that of the hind-wheel by $\frac{1}{5}$ of its present length, the fore-wheel will make 4 revolutions more than the hind-wheel in 120 yards. Find the circumferences of the wheels.

41. Two parties have a joint capital of £2000: one withdraws his money after 12 months, and receives for capital and profit £1040; the other lets his remain 17 months, and then finds it amount to £1710. What was the original capital of each partner, the profit being uniform throughout?

42. A sum of £6 is to be divided among a certain number of men, and £3 among as many women; and it is

found that if there were 5 more men, and 6 fewer women, each woman would receive as much as a man. How many of each were there?

43. A passenger train sets out from L at the same time that a luggage train, travelling 10 miles an hour slower, sets out from S, and they meet at a certain place on the road. On their return they again set out at the same time, and meet at a place 30 miles nearer L. What is the distance of this place from L, and at what rates do the trains travel? The distance from L to S=80 miles.

44. A merchant made a mixture of wine, at 28*s.* a gallon, with brandy at 42*s.* a gallon; and he found that by selling the mixture at 35*s.* a gallon he gained 15 per cent. on the price of the wine, and 20 per cent. on the price of the brandy. In what ratio were the wine and brandy mixed together?

45. In a garrison of 990 men there was a certain quantity of gunpowder, which was calculated to last a certain number of weeks, at the rate of $\frac{1}{2}$ lb. a week for each man, but after the first weekly distribution 10 men died weekly, and it was found that the powder lasted twice as long as was at first calculated. Find the quantity of powder at first. How long did it last?

46. In a regiment consisting of English, Irish, and Scotch, ·4 of the regiment were Irish and ·3 Scotch; but after 200 Irish and 200 Scotch were added to the regiment, ·225 were English. What was the original strength of the regiment and the number of men of each nation?

47. A person, after paying a poor-rate and also an income-tax of 7*d.* in the pound, has £486 remaining. The poor-rate amounts to £22 10*s.* more than the income-tax. Find the original income, and the number of pence per £ in the poor-rate.

48. A plays at chess with B, winning 3 games out of 4; and afterwards with C, winning 2 out of 3; at the end of 21 games he has won 15. How many did he play with each?

49. A garrison was provisioned for 30 days; after 10 days the garrison was reinforced by 3000 men, and the provisions were then exhausted in 5 days. Find the number of men in garrison at first.

50. A person having to walk 10 miles finds that by increasing his speed $\frac{1}{2}$ a mile an hour he might reach his journey's end $16\frac{2}{3}$ minutes sooner than he otherwise would. What time will he take if he quicken his pace halfway?

51. A detachment from an army was marching in regular column, with 7 men more in depth than in front; but the front being increased by 336 men, the detachment was drawn up in 5 lines. Find the number of men.

52. The number of men in both fronts of 2 columns of troops, H and K, when each consisted of as many ranks as it had men in front, was 84; but when the columns changed ground, and H was drawn up with the front K had, and K with the front H had, the number of ranks in both columns was 91. Find the number of men in each column.

53. A certain number of sovereigns, shillings, and sixpences, together amount to £8 6*s.* 6*d.*, and the amount of the shillings is a guinea less than that of the sovereigns, and a guinea and a half more than that of the sixpences. Find the number of each coin.

54. There is a number composed of 2 figures, of which the figure in the tens' place is 4 times that in the units', and if 54 be subtracted from the number the difference is expressed by the same digits reversed. What is the number?

55. Each one of a company of persons bows once to each of the others. If the company had been $\frac{1}{3}$ less, the number of bows would have been less by 18. Find the number of persons present.

CXI.

PROGRESSIONS, &c.

1. Define arithmetical progression and harmonical progression. Show that if a^2, b^2, c^2 are in A.P., $b+c$, $c+a$, $a+b$ are in H.P.

2. The first term of an arithmetical progression is 2, and the fifth is 18. How many terms must be taken to make the sum 800?

3. Sum (1) the arithmetical series $58+54+\ldots$ to 30 terms;
(2) the geometric series $\sqrt{\frac{2}{3}}+\sqrt{\frac{3}{2}}\ldots$ to 5 terms;
(3) the series $1+2x+3x^2+\ldots$ to n terms.

4. Sum the series—
(1) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, &c., to 7 terms;
(2) $1-\frac{\sqrt{3}}{2}$, $1+\sqrt{3}$, $1+\frac{5\sqrt{3}}{2}$, &c., to 9 terms;
(3) 5, 4, $3\frac{1}{5}$, &c., to infinity.

5. If 3 numbers be in arithmetical progression, and a fourth proportional to them be found, the last 3 terms of the proportion will be in harmonical progression.

6. The first term of a geometric series is 1, and if the series be continued indefinitely, any term is the limit of all the following terms. Find the series.

7. (1) Insert 4 arithmetical means between a and b. Sum the series—
(2) 12, $11\frac{1}{4}$, $10\frac{1}{2}$, to 9 terms;
(3) $\frac{5}{7}$, $\frac{15}{14}$, $\frac{45}{28}$, to 6 terms;
(4) $\sqrt{6}$, $\sqrt{2}$, $\sqrt{\frac{2}{3}}$, to infinity.

8. A sets out from a certain place, and travels 1 mile the first day, 2 miles the second day, 3 miles the third day, 4 miles the fourth day, and so on; B sets out 5 days after A, and travels 12 miles a day. How far will B travel before overtaking A?

9. Sum the series—

(1) $9+8\frac{1}{3}+7\frac{2}{3}+$ .....to 50 terms;

(2) $\frac{1}{3}+2+12+$ .....to 5 terms.

10. Sum the series—

(1) 1, $2\frac{1}{2}$, 4, &c., to 10 terms;

(2) $\frac{3}{8}$, $\frac{1}{4}$, $\frac{1}{6}$, &c., to 6 terms;

(3) ·23, ·0023, ·000023, to infinity.

11. Find the harmonic mean between 2 and 7.

12. Insert (1) 2 arithmetic means, (2) 3 geometric means, between a and b.

13. Sum the series—

(1) 8, 5, 2, &c., to 12 terms;

(2) $\frac{32}{105}, \frac{16}{35}, \frac{24}{35}$, &c., to 6 terms;

(3) $\frac{\sqrt{3}}{2}, \frac{4\sqrt{3}-3}{8}, \frac{19\sqrt{3}-24}{32}$, &c., to infinity.

14. Assuming $1^2+2^2+3^2+ \ldots . n^2=\frac{n(n+1)(2n+1)}{1\cdot 2\cdot 3}$, find $1+3+6+10+$ to n terms.

15. Prove the formula for the sum of a geometrical progression, and apply it in finding the value of the recurring decimal ·77777 &c.

16. Find the sum of—

(1) $16\frac{1}{3}+14\frac{2}{3}+13+$ &c. to 21 terms,

(2) $\frac{5}{6}+\frac{5}{9}+\frac{10}{27}+$ &c. to n terms;

(3) $\frac{n-1}{n}, \frac{n-2}{n}, \frac{n-3}{n}$, &c., to n terms;

(4) 2, −4, 8, &c., to 9 terms;

(5) 2, $-\frac{4}{3}, \frac{8}{9}$, &c., to infinity.

17. Sum the series—

(1) 27, $\frac{81}{3}$, 24, &c., to 21 terms;

(2) $\frac{x}{y}, \frac{x}{2y}, \frac{x}{4y}$, &c., to infinity;

(3) 20, 10, 5, &c., to 6 terms.

18. Insert 5 harmonic means between -1 and 2^{-1}.

19. Sum the series—

(1) $1, \frac{1}{2}, \frac{1}{4}$, &c.;

(2) $1, -\frac{1}{5}, \frac{1}{25}$, &c., ad infinitum.

20. Sum the series—

(1) $2\frac{1}{2}, 3\frac{3}{4}, 5$, &c., to 12 terms;

(2) $15-3+\frac{3}{5}-$&c. to infinity;

$\frac{a+b}{2}\ a, \frac{3a-b}{2}$ &c. to n terms.

21. When are magnitudes in harmonic progression? Find the H.M. between $a+b$ and $a-b$.

22. If x, y, z, be in geometrical progression, prove that $x^2y^2z^2(x^{-3}+y^{-3}+z^{-3})=x^3+y^3+z^3$.

23. When are four quantities said to be in proportion? What value must be given to x to make $1+x, 2+x, 8-x, 10-x$, proportionals?

24. If $a : b :: c : d$, prove the equality—

$$\frac{a^3+b^3}{c^3+d^3}\times\frac{b}{d}=\left(\frac{a+b}{c+d}\right)^4.$$

25. If $a : b :: c : d : e : f$, prove

$$(a^2+c^2+e^2)^{\frac{1}{2}} : (b^2+c^2+d^2)^{\frac{1}{2}} :: a : b.$$

26. If $x : a :: y : b :: z : c$ &c., prove that

$$mx+ny+pz+\text{&c.} : ma+nb+pc+\text{&c.} :: x : a.$$

27. If $y+z : 3b-c :: z+x : 3c-a :: x+y : 3a-b$,

then $x+y+z : ax+by+cz :: a+b+c : a^2+b^2+c^2$.

28. If $\frac{a}{b}$ and $\frac{a'}{b'}$ be two unequal fractions, prove that $\frac{a+a'}{b+b'}$ is intermediate in value between them.

29. If $a : b = c : d$,

$$a(a+b+c+d) = (a+b)(c+a).$$

30. If $a : b = b : c = c : d$,

$$\sqrt{ab} + \sqrt{bc} + \sqrt{cd} = \sqrt{(a+b+c)(b+c+d)}.$$

31. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$,

$$\frac{a^3+b^3}{c^3e+d^3f} = \frac{b^3(ae+bf)}{d^3(a^2+b^2)}.$$

32. If $a : b = c : d = e : f$,

$$ac : ace+bdf+acd = e^2 : e^3+f^3+e^2d.$$

33. Which is greater—

$$\frac{a^3+b^3}{a^2+b^2} \text{ or } \frac{a^2+b^2}{a+b}?$$

34. Explain fully the meanings of the terms *Ratio* and *Proportion*.

If $a : b = c : d$,

$$a : b = (\sqrt{a}+\sqrt{c})^2 : (\sqrt{b}+\sqrt{d})^2.$$

35. Show that a ratio of greater inequality is increased by subtracting the same quantity from both terms.

CXII.

MISCELLANEOUS QUESTIONS.

1. If $xy+yz+xz=1$, show that

$$\frac{x}{1-x^2} + \frac{y}{1-y^2} + \frac{z}{1-z^2} = \frac{4xyz}{(1-x^2)(1-y^2)(1-z^2)}.$$

2. Prove that when n is a positive integer x^n-y^n is divisible by $x-y$, and $x^{2n}-y^{2n}$ by $x-y$.

3. If $a : b = c : d = e : f$, show that each of these ratios $=$

$$\sqrt{ac+ae+ce} : \sqrt{bd+bf+df} = \sqrt[n]{a^n+c^n+e^n} : \sqrt{b^n+d^n+f^n}.$$

4. Extract the square root of

$$x^{\frac{2}{3}}+2\sqrt{x}+3\sqrt[3]{x}-2x^{-\frac{1}{6}}+x^{-\frac{1}{3}}-1.$$

5. Show that $\frac{a+b+c+d}{p+q+r+s}$ is greater than the least, and less than the greatest, of the fractions $\frac{a}{b}, \frac{p}{q}, \frac{c}{r}, \frac{d}{s}$.

6. If $a : b :: c : d :: e : f$, show that

$$\frac{a-b}{a+b}=\frac{c+d}{c-d}=\frac{e+f}{e-f} \text{ and } \frac{a+c+e}{b+d+f}=\frac{a}{b}.$$

7. Prove that when $a>b$ and $c>d$,

$$(a-b)(c-d)=ac-bc-ad+bd.$$

8. If $\left(\frac{y}{x}+\frac{b}{a}\right)\left(\frac{z}{x}+\frac{c}{a}\right)=\frac{bc}{a^2}$, then $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=0$.

9. Extract the square root of

$$(x+y)^2-\frac{2(x+y)^2}{xy}+\left(\frac{1}{x}+\frac{1}{y}\right)^2.$$

10. If a, b, c, are positive and unequal numbers, show that $\frac{a}{b}+\frac{b}{a}>2$; and that

$$(a+b+c)(ab+ac+bc)>9abc.$$

11. If $x : a :: y : b :: z : c$ &c., show that

$$mx+ny+pz+\ldots : ma+nb+pc :: x : a.$$

12. If $x^2-yz=a^2$, $y^2-xz=b^2$, $z^2-xy=c^2$, show that $x+y+z=\frac{a^2x+b^2y+c^2z}{a^2+b^2+c^2}$.

13. If $y+z : 3b-c :: x+z : 3c-a :: x+y : 3a-b$, then $x+y+z : ax+by+cz :: a+b+c : a^2+b^2+c^2$.

14. Prove that

$$\frac{a^2d^2-b^2c^2}{(ac-bd)^2+(ad-bc)^2}=\frac{a^2}{a^2+b^2}-\frac{c^2}{c^2+d^2}.$$

15. Prove that

$$\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(z-x)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}=1.$$

16. Prove that

$$\frac{(ac+bd)^2+(bc-da)^2}{(a^2+b^2)(c^2+d^2)}=1.$$

17. Show that $(a+b-c)(b+c-a)+(b+c-a)(c+a-b)+(c+a-b)(a+b-c)+(a+b+c)^2=4(ab+ac+bc)$.

18. Divide $8a^{\frac{3}{2}}-12ab^{-\frac{1}{3}}+4a^{\frac{1}{2}}b^{-\frac{2}{3}}$ by $2a^{\frac{1}{2}}-b^{-\frac{1}{3}}$.

19. Prove that

$$(b+c-a)\sqrt{a}+(a+c-b)\sqrt{b}+(a+b-c)\sqrt{c}$$

$$=(a+b+c)(\sqrt{a}+\sqrt{b}+\sqrt{c})-2(a^{\frac{3}{2}}+b^{\frac{3}{2}}+c^{\frac{3}{2}}).$$

20. Find the time in which a sum of money will be doubled at $3\frac{1}{4}$ per cent. compound interest.

21. Find the greatest term in the expansion of $(1+x)^n$.

Ex. $n=-12$, $x=\frac{1}{5}$.

22. Show how to find the present value of an annuity of £A for a given number of years at a given rate compound interest. From the expression obtained, determine the number of years' purchase that should be given for a freehold estate, calculating the interest at 5 per cent.

23. Account for the formula

$$\frac{n(n+1)(2n+1)}{1\cdot 2\cdot 3}$$

always being integral when n is a positive integer.

24. Find n, a whole number, such that $5^{\frac{1}{n}}$ shall have its nearest value to 1·307.

25. If n be a whole number, show that the fourth coefficient of $(x+a)^n$ is the same as the number of combinations of n things taken 3 at a time.

26. Find the eighth term of $\left(x+\frac{1}{x}\right)^{\frac{7}{2}}$.

27. Prove that

$$\left(\frac{1+x}{1-x}\right)^n = 1+n\cdot\frac{2x}{1+x}+\frac{n(n+1)}{1\cdot 2}\left(\frac{2x}{1+x}\right)^2+ \Bigg\} \\ \frac{n(n+1)(n+2)}{1\cdot 2\cdot 3}\cdot\left(\frac{2x}{1+x}\right)^3+\&c.$$

28. There are two towns, A and B; a traveller sets out from A towards B, another from B towards A on the same day, and they meet halfway between A and B. One travels 5 miles the first day, 10 the second, 15 the third, and so on; the other travels 20 miles a day for 2 days, and 35 miles a day afterwards, until they meet. Find the number of days they travelled, and the distance from A to B.

29. Write down as a series in geometric progression the recurring decimal ·PPQQQ. . . . where P, Q contain p, q digits respectively: hence find its value in the form of a fraction. Deduce the ordinary rule for obtaining such value.

30. Prove that a number is divisible by 9, if the sum of its digits is divisible by 9.

31. Find the present value of £1 due 12 years hence, money bearing $4\frac{1}{2}$ per cent. compound interest.

32. (1) Expand $(3a-4x)^{\frac{4}{3}}$ to 4 terms; (2) write down the $r+1$th term of $(1-x)^{-3}$.

33. Explain what is meant by variation, and if $a \propto b$ and $c \propto d$, prove that $ac \propto bd$ and $\frac{a}{c} \propto \frac{b}{d}$.

34. How many permutations can be formed of the letters in the word *Examination*, taken altogether?

35. On how many nights may a different patrol of 5 men be drafted from a company of 36 men? and on how many of these would any one man be taken?

36. Find by Horner's method one value of x from the equation $x^3+8x-15=0$, to 4 places of decimals; first demonstrating that one root at least must lie between the numbers 1 and 2.

37. If $b-a$ be the harmonic mean between $c-a$ and $d-a$, show that $d-c$ is the harmonic mean between $a-c$ and $b-c$.

38. If $\frac{1}{10}$ be the harmonic and $\frac{1}{8}$ the geometric mean between two fractions, determine the fractions.

39. If $a : b :: c : d$, show that $\frac{ax+b}{cx+d}$ has always the same value, whatever be the value of x; also show that

$$\sqrt{a+b} : \sqrt{b} :: \sqrt{c+d} : \sqrt{d}.$$

40. If $A \propto B+\frac{1}{C}$ and $B \propto C$, and if, when $C=1$, $B=1$, $A=4$, find the value of B when $A=5$.

41. How many different numbers can be formed with the digits 1, 2, 3, 4? If a decimal point be put before each of them, what sum will be obtained by adding them together?

42. Calculate the sum which must be paid to purchase an annuity of £100 to begin immediately and to go on for ever, each payment being supposed to be half the preceding one: reckoning compound interest at 5 per cent.

43. Prove the binomial theorem when n is a positive fraction.

44. Show how to find the number of numbers less than any proposed number and prime to it.

45. If 5 be subtracted from the sum of the squares of 4 consecutive numbers all prime to 5, the remainder will be a perfect square.

46. Find the present value of an annuity of £90, to continue 17 years, allowing interest at 5 per cent.

47. Find the number of positive integral solutions of the equation $7x+34y=4030$.

48. Assuming $a^x=1+Ax+Bx^2+Cx^3+$ &c., find B, C, D, &c., in terms of A.

49. Prove that the number of combinations of n things taken r together is the same as the number taken $n-r$ together. How many words can be made of the letters in *the* word *rotation*?

50. In what scale of notation will the common number 5261 be expressed by 40205? What are the greatest and least numbers that can be formed with 5 digits in that scale?

51. Express $\sqrt{50}$ in the form of a continued fraction.

52. If $x=p+1$, show that $x^6>p(x^5+x^4+x^3+x^2+x+1)$.

53. A rectangle is contained by two sides a and b: if a is shortened by 2 feet and b is lengthened by 3 feet, the area is made 3 square feet less; but if a is lengthened by 4 feet and b is shortened by 3 feet, the area is made 3 square feet greater. Find the lengths of the sides.

54. What are eggs per dozen when, if there are two less in a shilling's worth, the price is increased one penny per dozen?

55. Assuming the binomial theorem, examine the condition that $(1+x)^n$ may converge.

56. Develope $(1+\sqrt{-3})^6$; and exhibit the result in its simplest form.

57. If $a:b=c:d=e:f$, show that each of these ratios

$$=\sqrt{ac+ae+ce}:\sqrt{bd+bf+df}$$

$$=(a^n+c^n+e^n)^{\frac{1}{n}}:(b^n+d^n+f^n)^{\frac{1}{n}}.$$

58. Having given the first and the last terms and the number of terms in a geometric progression, find the intermediate terms.

59. Divide the number n into 4 parts in A.P. such that the sum of their cubes shall be $\frac{n^3}{10}$.

60. Extract the $\sqrt{12}$ to 7 places of decimals, and show that, after $n+1$ figures have been obtained by the ordinary method, n more may be obtained by division only; $2n+1$ being a whole number.

61. If 3 dice be thrown, what is the chance that they will all turn up aces?

62. If $y=x-\frac{x^2}{2}+\frac{x^3}{3}-$&c., show how to determine x.

63. If the sum of the nth and $2n$th terms of a geometrical progression be given, and also the sum of the $2n$th and $3n$th terms, find the first term and the common ratio.

64. Sum the series of which $2r+3+2\times 3^r$ is the rth term.

65. Find the present value of an annuity of £A to be continued for n years at r per cent. compound interest.

66. If the annuity become a perpetuity, find the number of years' purchase that must be paid for it at 5 per cent.

67. Show how to find the number of shot in a complete rectangular pile; and apply the result to find the number of shot in an incomplete pile having 200 and 160 shot in the sides of its base, 70 and 30 in the sides of its top course.

68. The product of 5 numbers in A.P. is 945, and their sum is 25; find the numbers.

69. Expand $(a^2-x^2)^{-2}$ to 6 terms by the binomial theorem.

70. If four magnitudes be proportionals, the sum of the greatest and least is greater than the sum of the other two.

71. The number of combinations of n things, taken 4 together, is to the number taken 2 together as 51 to 2. Find n.

72. Write down all the numbers that can be composed of the four digits 3, 4, 5, 6, which are divisible by 11.

73. Six papers are to be set in an examination, two of them in mathematics; in how many different orders may the papers be given, provided only that the two mathematical papers do not come together?

74. Find a third proportional to $\frac{2a^2b^2}{a^2+b^2}$ and ab.

75. Prove that the coefficient of x^{2n} in the expansion of $\frac{1}{(1-x)(1+x)^4}$ is $\frac{(n+1)(2n^2+n+3)}{3}$.

76. The difference between the numbers of shot in the two sides of the base of an incomplete rectangular pile is 7, and the number in the longer side of the top course is 15. How many shot are required to complete the pile?

77. A dealer having laid in a stock of a certain article, began to sell it by retail. The first day he made a profit of $3d.$, the second of $4{\cdot}2d.$, and so on, the profit increasing by $1{\cdot}2d.$ a day, until the stock was disposed of; he then found that he had realised a profit of $14s.\ 3d.$ How many days did he continue to sell the article?

78. Prove that the difference of the squares of any two odd numbers is divisible by 8.

79. Expand $\sqrt{11}$ in the form of a continued fraction.

80. If n be a very large number, prove that $\log(n+1) - \log n$ is very nearly equal to $\frac{M}{n}$, M being the modulus of the system of logarithms.

81. Write down the first five terms of the expansion of $(1+x)^{2x}$, and show, when n is a whole number, that the sum of the coefficients of the expansion $=(2^n)^2$. Also assuming the $(p+1)$th coefficient of the expansion equal to the $(p+3)$th coefficient, prove $p=n-1$.

LOGARITHMS.

CXIII.

Find by the tables the value of—

1. $\cdot03571 \times \cdot2568$.

2. $\dfrac{8352 \times 3{\cdot}69}{(30{\cdot}57)^3}$.

3. $(\cdot0874)^{\frac{1}{5}}$.

4. $\sqrt[3]{\cdot00061475}$.

5. 8^{2^6}.

6. $\dfrac{35^{\frac{1}{2}} \times 12^{17}}{13}$.

7. $\left\{\dfrac{(\cdot21)^{\frac{1}{2}}\times(\cdot21)^{\frac{1}{3}}\times(\cdot21)^{\frac{3}{4}}\times365^{2}}{(\cdot00416)^{2}\times(\cdot3125)^{3}\sqrt[4]{365}}\right\}^{\frac{2}{5}}$.

8. $(34)^{\frac{1}{5}}\times(24\cdot68)^{\frac{1}{3}}$.

9. A fourth proportional to 254·5, 4000·6, ·028.

10. $\dfrac{\cdot6325\times\cdot001355}{1\cdot191\times5\cdot355}$.

11. $\dfrac{(\cdot0476)^{3}}{(3\cdot005)^{\frac{1}{3}}}$.

12. A mean proportional to $\sqrt{4\cdot756}$ and $\sqrt[3]{\cdot0078^{2}}$.

13. $\dfrac{15\cdot6^{\frac{1}{3}}\times(\cdot0045)^{\frac{3}{4}}}{(\cdot00065)^{\frac{1}{2}}}$.

14. $\left(\dfrac{347}{123}\right)^{\frac{1}{5}}$.

15. $\sqrt[5]{\cdot023476}$.

16. $\left(\dfrac{5}{7}\right)^{\cdot1345}$.

17. $\dfrac{1\cdot265\times\cdot01628}{2\cdot2825\times64\cdot28}$.

18. $\sqrt{2543\times\cdot1726}$.

19. $\left\{\dfrac{7432}{\sqrt{29}}\right\}^{\frac{3}{7}}$.

20. $\left\{\dfrac{5\sqrt[3]{138}}{\sqrt[5]{\cdot01}}\right\}^{\frac{1}{2}}$.

21. The square root of $\dfrac{\sqrt[3]{\cdot0125}\times\sqrt{31\cdot15}}{\cdot00081}$.

22. $\dfrac{(42\cdot666)^{12}}{1\cdot147}\times\dfrac{(\cdot0765)^{10}}{194\cdot32}$.

23. $\sqrt{4^{\frac{1}{3}}+5^{\frac{1}{3}}}$.

24. A fourth proportional to $\sqrt{1\cdot027}$, $\sqrt[3]{2\cdot546}$, and $(31\cdot027)^{2}$.

25. A third proportional to 1·6 and $\dfrac{\sqrt[3]{44}}{794\cdot81}$.

26. A fourth proportional to $\sqrt[3]{8\cdot37}$, $(\cdot84)^{2}$, $\sqrt[5]{\cdot054321}$

27. The sixth root of ·000000004096.

28. The 5th term of 5, 15, 45...

29. $\dfrac{\sqrt[3]{172500}}{\sqrt[5]{\cdot01}}$.

30. A mean proportional between

$$(2{\cdot}0736)^{\frac{1}{2}} \text{ and } (\cdot 000000262144)^{\frac{1}{3}}.$$

31. $\dfrac{70{\cdot}25 \times {\cdot}67957}{21000 \times {\cdot}018252}$.

32. $\dfrac{42 \times ({\cdot}0016)^{\frac{7}{4}}}{\sqrt[3]{108}}$.

33. $\dfrac{250}{{\cdot}045}\left\{1-1{\cdot}045^{-20}\right\}$.

34. $\dfrac{7{\cdot}014^3-1}{7{\cdot}014^3+1}$.

35. A fourth proportional to $41{\cdot}06\frac{1}{3}$, ${\cdot}00056^2$ and $120\frac{2}{3}$.

36. x^3-2x-5 when $x=2{\cdot}0946$.

37. Value of x when $10^{\frac{1}{x}}=2{\cdot}45$.

38. Value of x when $\left(\frac{5}{3}\right)^x=17{\cdot}4$.

39. $\dfrac{1}{21979}$.

40. $1{\cdot}56^{1{\cdot}5} \times 1{\cdot}56^2 \times 1{\cdot}56^{2{\cdot}5}$ to 6 factors.

41. $1+\frac{19}{20}+\left(\frac{19}{20}\right)^2+\left(\frac{19}{20}\right)^3+$ to 100 terms.

42. £1875 17*s.* 6*d.* $\times$ $1{\cdot}035^{20}$.

CXIV.

1. Define the terms 'Logarithm,' 'Base,' 'Mantissa,' 'Characteristic,' 'Modulus.'

2. Prove that $\log(3^5 \times 4^6)=5\log 3+6\log 4$.

3. Prove that $\log_{10}N=\log_e N \times \dfrac{1}{\log_e 10}$.

4. Explain the method of finding the characteristic of $\log_{10}$ ·0005.

5. What are the logs of 10, 100, ·001 to the base 10; and of 1000 to the base ·01?

6. Given log 2=·30103, log 18=1·2552725, log 21=1·3222193: find (1) log ·0075, (2) log 31·5, (3) log 128, (4) log 125, (5) log 2500.

7. In any system, what are the logs (1) of the base, (2) of 1, (3) of 0?

8. Prove that in the common system of logs the log of a number having n digits in the integral part is $n-1$; and that the characteristic of the log of a decimal having n ciphers between the point and the first significant digit is $-(n+1)$. Also show that the same mantissa serves for all numbers consisting of the same digits in the same order.

9. Given log 2=·30103, log 3=·4771213, log 7=·8450980, find (1) log 60, (2) log ·03, (3) log 1·05.

10. Given log 1752=3·2435341, log 1752·1=3·2435589, construct a table of proportional parts, and find

log 17·520−87. Find without tables $\log_{10}\sqrt{1000}$.

11. Divide 14·326847 by 9.

12. What is the logarithm of $81\sqrt[3]{3}$ to the base $3\sqrt{3}$? How would you transform a system of logs from the base 8 to the base 4? What advantages belong to a system with 10 at the base?

13. Prove that $\log_b a \times \log_a b = 1$. Can the same number have more than one logarithm, real or imaginary? If log 25=·6, find the base.

14. Prove that $\log_{10} N = \log_{10} 5 \times \log_5 N$, and find without tables $\log_{25} 8$, having given $\log_{10} 2 = ·30103$.

15. Given $\log_{10} 2 = ·30103$, find $\log_{10} 125$, and the $\log_{10}$ of the mean proportional between

$$\frac{(1·25)^{\frac{1}{3}}}{(·00016)^6} \text{ and } \frac{(1·25)^{\frac{2}{3}}}{1000}.$$

16. Of what number is −5 the log to the base 10? What is the log of 256 to the base 2? If $\log_{10} 2 = ·30103$, and $\log_{10} 7 = ·845098$, find $\log_{10} ·0035$; $\log_{10} N = 1·5$, find N without tables.

17. If $\log_{10} 125 = 2·09691$, find $\log_{10} 2$. Find without

tables, to 3 places of decimals, the number of which 1·3 is the common log.

18. Solve the equations :—

(1) $\left.\begin{matrix} a^{2x}-b^{2y}=c \\ a^{x}-b^{y}=d \end{matrix}\right\}$ (2) $\left.\begin{matrix} \dfrac{3^{x}}{2^{x+y}}=8 \\ x=3y \end{matrix}\right\}$

(3) $\left(\dfrac{6}{11}\right)^{2x^2}=\left(\dfrac{3}{4}\right)^{4x}$.

(4) $2^{3x}\times 6^{2x-5}=3^{x(x-1)}\times 2^{5x}$.

(5) $\left.\begin{matrix} 3^{2x}+3^{x}=6 \\ 4^{2xy}-2\cdot 4^{xy}=8 \end{matrix}\right\}$ (6) $\left(\dfrac{7}{3}\right)^{x}=100$.

19. Find the number of digits in $\dfrac{3^{20}\times 5^{15}}{2^{11}}$.

20. Explain what is meant by systems of logs calculated to different bases ; what is the base of each of the systems most commonly used in mathematical calculations? State briefly any advantage belonging to these systems. Show $\log_2 N=3\log_8 N$.

21. If all the logs in the common tables were doubled, they would still be the logs of the same numbers as before, but the base would now be $\sqrt{10}$. Prove this. What would the bases be if the existing logs were (α) multiplied by n, (β) divided by n?

22. Find $\log a^{\frac{p}{q}}$ to the base $a^{\frac{m}{n}}$.

23. Show that the characteristic of the log of a number can be determined by inspection when the base is 10. From $\log_{10}2$ and $\log_{10}7$, find $\log_{10}\cdot 0000432$.

24. Explain why $\log_{10}3\cdot 56$ and $\log_{10}\cdot 0356$ have the same mantissa : $\log_{10}\cdot 0356$ and $\log_{10}\cdot 04$ have the same characteristic.

Show that

$$\log P^{\frac{1}{n}}=\frac{1}{n}\log P\,;\quad \log_a P=\frac{\log_e P}{\log_e a}.$$

25. Assuming that

$\log_e(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+$ &c., show that $\log_e(n+1)$

$=\log_e n+\frac{2}{2n+1}+\frac{1}{3}\cdot\frac{2}{(2n+1)^3}+\frac{1}{5}\cdot\frac{2}{(2n+1)^5}+$ &c.

26. Investigate the expansion for $\log(1+x)$ in powers of x, and show that the series is convergent when x is less than unity.

27. If e be the base of Napier's system of logarithms, expand e^x, and show that the series for calculating e converges.

28. Investigate an expression for the logarithm of a number to a given base, in the form of a converging series.

29. Obtain the series—

$$\log_e m=2\left\{\frac{m-1}{m+1}+\frac{1}{3}\left(\frac{m-1}{m+1}\right)^3+\frac{1}{5}\left(\frac{m-1}{m+1}\right)^5+\text{ &c.}\right\}.$$

30. Show that

$$\log_e\left(1+\frac{p}{q}\right)=2\left\{\frac{p}{2q+p}+\frac{1}{3}\left(\frac{p}{2q+p}\right)^3+\frac{1}{5}\left(\frac{p}{2q+p}\right)^5+\ldots\right\}.$$

PLANE GEOMETRY

CXV.

1. AB is the side of an equilateral triangle inscribed in a circle whose diameter is AOC and centre O.

Prove (1) the triangle BOC is equilateral;

(2) $AB^2=3\,AO^2$.

2. If two chords AB, CD in a circle cut each other at right angles, the sum of the opposite arcs AC, BD will be a semicircle.

3. The angle between the lines bisecting two adjacent exterior angles of any quadrilateral is equal to the angle between the lines bisecting the two interior opposite angles.

4. If through the extremities of a chord common to any number of circles two lines be drawn cutting the circles, the lines joining their other points of intersection with each circle are parallel.

5. In a triangle the sides about the vertical angle are 25 and 16, the line bisecting the vertical angle is 12. Find the base.

6. If AB, CD be the opposite sides of a quadrilateral circumscribing a circle whose centre is E, prove that the angles AEB and CED are together equal to two right angles.

7. The sides AB, AC of a triangle are bisected in E, F. AD is drawn perpendicular to the base BC. If ED, DF be joined, the angle FDE is equal to the angle BAC, and the area of the triangle ABC is double that of the quadrilateral AFDE.

8. Two circles touch each other; a common tangent is drawn to them at their point of contact; from any point in this tangent a tangent is drawn to each circle. Show that these latter tangents are equal to each other.

9. Through any point D in the base of a triangle ABC, straight lines DE, DF are drawn parallel to the sides AB, AC, and meeting the sides in E, F, and EF is joined. Prove that the triangle AEF is a mean proportional between the triangles FBD, EDC.

10. Given two straight lines, construct the rhombus of which they are the diagonals, and show that it is greater than any other parallelogram having the same straight lines for diagonals.

11. Prove that if a quadrilateral be bisected by both of its diagonals, it must be a parallelogram.

12. Prove that the area of the isosceles triangle is greater than that of any other triangle on the same base, and having an equal perimeter.

13. ABC is a triangle inscribed in a circle, BD is drawn

parallel to the tangent to the circle at the point A, and meets AC in D. Prove AC : AB :: CB : DB.

14. From any point in the base of a triangle lines are drawn parallel to the sides. Show that the intersection of the diagonals of every parallelogram thus formed lies in a certain straight line.

15. If two sides AD, BC of a quadrilateral inscribed in a circle ABCD be produced to meet in E, the circle described about ECD will have the tangent at E parallel to AB.

16. If from the angles at the base of any triangle perpendiculars be drawn to the opposite sides, produced if necessary, the line joining the points of intersection will be bisected by the perpendicular drawn to it from the middle of the base.

17. ABC is a triangle, C an obtuse angle, BF the perpendicular on AC produced. If in the base AC a point E be taken, such that $AC = 2\ EF$, prove $AB^2 + BC^2 = 2(AE^2 + BE^2)$.

18. Prove that the rectangle contained by a side of the isosceles triangle, Euc. IV. 10, and the base, together with the square on the base, is equal to the square on a side of the triangle.

19. In an isosceles triangle show that the base is a mean proportional between the diameter of the circle inscribed in the triangle and the diameter of the circle that is escribed touching the base, and the equal sides produced.

20. The sides of any quadrilateral figure are together greater than the sum of its diagonals.

21. The sides AB, AC of a triangle are bisected in D and E, and BE, CD are produced until EF=EB and GD=DC. Show that the line GF passes through A.

22. Perpendiculars are let fall from two opposite angles of a rectangle upon a diagonal. Show that they will divide the diagonal into equal parts if the square on one side of the rectangle be double that on the other.

23. Describe a circle touching one side of a triangle and the other two sides produced.

24. Given the centres of the three escribed circles of a triangle; construct the triangle.

25. If E be a point without a circle, and straight lines EAB, ECD are drawn to cut the circle in A, B and C, D, show that the angle CEA is equal to half the difference of the angles subtended by AC and BD at the centre of the circle.

26. Show that the square on the sum of two lines, together with the square on their difference, is double the squares on the two lines.

27. If two circles intersect in A and B, and through P, any point in the circumference of one of them, the chords PA, PB be drawn to cut the other circle in C and D, show that CD is parallel to the tangent at P.

28. The sum of the alternate angles of any hexagon inscribed in a circle is equal to four right angles.

29. ABC is an isosceles triangle, C being an obtuse angle, AD a perpendicular drawn from A to BC produced; the square on AB is equal to twice the rectangle contained by BC and BD.

30. If two circles intersect in A and B, and AC, AD be two diameters, prove that the line CD will pass through B.

31. If two circles cut each other, show that there is a certain straight line from any point of which exterior to the circles the tangents drawn to both circles will be equal.

32. A pyramid is cut by a plane parallel to the base. Show that the section is a figure which is similar to the base.

33. P is a point without any number of concentric circles, O is their common centre; from P any number of tangents are drawn to the circles. Show that the circle described on PO as a diameter will pass through all the points of contact.

34. If the straight line AB be divided in C, so that the rectangle contained by AB and BC is equal to the square

on AC, show that if in AC, CD be taken equal to CB, the rectangle AC . AD=CD^2.

35. If c, c_1 be two chords of a circle that intersect at right angles, d its diameter, x, y the segments of the chord c, prove that $2(x^2+y^2)=d^2+c^2-c_1^2$.

36. A circle is described about a triangle ABC, which has the side AB=side AC; from A a line is drawn meeting the base of the triangle in D and the circumference of the circle in E. Prove that the circle which passes through B, D, E touches AB.

37. ABCD is a parallelogram, and F a point in the diagonal AC; through F a line is drawn parallel to AB, meeting AD in G, and BC in H; and through F a line is drawn parallel to AD, meeting AB in E, and DC in K. Show that GK and EH produced meet in AC produced.

38. If two straight lines, AEB, CED, in a circle intersect in E, the angles subtended by AC and BD at the centre are together double of the angle AEC.

39. If a quadrilateral ABCD be inscribed in a circle,

$$AB.BC+AD.DC : BC.CD+BA.AD :: BD : AC.$$

40. Through a point K, within a parallelogram ABCD, straight lines are drawn parallel to the sides. Show that the difference of the parallelograms of which KA, KC are diagonals is twice the triangle BKD.

41. Show that the difference of the squares on the side of a regular pentagon and a regular decagon, inscribed in the same circle, is equal to the square on the radius.

42. Show that the equilateral triangle described about a circle has double the perimeter of that inscribed in the same circle.

43. If any number of triangles on the same base BC and on the same side of it have their vertical angles equal, and perpendiculars meeting in D be drawn from BC on the opposite sides, find the locus of D; and show that all the lines which bisect the angle BDC pass through the same point.

44. O is the middle point of the hypothenuse AB of a right-angled triangle ABC. Show that OA, OB, OC are all equal to one another.

45. Inscribe a sphere in a triangular pyramid.

46. Construct a square three times as large as a given square.

47. ACB is a triangle whose base AB is divided in E and produced to F, so that AE : EB :: AF : FB :: AC : CB. Prove that $(EF-CF)(EF+CF)=EC^2$.

PLANE TRIGONOMETRY.

CXVI.

1. State the different methods of measuring angles. Show that English seconds may be reduced to French seconds by dividing by ·324.

2. Define the sine, secant, and tangent of an angle. Show that the versed sine of an angle lies between 0 and 2.

3. Find cos 30° and sin 45°. Express all the trigonometrical ratios of an angle in terms of the versed sine.

4. Trace the changes of cot A in sign and magnitude, as A changes continuously from 0° to 360°.

5. Explain the terms complement and supplement, illustrating them by examples. Show, by example, how to convert English degrees into French. Explain the symbol π and the No. 3·1416 ; and exhibit the algebraical signs of the sine, cosine, and tangent, in the third quadrant.

6. Show, geometrically, that $\sin(90°+A)=\cos A$; $\cos(90°+A)=-\sin A$; $\tan(-A)=-\tan A$.

7. Find the numerical values of sin 60°, cos 60°, tan 75°.

8. Prove that $\cos(A-B)=\cos A \cdot \cos B+\sin A \cdot \sin B$. Deduce the expression for $\sin(A-B)$.

If $A+B$ is $<90°$, then $\cot A \cdot \cot B$ is >1.

9. Define the cosecant of an angle, and trace its variation in sign and magnitude through the four quadrants.

10. Prove—

(1) $\sin A+\sin B=2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$;

(2) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$.

11. Show that $\dfrac{\tan(45°+A)}{\tan(45°-A)}=\dfrac{2\cos A+\sin A+\sin 3A}{2\cos A-\sin A-\sin 3A}$.

12. Prove the formulæ—

(1) $4\cos^3 A\sin 3A+4\sin^3 A\cos 3A=3\sin 4A$;

(2) $\sin^3 A\sin 3A+\cos^3 A\cos 3A=\cos^3 2A$.

13. Prove that, (1) $\dfrac{\cos A-\cos 3A}{\sin 3A-\sin A}=\tan 2A$;

(2) $(\cos A+\cos B)^2+(\sin A+\sin B)^2=4\cos^2\frac{1}{2}(A-B)$;

(3) $\cot(A+B)=\dfrac{1-\tan A\tan B}{\tan A+\tan B}$, and hence find $\cot 75°$;

(4) $4\cos A\cos(120°-A)\cos(120°+A)=\cos 3A$;

(5) $\tan 2A+\tan A=\dfrac{2\sin 3A}{\cos A+\cos 3A}$.

14. Prove that—

(1) $\dfrac{\sin A\pm\sin B}{\cos A+\cos B}=\tan\frac{1}{2}(A\pm B)$.

(2) $\cos A=\frac{1}{2}\{\sqrt{1+\sin 2A}+\sqrt{1-\sin 2A}\}$, A being less than 45°.

(3) $\sin nA+\sin(n-2)A=2\sin(n-1)A\cos A$.

15. Prove that—

(1) $(\sin A+\sec A)^2+(\cos A+\operatorname{cosec} A)^2=(1+\sec A\operatorname{cosec} A)^2$.

(2) $\sin 2A\sin A=\cos A-\cos A\cos 2A$; and express $\cot 2A$ in terms of $\cot A$.

16. Trace the changes in the sign of $\sin A \cos (A-15°)$ as the angle A increases from 0° to 360°.

17. Prove the formulæ—

(1) $\cos (A+B)=\cos A \cos B-\sin A \sin B$;

(2) $\dfrac{\sin A+\sin B}{\sin A-\sin B}=\tan \frac{1}{2}(A+B) \cot \frac{1}{2}(A-B)$.

18. Express sin A in terms of sin 2 A, and apply the result to find sin 105° from sin 210° $=-\frac{1}{2}$.

19. Prove the formula :—

$$\tan \frac{A+B}{2} \cdot \tan \frac{A-B}{2}=\frac{\cos B-\cos A}{\cos B+\cos A}.$$

20. Prove that $\sin \frac{1}{2}A=\frac{1}{2}\{\pm\sqrt{1+\sin A}\pm\sqrt{1-\sin A}\}$

$$\cos \tfrac{1}{2}A=\tfrac{1}{2}\{+|\sqrt{1+\sin A}\mp\sqrt{1-\sin A}\}$$

Express the signs correctly when A lies between 315° and 360° ; and find the sine of 9°.

21. (1) Prove $\cos 3A=4\cos^3 A-3\cos A$. Find sin 18°, and thence deduce cos 36°.

(2) $(\cos A-\cos B)^2+(\sin A-\sin B)^2=4\sin^2 \dfrac{A-B}{2}$.

(3) $\tan A+\cot A=2 \operatorname{cosec} 2A$.

(4) $\cos 10A+\cos 8A+3\cos 4A+3\cos 2A$
$=8\cos A \cos^3 3A$.

22. Obtain formulæ, adapted to logarithmic computation, for determining the angles of a triangle when its sides are known. If the sides be 2 ft., 8 ft., and 9·99992 ft., show that the cosine of $\frac{1}{2}$ the greatest angle is approximately $\frac{1}{200}$, and that the angle contains approximately $\dfrac{180\pi-1\cdot8}{\pi}$ degrees.

23. Prove that $\tan A=\dfrac{3\tan \frac{1}{3}A-\tan^3 \frac{1}{3}A}{1-3\tan^2 \frac{1}{3}A}$.

24. Without assuming the formula for $\cos (A+B)$, prove that $\cos 2A=\cos^2 A-\sin^2 A$.

25. (1) Show that $\tan A=\frac{\cos A-\cos 3A}{\sin A+\sin 3A}$.

(2) $\frac{\sin 5A-\sin 3A}{\cos 5A-\cos 3A}=-\cot 4A$.

26. Given $\tan(a+x)\tan(a-x)=\frac{1-2\cos 2a}{1+2\cos 2a}$, find $\sin x$.

27. What are the values of sin 30°, cos 30°, sin 45°, cos 45°? Deduce from them the value of sin 15° to 6 places of decimals.

28. In the triangle ABC, given $a=3492{\cdot}76$, $b=2471{\cdot}34$, $B=20°\ 21'$, to find c.

29. If a, b, B are given to solve a triangle, $b<a$, and if c_1, c_2 be the two values found for determining c, the third side, prove that $b^2+c_1c_2=a^2$.

30. Prove that (1) the sine of an angle, or of an arc, is equal to the cosine of its complement;

(2) $\cos A-\cos B=-2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$;

(3) $\frac{1-\cos A}{1+\cos A}=\tan^2\frac{A}{2}$;

(4) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$;

(5) $\frac{\sin 2A+\sin A}{\cos 2A+\cos A}=\tan\frac{3}{2}A$;

(6) $\tan A=2-\sqrt{3}$, when $\sin 2A=\cdot5$.

31. In a plane triangle $c=732$, $b=846$, $a=945$. Find the greatest angle.

32. Find x from the equation $\cos^{-1}\frac{1-x^2}{1+x^2}+\tan^{-1}\frac{2x}{1-x^2}=\frac{4\pi}{3}$, and prove that $\sin(36°+A)-\sin(36°-A)$ $=\sin A+\sin(72°+A)-\sin(72°-A)$.

33. Investigate the area of a circle, and of any sector of a circle.

34. Prove that $\sin 4A=4\sin A\cos^3A-4\cos A\sin^3A$.

35. In a right-angled triangle, given the side $a=5618{\cdot}97$, and the opposite angle $A=61°\ 40'$, find the hypothenuse.

36. The sides of a right-angled triangle are 13 and 7543; obtain the formulæ for determining the angles with accuracy.

37. If R, r be the radii of the circles described about, and inscribed in, a given triangle, O, O′ their centres, show that

$$OO'^2=R^2-2Rr.$$

38. Express all the trigonometrical ratios in terms of the sine.

39. Prove the following formulæ:—

(1) $\sin(A+B)=\sin A\cos B+\cos A\sin B$, where A is $>90°$ and $A+B<180°$;

(2) $\tan\frac{A}{2}=\frac{1-\cos A}{\sin A}$;

(3) $\sin(A+B)+\sin(A-B)=2\sin A\cos B$;

(4) $\sin(A+B)\times\sin(A-B)=\sin^2 A-\sin^2 B$, or $\cos^2 B-\cos^2 A$.

40. Write down the four least values of θ which satisfy the equation $\tan\theta=1$.

41. Prove that $\tan\left(30°+\frac{\theta}{2}\right)\tan\left(30°-\frac{\theta}{2}\right)=\frac{2-\sec\theta}{2+\sec\theta}$.

42. Given $\sin 60°\ 27'\ 30''=\cdot87$, to find $\cos 14°\ 46'\ 15''$ to 4 places of decimals.

43. The three sides of a triangle being 357, 476, 595 respectively, find each of the angles independently, and verify the accuracy of the solution.

44. Given the base, a, the vertical angle, A, and the difference, d, of the sides of a plane triangle, to find the sides.

45. Prove the formulæ:—

(1) $\tan(45°+\frac{1}{2}A)+\cot(45°+\frac{1}{2}A)=2\sec A$;

(2) $\frac{\sin 60°+\sin 30°}{\sin 60°-\sin 30°}=\frac{\tan 60°+\tan 45°}{\tan 60°-\tan 45°}$.

46. If $\sin\theta=m\sin\phi$ and $\tan\theta=n\tan\phi$, find the sines and cosines of θ and ϕ.

47. In a triangle, given $a=215$, $b=248$, $C=30°$, find its area.

48. In a triangle, given $c=1124$, $b=640$, $A=128° 4'$, find B and C.

49. Prove $\tan 2A=\frac{2\tan A}{1-\tan^2 A}$.

50. Two sides of a triangle are 850, 960 feet respectively, and the included angle is 56° 13′. Find the remaining angles.

51. Find the greatest angle of the triangle whose sides are 299, 325, and 442 yards.

52. $\cot 5A=\frac{1}{\sqrt{3}}$; find $\sin 5A$, and express generally the values of the angle A.

53. The three sides of a triangle are 460, 575, 805; find the greatest angle.

54. In general, if a, b, c be the sides of a triangle, how may it be determined whether or not the triangle contains an obtuse angle?

55. Show that $\tan^{-1}\frac{a}{b+c}+\tan^{-1}\frac{b}{a+c}=\frac{\pi}{4}$.

56. From any point O draw straight lines to the vertices of a plane triangle ABC; prove that

$$\sin OAC \sin OCB \sin OBA=\sin OCA \sin OBC \sin OAB.$$

57. Find the remaining parts of the triangle ABC, when the parts given are $c=24992$, $a=7946$, $A=8° 19' 35''$.

58. Find x from the formula

$$\sin 3A=4 \sin A \sin (x+A) \sin (x-A).$$

59. If in a triangle the angle A and the sides a and b are given to solve it, show that the third side c may be obtained from the expression $c=b\cos A\pm\sqrt{a^2-b^2\sin^2 A}$, and thence explain the ambiguity of the expression when b is greater than a.

60. Prove that $\cot 3A = \frac{1-3\tan^2 A}{3\tan A - \tan^3 A}$.

61. If p_1, p_2, p_3 be the perpendiculars from the angles of a triangle upon the opposite sides, r the radius of the inscribed circle, r_1, r_2, r_3 the radii of the escribed circles, prove $\frac{1}{p_1}+\frac{1}{p_2}+\frac{1}{p_3}=\frac{1}{r}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}$.

62. For what angles, less than 360°, is the tangent = sin 90°?

63. Investigate a formula from which tan A may be found in terms of tan 2 A, and find tan 22° 30′.

64. Prove

$$\frac{\tan A+\tan B}{\cot A+\cot B}=\frac{\cos(A-B)-\cos(A+B)}{\cos(A-B)+\cos(A+B)}.$$

65. The three sides of a triangle are 508, 401, 299 ft.; find the greatest angle.

66. Express $\frac{4}{17}$ of a right angle, (1) in degrees &c. English; (2) in grades; (3) in circular measure.

67. Show that

$$\cos A=\cos(2n\,.\,180°+A)=-\cos\{(2n+1)\,180°-A\}$$
$$=\mp\cos\{(2n+1)\,180°+A\}=\cos(2n\,.\,180°-A),$$

n being a positive integer.

68. If $\sin A=\sqrt{2}\sin B$, and $\tan A=\sqrt{3}\tan B$, determine A and B.

69. Prove the sine of an angle equal to the sine of its supplement. For what angle is the sine of the supplement equal to the sine of the complement? Find sin 15°.

70. Express cos 3 A in terms of cos A.

71. Find (1) sin 18°; (2) cos 36°.

72. Having given $\frac{\tan^2\alpha}{\tan^2\beta}=\frac{\cos\beta(\cos x-\cos\alpha)}{\cos\alpha(\cos x-\cos\beta)}$, show that $\tan^2\frac{x}{2}=\tan^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}$.

73. Given $\sin 50° = \cdot 766$, find $\sin 25°$ without using the tables.

74. Find $\sin 15°$, and thence prove that $\tan 15° = 2 - \sqrt{3}$.

75. Show how to calculate $\sin 7° 30'$.

76. If c be the hypothenuse of a right-angled triangle, whose sides are a and b, prove that

$$\log_e c = \tfrac{1}{2}\log_e 2 + \tfrac{1}{2}\log_e a + \tfrac{1}{2}\log_e b +$$
$$\left(\frac{a-b}{a+b}\right)^2 + \frac{1}{3}\left(\frac{a-b}{a+b}\right)^6 + \frac{1}{5}\left(\frac{a-b}{a+b}\right)^{10} + \&c.$$

77. In any triangle right-angled at C

$$\sin^2 \tfrac{1}{2}A - \sin^2 \tfrac{1}{2}B = \frac{a-b}{2c}.$$

78. In a right-angled isosceles triangle the radius of the inscribed circle is 1 foot. Find the sides.

79. Adapt the formula $c^2 = a^2 + b^2 - 2ab \cos C$ to logarithms.

80. The angles of a plane triangle form a geometrical progression of which the ratio is $\frac{1}{2}$. Show that the greatest side is to the perimeter as 2 sin $12\frac{6}{7}°$ is to unity.

81. Show that unity is the limiting value of $\frac{\sin\theta}{\theta}$ as θ is indefinitely diminished. Investigate a method of obtaining an expression for the area of a circle of given radius.

82. Find an angle of which the sine is to the cosine as $\sqrt{3}$ to 1.

83. (1) Prove $\sin 2A = \dfrac{2\tan A}{1+\tan^2 A}$, and verify it when $A = 30°$;

(2) Prove that $\dfrac{\cos 3A + \sin 3A}{\cos A - \sin A} = 1 + 2\sin 2A$.

84. Find the number of degrees in the least positive angle that satisfies the equation $\tan A + \tan(45° + A) = 2$.

85. Two sides of a triangle are 175 feet and 105 feet, the included angle is 37°; calculate the remaining parts of the triangle and its area.

86. Prove the formula $\cos 2A = 2\cos^2 A - 1$; and show, by means of it, that $2\cos\frac{45°}{2^n} = \sqrt{2+\sqrt{2+\sqrt{2+}}}$ &c.

87. The sides of an isosceles triangle are 183, 183, and 197; find its angles.

88. Prove that $\frac{\cos A+\cos 3A+\cos 5A}{\sin A+\sin 3A+\sin 5A} = \cot 3A$.

89. Prove that $\sin^2 30° : \sin^2 45° : \sin^2 60° :: 1 : 2 : 3$.

90. Given $\sin(A-B) = \sin A\cos B - \cos A\sin B$, to prove that $\cos(A+B) = \cos A\cos B - \sin A\sin B$.

91. Prove that $\frac{\sin 45° - \sin 30°}{\sin 45° + \sin 30°} = \frac{\sec 45° - \tan 45°}{\sec 45° + \tan 45°}$.

92. Find x in the equations, (1) $\sin 2x = \tan x$;

$$(2)\quad \cos^{-1}\frac{\sqrt{3}}{2} + \sec^{-1}x = \frac{\pi}{4}.$$

93. Prove $\frac{1}{2}\sin^{-1}\frac{4}{5} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$.

94. Show that $\sin 2A = 2\sin A\cos A$, without assuming the formula for $\sin(A+B)$.

95. Prove that

$$\sec A + \tan A = \sqrt{\frac{1+\sin A}{1-\sin A}} = \frac{\sin(45° + \frac{1}{2}A)}{\sin(45° - \frac{1}{2}A)}.$$

96. In any triangle, ABC, (1) the radius of the circumscribing circle

$$= \frac{a+b+c}{2(\sin A+\sin B+\sin C)} = 2\left\{\frac{abc}{\sin A\sin B\sin C}\right\}^{\frac{1}{3}}$$
$$= \tfrac{1}{2}(a+b+c)\tan\tfrac{1}{2}A\tan\tfrac{1}{2}B\tan\tfrac{1}{2}C.$$

(2) The length of the perpendicular from A upon the opposite side $= \frac{b^2\sin C + c^2\sin B}{b+c}$.

(3) $\tan^2\frac{A}{2}=\frac{(a+b-c)(a+c-b)}{(a+b+c)(b+c-a)}$; and if $C=90°$, $\tan\frac{A}{2}=\frac{a}{b+c}$.

(4) $\frac{\sin A+\sin B+\sin C}{\sin A+\sin B-\sin C}=\cot\frac{1}{2}A\cot\frac{1}{2}B$.

(5) $\frac{\tan\frac{1}{2}A-\tan\frac{1}{2}B}{\tan\frac{1}{2}A+\tan\frac{1}{2}B}=\frac{a-b}{c}$.

(6) $\frac{a^2-b^2}{c^2}\sin C+\frac{b^2-c^2}{a^2}\sin A+\frac{c^2-a^2}{b^2}\sin B+4\sin\frac{A-B}{2}\sin\frac{B-C}{2}\sin\frac{C-A}{2}=0.$

(7) $\sin A-\sin B+\sin C=4\sin\frac{1}{2}A\cos\frac{1}{2}B\sin\frac{1}{2}C$.

(8) $\cot\frac{1}{2}A\cot\frac{1}{2}B=\frac{a+b+c}{a+b-c}$.

(9) $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$.

(10) $(a+b)\cos C+(a+c)\cos B+(b+c)\cos A=a+b+c$.

(11) $\frac{\tan A+\tan B+\tan C}{(\sin A+\sin B+\sin C)^2}=\frac{\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2}}{2\cos A\cos B\cos C}$.

(12) $\tan\frac{A+B}{2}=\cot\frac{1}{2}C$.

(13) $a\sin(B-C)+b\sin(C-A)+c\sin(A-B)=0$.

(14) $\cos A=\frac{b^2+c^2-a^2}{2bc}$, deduce the relation $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$.

(15) $\frac{\sin\left(\frac{A}{2}+B\right)}{\sin\frac{A}{2}}=\frac{b+c}{a}$.

(16) Area$=\sqrt{s.(s-a).(s-b).(s-c)}$.

(17) $(a+c)\sin\frac{1}{2}B=b\,.\cos\frac{1}{2}(A-C)$.

(18) $\sin^2A+\sin^2B+\sin^2C-2\cos A\cos B\cos C=2$.

(19) $\cot B-\cot A=\frac{a^2-b^2}{ab}\operatorname{cosec} C$;

(20) $\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$,

(21) $c=(a-b)\times\frac{\cos\frac{1}{2}C}{\sin\frac{1}{2}(A-B)}=(a+b)\times\frac{\sin\frac{1}{2}C}{\cos\frac{1}{2}(A-B)}$.

97. The sum of the squares of the cosines of the angles of a triangle is equal to unity; show that the triangle is right-angled.

98. Having given the perimeter $2s$, and the angles A, B, C, of a plane triangle, find its sides.

CXVII.

HEIGHTS AND DISTANCES

1. The angle of elevation of the top of a steeple is 60° from a point on the ground. That of the top of the tower on which the steeple rests is 45° from the same point. What proportion does the height of the steeple bear to that of the tower?

2. The angle of elevation of the top of a steeple is 45° from a point in the same horizontal plane as its base, and is 30° from a point 30 ft. directly above the former point. Find the height and distance of the steeple.

3. An object C being inaccessible from B, a line BA, 408 yards in length, is measured, and the angles CBA, CAB, observed to be 22° 37′ and 58° 7′ respectively. Find BC, having given L sin 80° 44′=9·994295, L sin 58° 7′ =9·928972, log 4·08=·610660, and log 3·51=·545337.

4. A pole is fixed on the top of a mound, and the angles

of elevation of the bottom and top of the pole are 30° and 60° respectively. Prove that the height of the pole=twice the height of the mound.

5. Two straight roads, which cross one another, meet a canal at angles of 60° and 30° respectively. If it be 3 miles by the longer of the two roads from the crossing to the canal, how far is it by the shorter? If there be a foot-path which goes the shortest way to the canal, what is the distance by it?

6. What is the angle of depression of an object? From the top of a hill the angles of depression of two consecutive milestones on a straight, level road, were found to be 12° 13′ and 2° 45′. Find the height of the hill.

7. A balloon ascends uniformly in a vertical line; an observer takes up a position in the horizontal plane at a distance of 350 yards from the point where the balloon left the earth; after a time he observes the angle of elevation of the balloon to be 55° 17′, and after an interval of three minutes he observes the angle of elevation to be 60° 18′. Find the rate of the ascent of the balloon.

8. A lighthouse was observed by a ship at sea to bear SE.; after the ship had sailed NE. for 12 miles, the lighthouse was observed to bear 15° E. of S. Find the distance of the lighthouse from each position of the ship.

9. An object 10 ft. high is placed on the top of a tower, and subtends an angle of 6° at a place which is in the same horizontal plane as the foot of the tower, and is 50 ft. distant from it. Determine the height of the tower.

10. From a boat on a river, the angle of elevation of the top of a column on the bank was observed to be 32° 15′, and the angle subtended by the top of the column and a boat down the river was 48° 12′; after sailing past the column towards the barge for a distance of 480 yds., the observer in the boat found that the angle subtended by the top of the column and the first position of the boat was 20° 20′. Find the height of the column.

11. A person, wishing to determine the length of an inaccessible wall, places himself due S. of one end and then due W. of the other, at such distances that the angle which the length of the wall subtends at each station is 30°. Find the length of the wall, the distance between the stations being 120 yds.

12. Four points A, B, C, D are in the same horizontal plane; if the distance CD be known, show how to determine AB by means of angles measured at A and B.

13. An object 12 ft. high, standing on the top of a tower, subtends an angle of 1° 54′ 10″, at a station which is 250 ft. from the base of the tower. Find the height of the tower.

14. From a ship sailing due SE. at the rate of seven miles per hour a lighthouse is observed to bear N. 30° E., and after two hours its bearing is due N. Find the distance of the ship from the lighthouse at each observation.

15. A man ascends a hill by a path which is the shortest distance between the base and the summit; the inclination of the path to the horizon is at first 20°, but afterwards suddenly increases to 40°; on arriving at the summit he observes the angle of depression of the point from which he started to be 60°. Supposing that he walked 800 yds. before the change of inclination of his path, determine the height of the hill.

16. B and C are two stations 3 miles apart; travellers start from B and C towards a third station, A: the traveller from B, walking 4 miles an hour, reaches A in $\frac{3}{4}$ hour; the traveller from C, at the rate of 5 miles an hour, reaches A in $\frac{1}{2}$ hour. Find either angle of the triangle ABC.

17. An observer, whose eye is 5 ft. above the ground, observes a vertical object 100 ft. high standing on the same horizontal plane with him, and finds the angular distance between its highest and lowest points to be 30°. What is the distance of the object from him?

18. An observer, standing on a horizontal plane, observes

the angle of elevation of the top of a mountain to be 63° 25′; walking 1 mile towards the mountain, on an ascent that makes an angle of 30° with the horizon, he finds the elevation of the mountain top to be 74° 25′. Find the height of the mountain above the horizontal plane.

19. AB, AC, are two railroads inclined at an angle of 50° 20′; a locomotive engine starts from A along AB at the rate of 30 miles an hour; after an interval of 1 hour, another locomotive engine starts from A along AC, at the rate of 45 miles an hour. Find the distance between the engines 3 hours after the first started.

20. Wishing to know the distance of an inaccessible object P, and having no instrument for measuring angles, I measured a horizontal base AB, 400 ft., and from A to B, in a line directly away from P, I measured AC, 300 ft., BD, 450 ft.; I also measured BC, 500 ft., and AD, 750 ft. Find PA or PB.

21. The elevation of a tower standing on a horizontal plane is observed; on advancing 240 ft. nearer to it, the angle of elevation is found to be the complement of the first observed angle, and on advancing 100 ft. nearer still to the tower, its angle of elevation is found to be double the first. Find the height of the tower.

If the given distances be a and b, between what limits must the first angle of elevation lie, that the angular data, as above, may be possible?

22. The angle of elevation of a tower 400 ft. high, when due N. of the observer, was 55° 14′; what will the elevation be when the observer has walked 720 ft. due E. on the horizontal plane?

23. A railroad AB runs N., and a railroad AC NE. from A. A train starts from A along AB with a velocity of 25 miles an hour; after half an hour a train starts from A along AC with a velocity of 30 miles an hour. Find the distance of the trains from each other three hours after the first train has started.

24. Two poles are of equal height; a person standing between them in the line joining their bases observes the elevation of the nearer one to be 60°. After walking 80 ft. in a direction at right angles to the line joining their bases he observes the elevations of the two to be 45° and 30°. Find their height and the distance between them.

25. A, B, C are three points at known distances from one another; AB=476, AC=513, BC=495. An observer stations himself so as to see B and C in the same straight line, and the side AB then subtends an angle of 23° 10′ at his eye. Find his distance from A.

26. In an oblique-angled triangle ABC, given AB=2700 ft., ∠A=50° 20′, ∠B=110° 12′, find BC.

27. To determine the height of the top, C, of a mountain, a base, AB, of 2700 ft. was measured in the horizontal plane; the angle subtended by CB at A was 50° 20′, the angle subtended by AC at B was 110° 12′, and the angle of elevation of C from B was 10° 7′. Find the height of the mountain.

28. From the top of a tower, whose height is 108 ft., the angles of depression of the top and bottom of a vertical column standing on the horizontal plane are 30° and 60° respectively. Find the height of the column.

29. From each of two ships a mile apart the angle which is subtended by the other ship and the beacon on shore is observed; these angles are 55° and 62° 30′. Determine the distances of the ships from the beacon.

30. Four trees, A, B, C, D, are in the same straight line. The distance AB=40 yards, BC=20 yards, CD=60 yards. P is a point, out of the line, such that the distances between the trees appear equal to each other. Find AP, BP, CP, DP.

31. The angle between a fort and a lighthouse observed from a ship at sea is 56° 30′, but after the ship has sailed half a mile in the direction of the fort, the angle between the same two objects is 70° 45′. What was the ship's distance

from the lighthouse midway between the two places of observation ?

32. The distance from A to B is 1·75 mile, from B to C 2·5 miles, and these three stations are in the same straight line. I travel, not in that straight line, from C to a station D, and then observe that the stations B and C subtend at my eye an angle of 23° 19′, and A and B an angle of 17° 49′. Find the distances of D from A and B.

33. The boundary of a breakwater seen from either A or B, two stations 1250 yards apart (of which A is due south of one extremity, B due east of the other), subtends an angle of 15°. Find the rectilineal distance between the ends of the breakwater.

34. At what distance may a light, which is 60 ft. above the level of the sea, be just seen from the deck of a ship 12 ft. above the level of the sea, the earth being considered a sphere of 4000 miles radius ?

35. At the foot of a hill a visible object has an elevation of 29° 12′ 40″; when the observer has walked 300 yds. up the hill away from the object, he finds himself on a level with it. The slope of the hill being 16°, and the places of observation in a vertical plane with the object, find the distance of the object from the first place of observation.

36. An observer at A wishes to determine the distance between two inaccessible objects B, C. He measures a base AD, 1763 yds., and observes the angles, BAC=45° 1′, CAD=30°, BDC=36° 15′, BDA=33° 8′. Find BC.

37. A captain, marching on the left of his company, observes another company, of equal strength with his own, advancing from the right in a direction cutting his line of march at right angles ; he observes the angle subtended by this company at his eye to be 7° 30′, and the angle between his own line of march and the line joining himself with the other captain, who is also on the left of his company, to be 60°. Find the distance of each captain from the line of

march of the other, the breadth of each company being 52 ft.

38. In a plane triangle ABC, given AB=1114·5 ft., $\angle$CAB=38° 41′, $\angle$CBA=45° 39′, find BC.

39. A tower 300 ft. high, of which C is the top, stands on a horizontal plane; a horizontal base AB=1114½ ft. is measured, but not in the same vertical plane as the tower; the $\angle s$ CAB and CBA are observed and found to be respectively 38° 41′ and 45° 39′. Find the nearest distance of the foot of the tower from AB.

40. The elevations of two mountains in the same line with the observer are 9° 30′ and 18° 20′; on approaching 4 miles nearer, they have both an elevation of 37°. Find the heights of the mountains in yards.

41. A and B are two stations a mile apart, A due north of B. At the same instant a balloon is seen from A to bear 60° 15′ west of south, and as seen from B to bear 54° 30′ west of north; also, the angle of elevation of the balloon as seen from A at the same time was 35° 25′ 25″. Find the perpendicular height of the balloon above the horizontal plane passing through A and B.

42. Given two sides of a triangle 7 and 9⅘ miles, and the included angle 30° 20′, find the third side.

43. P is a station on the coast, A and B, two headlands at sea, PA 7 miles, PB 9⅘ miles, and the angle which the headlands subtend at P 30° 20′; in the line PA, 3 miles from P, a steamer is seen to start at the rate of 9 miles an hour. In what direction relatively to AP must it sail to be seen from P in the line PB after 40 minutes?

44. The plane of the side of a hill running E. and W. is inclined to the horizon at an angle α. It is required to construct a straight road upon it inclined at an angle β to the horizon. Determine the point of the compass to which it must be directed.

45. Two railways intersect at an angle of 35° 20′; from the

point of intersection two trains start together, one at the rate of 30 miles an hour. Find the rate of the other train so that after $2\frac{1}{2}$ hrs. the trains may be 50 miles apart.

46. Show that there are two velocities that will satisfy the condition in *Quest.* 45.

47. In an oblique plane triangle ABC, given the angle ACB=139° 58′, ABC=22° 10′, BC=840·5 yds., determine by how much AB differs from a mile.

48. A is the summit of a hill of which B is the foot; the lower part, BC, of the hill (840·5 yds.) is a plane inclined to the horizon at 12° 10′; the elevation of the top of the hill, seen from B, is 34° 20′, and seen from C is 52° 12′. Find the perpendicular height of A above the horizontal plane through B.

49. On the bank of a river stands a tower 200 ft. high, on which is a flagstaff 30 ft. high, and at the bottom of the tower stands a man 6 ft. high. To an observer on the opposite bank the man and the flagstaff subtend equal angles. Find the width of the river.

50. An observer stands on a wall, so that his eye is 10 ft. above the horizontal plane on which a tower, whose height is 250 ft., stands; the tower subtends, at the eye of the observer, an angle of 56° 40′. Find its horizontal distance from the wall.

51. A tower standing on a horizontal plane is surrounded by a moat, which is as wide as the tower is high; a person on the top of another tower whose height is a, and whose distance from the moat is c, observes that the first tower subtends an angle of 45°. Find the height of the first tower.

52. Two hills rise at the same point with inclinations of 60° and 40° to the horizon, at a distance of 64 feet from the base of the lower hill; the angles of elevation of the bottom and top of a vertical object on the other hill are 40° and 70°. Find the height of the object.

53. From the top of a tower by the seaside 150 ft. high,

it was found that the angle of depression of a ship's hull was 35° 18′. Find the distance of the ship from the foot of the tower.

54. From the top of a tower 150 feet high, the angles of depression of the top and bottom of a vertical column, standing on the same horizontal plane, were observed 22° 15′ and 44° 30′. Find the height of the column.

55. A lighthouse was observed from a ship to bear N. 45° E.; and after the ship had sailed 4 miles SE. the bearing of the lighthouse was observed to be N. 15° E. Find the distance of the lighthouse from each position of the ship.

56. X and Y are two stations on a coast; X is N. 45° E. of Y, and 3 miles distant from it: a ship at sea observes at the same instant the bearing of X to be N. 20° 15′ E., and that of Y to be N. 42° 20′ W. Find the distance of the ship from the nearer station.

57. A spectator observes the explosion of a meteor due north of him, and at an altitude of 30°. To another spectator, at a place 10 miles NNE. of the former, it appears at the same moment to have an altitude of 45°. Investigate its height from the earth at the time.

58. From C, the foot of a tower, AC is drawn in the horizontal plane; the angle of elevation of B, the top of the tower, as seen from A, is α; AD ($=a$ ft.) is measured from A, in the horizontal plane, perpendicular to AC; at D the angle of elevation of the tower is β. Prove the height of the tower to be $\dfrac{a \sin \alpha \sin \beta}{\sqrt{\sin (\alpha+\beta) \sin (\alpha-\beta)}}$. Apply the formula, when $a=350$ ft., $\alpha=10°\ 45'$, $\beta=8°\ 20'$.

59. A, B, C are three points, the distances of which are, AB=320 yds., AC=600 yds., BC=435 yds. From S, a station, it is observed that ASB=15°, BSC=30°. Find the distances of S from A, B, and C; the point B being nearest to S, and the angle ASC being the sum of ASB and BSC.

60. A line AB, 450 yds. long, is measured close by the

brink of a river, and a point C, close to the bank of the river on the other side, is observed both from A and B ; the angle CAB is 52°, CBA 70°. Find the width of the river.

61. Two towers stand on a horizontal plane 144 ft. apart; a person standing at the foot of each tower observes the elevation of one tower to be double that of the other, but when he is halfway between them the angle of elevation of one tower is the complement of that of the other. Show that the heights of the towers are 108 feet and 48 feet respectively.

62. From the edge of one bank of a river, a person ascends 100 yards up a slope of 1 in $4\frac{1}{2}$, and observes the angle of depression of an object on the opposite bank close to the edge of the river to be $1\frac{3}{4}°$. Find the breadth of the river.

CXVIII.

MENSURATION.

1. When two diagonals of a trapezium and the angle at which they intersect are given, show how to find its area.

Ex. ABCD is a trapezium, AC=312·4, BD=612·5, the angle at which AC, BD intersect is 105° 20′. Find the area.

2. A regular triangular pyramid is contained by 4 equilateral triangles, the side of each triangle being 20 ft. Find the content of the pyramid.

3. AC is the diameter of a circle, and a diagonal of the inscribed quadrilateral ABCD. Given AB=30, BC=40, CD=10, find AD and the area of the figure.

4. A slice is cut from a sphere by two parallel planes: the distance between the planes is 4 feet, the radius of one circular section is 9 feet, and of the other 12 feet. Determine the radius of the sphere and the volume of the slice.

5. Two sides of a triangular field, the lengths of which

are 16 and 40 poles respectively, are inclined to each other at an obtuse angle. Find that angle, and the third side of the triangle, when the field contains exactly an acre.

6. The sides of a quadrilateral are 10, 7, 8, 9, and the angle between the second and third sides is 120°. Find the area of the figure.

7. The perpendicular height of the frustum of a cone is 7 feet, and the radii of the ends are 4 and 5 feet. Find its volume.

8. Find the whole surface and solid content of a square pyramid, each side of the base being 12 ft., and the slant height 25 ft.

9. The radius of a sphere is 10 ft. Find the volumes of the two segments into which it is divided by a plane, the perpendicular on which from the centre is 5 ft.

10. A pyramid has a square base, the area of which is 20·25 square feet; each of the edges of the pyramid passing through the vertex is $30\frac{3}{4}$ ft. Find the inclination of either of the triangular faces to the base, and determine by how much the volume of the pyramid differs from 3800 cubic feet.

11. How long will it take to fill a hemispherical tank of 6 ft. radius from a cistern which supplies, by a pipe, 6 gallons of water per minute, a gallon of water containing 277·27 cubic inches?

12. The frustum of a right cone is 6 ft. high, the radius of the smaller end is 2 ft., the radius of the larger end is 3 ft. Find its volume. Find the position of the plane parallel to the ends which will bisect the frustum.

13. What length of a gun of 9 inch bore will a charge of 15 lbs. of powder fill, if 30 cub. in. of powder weigh 1 lb.?

14. A hemispherical bell of 10 ft. diameter is partially buried with its mouth downwards and in a horizontal position, so that only $\frac{1}{3}$ of the vertical radius of the bell appears above ground. What quantity of earth must be dug out in order to leave the bell entirely uncovered and surrounded by a cylindrical wall of earth?

15. If a cubic foot of metal weighs 4 cwt. 1 qr., and is worth 10 guineas a ton, what will be the cost of 1 mile of piping made out of it, with a 9-inch bore, and $\frac{3}{8}$ in. thick?

16. A solid sphere, whose radius is a, has a cylindrical hole bored through it, whose axis passes through the centre, and whose diameter is half that of the sphere. Determine the volume of the remaining portion.

17. The total length of the edges of a rectangular parallelopiped is 72 lineal feet, its entire surface measures 174 square feet, and its volume is 110 cubic feet. Find its length, breadth, and height.

18. A regular hexagon, each side of which is 10 ft., revolves about a line which joins the points of bisection of two opposite sides. Find the whole surface of the solid thus generated.

19. If the middle points of the adjacent sides of two similar trapeziums be joined, show that the figures thus formed are parallelograms, whose areas are in the same ratio as those of the trapeziums.

20. Two concentric circles have radii of 10 ft. and 15 ft. respectively. Calculate the area of the figure bounded by these circles and by radii inclined at an angle of 40° to each other.

21. If 30 cubic inches of powder weigh 1 lb., show that it will require nearly 9 lbs. to fill a shell whose internal diameter is 8 inches.

22. One side of a quadrilateral inscribed in a circle is 40 ft., the angles which it makes with its adjacent sides are 90° and 75° respectively; if the radius of the circle be 25 ft., find the area of the figure.

23. If $\pi a^2 \left(b - \frac{a}{3}\right)$ be given as the expression for finding the content of the segment of a sphere, what do a and b represent?

24. What length of a gun 6 inches bore will be filled with 20 lbs of powder of which 30 cubic inches weigh 1 lb.?

25. One side of a rectangle is 6 in. Find the other side, when the area is equal to that of a trapezoid whose parallel sides are 7 in. and 8 in., the perpendicular distance between them being 12 in.

26. Regular hexagons are inscribed in circles whose radii are 3 and 5 in. What is the radius of the circle in which the area of the inscribed hexagon is a mean proportional between the areas of the other inscribed hexagons?

27. Define a pyramid. Find the content of a hexagonal pyramid; side of base 10 ft., perpendicular height 15 ft.

28. The edge of a cube is 12 in.; one of the angles of the cube is cut off, so that the part cut off forms a pyramid with each of its edges (terminating in the angle of the cube) 5 in. long. Find the volume of the remaining solid.

29. Find the number of cubic feet in a hexagonal room, each side of which is 20 ft. and its height 30 ft.; and which is finished above with a roof in the form of a hexagonal pyramid 15 ft. high.

30. What is the solid content of a sphere, when its surface is equal to that of a circle 4 ft. in diameter?

31. Find the solid content of a log of timber 10 yds. 2 ft. 7 in. long, 2 ft. 11 in. broad, and 2 ft. 5 in. thick.

32. If a pressure of 15 lbs. to the square inch be applied to a circular plate 3 feet in diameter, what is the total pressure?

33. Find the area of the equilateral triangle circumscribing the circle whose radius is 60 ft.

34. Show how to find the area of a trapezoid.

35. Find the volume of a pyramid.

Ex.: A pentagonal pyramid, side of base 7·25 ft., height 12·6 ft.

36. The paving of a semicircular courtyard at 5*s.* a foot cost £20. Find the radius of the semicircle.

37. Find the volume of a segment of a sphere 7·5 in. high; the radius of the sphere being 12 in.

38. One side of a triangle is 200 yds., another is 88 yds. Determine the third side, when the area is 5095 square yds.

39. An inverted conical vessel, whose height is equal to the diameter of its rim (4 in.), is supported when filled with water; on the top of it is placed a heavy sphere with a diameter of 5 in. How much water will be left in the vessel?

40. A quadrilateral has two of its sides parallel; these sides are 10 and 12 ft. respectively; the perpendicular distance between them is 4 feet. Find the area of the figure.

41. The perimeters of two similar trapeziums are as 3 : 7, a side of one is 3 in., its area 774 square inches. Find the area of the other.

42. A hemispherical basin holds 1 gal. of water. Find its diameter, if 1 gal.=277·27 cubic inches.

43. Show that the square described round a circle is equal to $\frac{4}{3}$ of the inscribed dodecagon.

44. The sides of a quadrilateral taken in order are 27, 36, 30, 25 ft.; the angle between the first two is 90°. Find its area.

45. Find the content of a right cone whose height is 210·6 ft., and radius of base $23\frac{1}{2}$ ft.

46. How many cubic feet of water are contained in a ditch shaped like the frustum of a wedge, 120 yds. long, 6 ft. deep, 10 yds. broad at the top and 4 at the bottom?

47. A ditch is 5000 ft. long, 9 ft. deep, 14 ft. broad at top, and 11 ft. broad at the bottom. How many cubic feet of water will fill it? If half that quantity of water is supplied, how high will it rise?

48. The base of a pyramid is a regular hexagon, each side =40 ft.; what must be its height that its cubical content may be the same as that of a sphere whose radius is 21·5 ft.?

CXIX.

SPHERICAL TRIGONOMETRY.

1. In a spherical triangle, having given $a=70°$, $b=40°$, $c=38°\ 30'$, calculate numerically the angle A; assuming a suitable formula.

2. In a right-angled spherical triangle, if c denote the side opposite to the right angle, show that

$$\cos^2\tfrac{1}{2}c=\cos^2\tfrac{1}{2}a\cos^2\tfrac{1}{2}b+\sin^2\tfrac{1}{2}a\sin^2\tfrac{1}{2}b.$$

Also that $\dfrac{\sin(c-b)}{\sin(c+b)}=\tan^2\tfrac{1}{2}\mathrm{A}.$

3. Are the three angles of a spherical triangle sufficient data to enable us to determine the angular measure of the arcs which are the sides of the triangle? Are they sufficient to enable us to determine the lineal measure?

4. In any spherical triangle, a, b, c being the sides, A, B, C the angles, prove that

$$\tan\tfrac{1}{2}(a-b)=\frac{\sin\tfrac{1}{2}(\mathrm{A}-\mathrm{B})}{\sin\tfrac{1}{2}(\mathrm{A}+\mathrm{B})}\cdot\tan\tfrac{1}{2}c.$$

5. Find the area of a spherical triangle in terms of the spherical excess; and obtain an expression for the latter, in a form adapted to logarithmic calculation, in terms of the sides.

6. Prove the following formula of spherical trigonometry, and adapt it to logarithmic computation:—

$$\cos c=\cos a\cos b+\sin a\sin b\cos\mathrm{C}.$$

7. In any right-angled spherical triangle—

(1) $\tan^2\tfrac{1}{2}a=\dfrac{\tan\tfrac{1}{2}(c+b)}{\cot\tfrac{1}{2}(c-b)}.$

(2) $\tan^2(45°+\tfrac{1}{2}c)=\dfrac{\tan\tfrac{1}{2}(\mathrm{B}+b)}{\tan\tfrac{1}{2}(\mathrm{B}-b)}.$

8. Solve the spherical triangle ABC from these given parts:—

$$\mathrm{A}=51°\ 30',\ b=80°\ 19',\ a=63°\ 50'.$$

9. If a and b are the sides, c the hypothenuse of a right-angled spherical triangle, prove that $\cos c = \cos a \cdot \cos b$.

10. In any spherical triangle show that the sines of the angles are proportional to the sines of the opposite sides.

11. Give Napier's rules for the solution of right-angled spherical triangles.

12. If a, b, c, be the sides of a spherical triangle, and if the arc δ be drawn from the angle A to bisect the side a,

$$\text{Prove } \cos\frac{a}{2}\cos\delta = \cos\frac{b+c}{2}\cos\frac{b-c}{2}.$$

13. In spherical trigonometry, state and prove the fundamental properties by which the primitive and polar triangles are related to each other.

14. Find generally the cosine of an angle of a spherical triangle in terms of the sides, and thence the cosine of a side in terms of the angles.

15. The angles of a spherical triangle of the surface of the earth are 42° 2′ 35″, 67° 55′ 38″, 70° 1′ 48″. Find the area of the triangle in square miles, taking the earth's radius at 4000 miles.

16. Define the polar triangle of a spherical triangle, and establish the relation between the sides and angles of the two triangles.

17. Prove that in a triangle ABC

$$\cos a = \cos b \cos c + \sin b \sin c \cos A;$$

and, by help of the polar triangle, deduce a relation between two angles and a side of the triangle.

18. In a spherical triangle, prove the formulæ—

(1) $\cot a \sin b = \cot A \sin C + \cos b \cos C$.

(2) $\cos a \sin b = \cos A \sin C + \cos b \cos C \sin a$.

(3) $\sin a \sin b \cos^2\frac{C}{2} = \sin\frac{a+b+c}{2}\sin\frac{a+b-c}{2}$.

19. Enunciate Napier's rules for the solution of a right-angled triangle, and prove these rules for the case in which one of the angles is the middle part.

20. In a triangle ABC, right-angled at C, the length of the arc of a great circle from C, perpendicular to AB, is 8. Prove that

$$\cot^2 8 = \cot^2 a + \cot^2 b.$$

21. If two sides and the included angle of a spherical triangle be given, show how to find the other parts of the triangle.

22. If the sides of a spherical triangle be small compared with the radius of a sphere, prove that each angle of the spherical triangle exceeds by one-third of the spherical excess the corresponding angle of the plane triangle, the sides of which are of the same length as the arcs of the spherical triangle. Explain the use of this theorem in the case of a spherical triangle in which two sides and the included angle are known.

23. If two sides of a spherical triangle be given, determine the relation between the small variations of any other pair of elements of the triangle.

24. Define a spherical triangle, and prove that the three angles of any spherical triangle are together greater than two right angles, and less than six right angles. Show also that any two sides are greater than the third side.

25. Prove the following relations in any spherical triangle:—

(1) $\dfrac{\sin A}{\sin a} = \dfrac{\sin B}{\sin b} = \dfrac{\sin C}{\sin c}$.

(2) $\tan \frac{1}{2}(a-b) \sin \frac{1}{2}(A+B) = \tan \frac{1}{2} c \sin \frac{1}{2}(A-B)$.

26. Explain fully the ambiguous case in the solution of a right-angled spherical triangle.

COORDINATE GEOMETRY.

CXX.

1. Find the equation to a straight line passing through a given point, and at right angles to a given straight line.

2. All the parallelograms circumscribing a given ellipse are equal to one another.

3. Find the coordinates of the points in which the line $3x+7y=1$ meets the two rectangular axes.

4. If $\frac{x}{a}+\frac{y}{b}=1$ be the equation to a straight line, what do a and b represent? What is the condition that two straight lines shall be at right angles to each other?

5. Show that $2y^2-3xy-2x^2-7y-x+3=0$ represents two straight lines at right angles to each other.

6. Find the length of the perpendicular let fall from the point $(3, -5)$ upon the line $7x-3y=9$.

7. What is the equation of a right line passing through the two given points (x', y'), (x'', y'')?

8. What is the length of the perpendicular drawn from the point (x', y') upon the line $x\cos\alpha+y\sin\alpha-p=0$, the axes being rectangular?

9. What is the area of the triangle formed by joining the three points (x', y'), (x'', y''), (x''', y''')?

10. Given the coordinates of the extremities of a right line, find the coordinates of its middle point.

11. Find the length of the perpendicular drawn from the point (3, 4) on the right line $\frac{x}{3}-\frac{y}{4}-1=0$, the axes being rectangular.

12. Find the equation to a straight line (1) drawn from the origin and making an angle of 45° with the straight line $y=ax+b$, (2) to a straight line bisecting the angle between two given straight lines passing through the origin, and explain why the problem admits two solutions.

13. The equation $3y^2-8xy-3x^2-9y+7x+6=0$ represents two straight lines at right angles to each other.

14. Determine the geometrical signification of the equation—

$$10x^2-xy-21y^2-9x-y+2=0.$$

15. Find the equation to the straight line passing through the origin and making an angle of 60° with the line $x+y\sqrt{3}-1=0$.

16. Draw the figure representing the position of the straight line whose equation referred to rectangular coordinates is $3y-5x+7=0$.

17. Find the angle between the lines whose equations referred to rectangular coordinates are $y-2x+3=0$ and $y+x-2=0$.

18. Find the equation to the right line perpendicular to the line whose equation is $y=mx$, the axes being rectangular.

19. Determine the angle between the lines $y=mx+n$ and $y=m'x+n'$.

20. Find the point of intersection of the two lines—

$$3x-2y+5=0,\ 4x+3y=0;$$

find also the distance of this point from the line $2y=x-1$.

CXXI.

1. Find the general rectangular equation to a given circle: construct the curve $x^2+y^2-6x-10y-15=0$; and find the equation to the diameter passing through the origin.

2. Find the equation to the circle with radius r having its centre on the axis of x, and passing through the origin.

3. When does the equation

$$Ay^2+Bxy+Cx^2+Dy+Ex+F=0$$

represent a circle, the axes being oblique ?

4. What is the equation to the circle circumscribing the

triangle whose sides have $\alpha=0$, $\beta=0$, $\gamma=0$, for their equations?

5. Obtain the equation of a circle of radius a referred to axes of coordinates which touch the circle at distances $a\sqrt{3}$ from the origin.

6. Ascertain whether the circles represented by the equations

$$x^2+y^2-6x-6y+13=0$$
$$x^2+y^2-14x-4y+43=0 \text{ intersect or not.}$$

7. If the circles in *Quest.* 6 intersect, find the equation to their common chord, and prove that it is perpendicular to the straight line joining the centres of the circles.

8. Construct the circle whose equation is

$$4x^2+4y^2-16y-4x-19=0,$$

and find the equation to the diameter passing through the origin.

9. The equation to a circle referred to rectangular co-ordinates being $2x^2+2y^2+8x-12y=6$, find its radius and the position of its centre.

10. Find the polar equation to a circle, the origin being an external point and the initial line a diameter.

11. Ascertain whether the right line $\frac{x}{5}+\frac{y}{4}-1=0$ cuts the circle $x^2+y^2-2x-4y+\frac{17}{4}=0$.

12. Find the equation of the right line which joins the points of contact of tangents drawn to the circle $x^2+y^2=r^2$, from the point (x', y').

13. $9x^2+9y^2-12y-32=0$ is the equation to a circle. Find the equation to the diameter which makes an angle of 45° with the axis of x.

CXXII.

1. In a parabola, show that the abscissa varies as the square of the semi-ordinate to the axis.

2. Find the area of the portion of a parabola included between two ordinates to the axis, one passing through the focus of the length of 12 ft., the other ordinate being 24 ft. in length.

3. Find the equation to the tangent at any point of a parabola.

4. Determine the coordinates of a point on a parabola such that the perpendicular, from the focus, on the tangent at that point shall be equal to the *latus rectum*.

5. Define a tangent to a curve, and, according to the definition, find the equation to the tangent at any proposed point of the parabola $y^2=4ax$. Show that tangents equal in length cannot be drawn to a parabola from an external point, unless that point lies in the axis produced.

6. Find the polar equation to a parabola, the focus being the pole.

7. Show that the tangent to a parabola will meet the directrix and *latus rectum* produced in two points equally distant from the focus.

8. Find the locus of the intersection of the tangent to a parabola, with the perpendicular drawn to it from a given point on the axis.

9. In the parabola prove that if (x', y'), (x'', y'') be points on the curve, the coordinates of the point of intersection of the tangents at those points will be respectively the arithmetic mean between y' and y'', and the geometric mean between x' and x''.

10. $x^2-ax=by$ being the equation of a parabola, find the equation of its direction, and the coordinates of its focus.

11. Prove that the tangents to a parabola from the same point on its directrix are at right angles to one another.

12. If S be the focus of a parabola, PM a semi-ordinate to the axis, show that the circle circumscribing the triangle SPM will touch the straight line which is a tangent at the

CXXIII.

1. Find the equation to the ellipse referred to polar coordinates, the origin being the focus.

2. PM is an ordinate at any point P of an ellipse ; MP is produced to meet in Q the circle described on the axis major of the ellipse as diameter. Find the distance from the centre of the ellipse, of the point of intersection of the normal to the ellipse at P, and the normal to the circle at Q.

3. What are conjugate diameters of an ellipse or hyperbola ?

4. Prove that, in an ellipse, the sum of the squares of two semi-conjugate diameters is equal to the sum of the squares of the semi-axes. State the corresponding property of the parabola.

5. In an ellipse the normal at any point bisects the angle between the focal distances.

6. Prove that the equation to the normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at the point (h, k) is $\frac{a^2x}{h}-\frac{b^2y}{k}=a^2-b^2$.

7. If the eccentricity of the ellipse be $\frac{1}{\sqrt{2}}$, find, in terms of a, the coordinates of the point where the normal at $\left(\frac{a}{3}, \frac{2b\sqrt{2}}{3}\right)$ again meets the ellipse.

8. Find the distance of any point on the ellipse from the focus.

If PSp be any focal chord of an ellipse, S the focus, L the *latus rectum* of the ellipse, prove

$$SP \cdot Sp=\frac{L}{4}(SP+Sp).$$

9. Investigate the equation to the tangent to an ellipse, the ellipse being referred to its axes as axes of coordinates.

10. Prove the *latus rectum* of an ellipse a third propor-

tional to the major and minor axes ; and find the *latus rectum* and area of the ellipse whose equation is

$$25y^2+16x^2=160x.$$

11. Find the equation to the normal at any point of an ellipse.

12. If C be the centre of an ellipse, CP its semi-axis minor, PT, PG, PN a tangent, a normal, and an ordinate, at any point P, cutting the axis major, produced if necessary, in T, G, N, respectively, prove that $CT \cdot NG=BC^2$.

13. Find a point in an ellipse such that the normal may bisect the angle included between the ordinate of that point and the radius drawn to the centre.

14. Find the locus of the intersection of tangents to an ellipse, which cut each other at right angles.

15. Show how to obtain the equation of the tangent to a curve represented by an equation between rectangular coordinates.

16. Show that the locus of the intersection of the perpendicular from the focus of an ellipse on a tangent is a circle. Show that the locus is also a circle, if the line from the focus, instead of being perpendicular to the tangent, makes a constant angle with it.

17. The distance between one vertex and the adjacent focus of an ellipse being given, prove that if the axis major increases without limit the curve will ultimately become a parabola.

18. Find the equation to the ellipse referred to any two conjugate diameters as axes.

If TPt be a tangent at any point P of an ellipse intersecting the major and minor axis in T and t, prove $PT \cdot Pt= CD^2$, where CD is the semiconjugate diameter to CP.

CXXIV.

1. Define the hyperbola. When are straight lines said to be asymptotes to curves ? How may such asymptotes

be detected algebraically? Find the equation to the hyperbola referred to its asymptotes.

2. The line $lx + my = c$ intersects the hyperbola $a^2y^2 - b^2x^2 = -a^2b^2$. Form the equation to the lines through the origin and points of intersection. Deduce the relation that must hold in order that the given line may touch the hyperbola.

3. If a right cone be cut through a given point, so tha its section is an hyperbola, find its axes.

If the angle of the cone be 90°, how must the cutting plane be drawn that the hyperbola may be equilateral?

4. Find by algebraic geometry the locus of a point the square of the distance of which from a given point is proportional to its distance from a given right line.

5. If from any point T on a tangent to a conic section TN, TM be drawn perpendicular to the focal distance and the directrix respectively, prove that $SN = e \cdot TM$.

Hence, show that the polar equation of the tangent at the point where the focal radius vector makes an angle α with the axis major is $\frac{l}{r} = e \cos\theta + \cos(\alpha - \theta)$, where $2l$ is the *latus rectum*.

6. Trace the curve given by the equation

$$4x^2 + 7xy + 4y^2 - 8x - 8y + 4 = 0.$$

7. If a right cone with a circular base be cut by a plane parallel to a slant side of the cone, show that the section is a parabola.

8. Construct the conic section represented by the equation $2x^2 + 3xy + 2y^2 - 8 = 0$, and find its area.

DIFFERENTIAL CALCULUS.

CXXV.

1. What is a function of an independent variable? Explain what is meant by the differential coefficient of such a function.

2. Exhibit, without proving it, the ordinary form of Taylor's theorem, and deduce from it the theorem known as Maclaurin's.

3. Expand $\sin x$ in terms of x to three terms by Maclaurin's theorem; and express the general term of the series for $\sin x$.

4. If a tangent be drawn at any point (x, y) of a plane curve, examine the trigonometrical meaning of $\frac{dy}{dx}$ with reference to that tangent.

5. Given (1) $y=\frac{(x-1)^{\frac{4}{3}}}{(x-5)^{\frac{2}{3}}(x-2)^{\frac{3}{5}}}$; (2) $y=\cos^{-1}\frac{3+5\cos x}{5+3\cos x}$;

(3) $y=2\tan^{-1}\sqrt{\frac{1-x}{1+x}}$; (4) $y=x^{x^2}$. Find $\frac{dy}{dx}$ in each case.

6. Prove that if $u=\frac{z}{y}$, $\frac{du}{dx}=\frac{y\frac{dz}{dx}-z\frac{dy}{dx}}{y^2}$.

Write down the first and second differential coefficients of

(1) x^x; (2) $xe^{\tan x}$; (3) $\tan^{-1}x$.

7. Find the differential coefficient of $\sin ax$ with respect to x.

8. Find the minimum value of $\frac{x^2-x-1}{x^2-x+1}$.

9. Find the subtangent of a spiral curve from its polar equation, and show how to determine whether the curve has an asymptote or not.

Ex. Find the subtangent and asymptote of the curve

$$r=\frac{a}{\theta}.$$

10. Differentiate $\tan^{-1}\frac{2x}{1-x^2}$; $\log\sqrt[4]{\frac{1+x}{1-x}}+\frac{1}{2}\tan^{-1}x$.

11. Expand $\tan^{-1}x$ in a series of ascending powers of x by Maclaurin's theorem.

12. Find what the area of the base of the roomiest tent of a conical form will be, which can be covered by 200 feet of canvas.

13. Explain how the form of a plane curve is determined by means of an equation, and illustrate the explanation by reference to some particular curve.

14. Find the differential coefficient in respect to x of x^m.

15. Find the equation to the tangent drawn to any point of the locus $x^2+y^2=c^2$, and prove that it is always perpendicular to the radius vector.

16. If ϕ be the angle which the normal to a curve makes with the radius vector r, from the pole at the point (r, θ), prove that $\sin\phi=\frac{dr}{ds}$, $\cos\phi=\frac{rd\theta}{ds}$.

17. Explain and illustrate the meaning of the limit of a variable ratio, and show how such a limit is applied in the fundamental principles of the differential calculus.

18. Investigate the differential coefficient of $\sin^{-1}ax$ with respect to x, and differentiate—

(1) $\log\frac{x-a}{x+a}$; (2) $\frac{\cos 3x+\cos x}{\sin 3x-\sin x}$;

(3) $(\tan x)^{\cot^{-1}x}$; (4) $\cos\{\cos(\cos x)\}$.

19. If $y=(a+x)^3(a-x)^2$, find the values of x which make y a maximum or a minimum.

20. Determine the position and magnitude of the radius of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$, at the point for which x and y are each equal to $\frac{a}{4}$.

21. Expand as far as x^4—

(1) $\log(x+\sqrt{x^2+a^2})$ · (2) $(e^x+e^{-x})^n$.

22. Prove the following differential formulæ which occur in the theory of plane curves :—

(1) Subtangent $=y\frac{dx}{dy}$; (2) subnormal $=y\frac{dy}{dx}$;

(3) $\frac{ds^2}{dx^2}=1+\frac{dy^2}{dx^2}$.

23. Differentiate $y=\frac{(x+1)^2}{x^2+1}$; $y=e^{\sin x}$; $y=\log_a \sin^{-1} x$.

24. Determine the maxima and minima of $f(x)$, when

$$f(x)=(x-2)^4\ (x-4)^2.$$

25. Determine the ratio between the height and radius of the base of a cylindrical quart cup so that its surface may be a minimum.

26. Show that if $y'=f(x')$ be the equation to a curve, the equation to the tangent is $y-y'=f'(x')\ (x-x')$.

27. Find the maximum or minimum value of

$$3x^4+4x^3-6x^2-12x+13.$$

28. Find the nth differential coefficients of (1) $x^2 \log x$, and (2) x^3e^x.

29. If $u=(x+1)\ (x^3+1)$, find the value of $\frac{du}{dx}$.

30. Find the values of x which will make

$\sin(x-a) \cos x$ a maximum or minimum.

31. Supposing $x=r \cos\theta$, $y=r \sin\theta$, find the value of $xdy-ydx$ in terms of r and θ.

32. Define the differential coefficient of a function, and investigate the differential coefficients with respect to x of $\sqrt{ax}$ and of $\tan ax$.

33. Expand $e^x \cos x$, by Maclaurin's theorem, to 3 terms, writing down the general term.

34. Assuming that $\frac{d \cos x}{dx}=-\sin x$, and $\frac{d \sin x}{dx}=\cos x$, develope $\sin x$ and $\cos x$ in ascending powers of x, by the method of indeterminate coefficients.

35. Expand to 4 terms in powers of x, by Maclaurin's theorem, $e^{\cos x}$.

36. Obtain the equation of the tangent to the curve $\phi(x, y)=0$, and show that it is one dimension lower, at least, than the equation to the curve.

37. Prove that $\frac{d(\tan x)}{dx}=\sec^2 x$.

38. Find the differentials of the following expressions:—

(1) $\cos\sqrt{x^2+a^2}$; (2) $\sin\sqrt{x^2+a^2}$; (3) $\log\sqrt{x^2+a^2}$.

39. A given sum £a is to be laid out in the purchase of a rectangular piece of ground and in building a wall round it. The wall on one side will cost £m per lineal foot, and on the other three sides £n per foot; and the land will cost £p per square foot. How may the greatest piece of land be enclosed subject to these conditions?

40. Find the differential coefficient of $\sqrt{a^2-x^2}$ with respect to x by first principles.

41. Find the values of x which make $x(a-x)^2(2a-x)^3$ a maximum or a minimum.

42. Find the value of the radius of curvature at any point of a curve; for example, a parabola at its vertex.

43. Expand $e^x \log_e(1+x)$ by Maclaurin's theorem.

44. Find the real value of

$$u=\frac{ax^2+ac^2-2acx}{bx^2-2bcx+bc^2}, \text{ when } x=c.$$

45. Evaluate $u=\dfrac{(a^2+ax+x^2)^{\frac{1}{2}}-(a^2-ax+x^2)^{\frac{1}{2}}}{(a+x)^{\frac{1}{2}}-(a-x)^{\frac{1}{2}}}$,

when $x=0$, $u=a^{\frac{1}{2}}$.

46. Find the value of $\frac{1-\cos x}{x\log(1+x)}$, when $x=0$.

47. Find the subtangent and subnormal to the curves

(1) $y^2=\frac{x^3}{2a-x}$; (2) $y=ce^{\frac{x}{a}}$; (3) $y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)$.

48. Prove that the greatest rectangle which can be inscribed in an ellipse is half of that contained by the axes.

49. If r be the radius vector at any point of a curve, and p the perpendicular from the pole upon the tangent, prove ρ the radius of curvature equal to $\frac{rdr}{dp}$.

In the ellipse, whose semi-axes are a, b, given $p^2=\frac{b^2r}{2a-r}$, find the radius of curvature drawn at the extremity of the major axis.

50. Two sides of a triangle being of given lengths, show that its area will be a maximum when they contain a right angle.

51. Hence, prove that if all the sides except one of a plane rectilineal figure be given, its area will be a maximum when the remaining side is the diameter of a semicircle circumscribing the figure.

52. And consequently if all the sides of a rectilineal polygon be given in length, its area will be a maximum when it admits of being inscribed in a circle.

53. Assuming the differential equation to the tangent to a plane curve referred to rectangular coordinates, show how the asymptotes to the curve may be drawn, if it has any.

Ex. $y^2=\frac{b^2}{a^2}(2ax+x^2)$.

54. Investigate the conditions under which a function of a single variable becomes a maximum or a minimum.

55. A rectangular plot of ground of given area is to be enclosed from the waste by a wall and divided into three equal areas, by partition walls parallel to one of its sides. What must be the dimensions of the rectangle that the length of walling may be a minimum ?

56. Expand, by Maclaurin's theorem, $\tan^4 x$ in terms of x to 3 terms.

57. Expand $(1+2x+3x^2)^{-\frac{1}{2}}$ to 5 terms by Maclaurin's theorem.

58. Write down the general expression for the equation to the normal at any point of a curve, explaining what each symbol indicates.

59. Find the equation to the tangent at any point of the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, and show that the portion of the tangent intercepted between the axes of coordinates is constant.

60. Find the equation to the tangent, at the point $(x'\,y')$, to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

61. (1) $u=\log\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}$;

(2) $u=\log\left\{\log\frac{1+\sqrt{\tan x}}{e^x(1-\sqrt{\tan x})}\right\}$.

Find $\frac{du}{dx}$.

62. A right circular cylinder is inscribed in a given right cone; find the radius of the cylinder when the whole surface of the solid is a maximum.

63. Find the radius of curvature at any point of an ellipse, and express it in terms of the focal distances of the point.

64. Find $\frac{du}{dx}$ in the following examples:—

(1) $u=\log_e(x+\sqrt{a^2+x^2})$; (2) $u=\frac{x^2+ax+a^2}{x^2-ax+a^2}$;

(3) $u=\sin\frac{x+a}{x-a}$; (4) $u=\tan^{-1}\frac{2x}{1-x^2}$;

(5) $u=\epsilon^{\sin x}\cos x$; (6) $u=(\sin x)^3(\cos x)^4$.

65. Show that $u=\phi(x)$ will have a maximum value when a root of $\frac{du}{dx}=0$ substituted in $\frac{d^2u}{dx^2}$ gives a negative result.

Can there be a maximum value of u if $\frac{d^2u}{dx^2}=0$, for the same value of x that makes $\frac{du}{dx}=0$?

66. Given the volume of a right cone, find its altitude when the surface is a maximum.

67. Find the equation of the tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$, and show that at the point (a, b) it is the same, whatever be the value of n.

68. Investigate the differential expression for determining the area of a plane curve referred to polar coordinates, and apply it to determine the whole area of the looped curve $r^2=a^2 \cos 2\theta$.

69. Expand $\sin^{-1}(x+h)$ to three terms by Taylor's theorem.

70. If $u=\frac{f(x)}{\phi(x)}$, deduce the rule for finding the differential coefficient of u with respect to x.

71. Investigate Maclaurin's theorem, and expand by means of it (1) $e^{\tan^{-1}x}$ and (2) $\epsilon^{a\tan^{-1}x}$ as far as the term involving x^3.

72. Determine the numerical value of the radius of curvature at the origin of the curve $y=x^4-4x^3-18x^2$.

73. Inscribe the greatest ellipse in a given semicircle.

74. Expand (1) $\tan^{-1}x$, (2) $\log_e(1+x)$, by Maclaurin's theorem.

75. Differentiate the functions—

(1) $u=(a^2-x^2)^{\frac{3}{2}}$; (2) $u=\log_e\frac{x^2+\sqrt{x^2-1}}{x^2-\sqrt{x^2-1}}$;

(3) $u=\frac{\sin mx}{(\cos x)^m}$; (4) $u=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$.

76. Find the least isosceles triangle which can circumscribe a given circle.

77. Find the radius of curvature, in polar coordinates, at the extremity of the *latus rectum* of a parabola.

78. Find the asymptotes and singular points of the curves

(1) $y^2=\frac{ax^2(x-a)}{x^2-4a^2}$; (2) $r=\frac{a(3\theta-\pi)(5\theta-\pi)}{4\theta-\pi}$.

79. Two ships are sailing uniformly with velocities u, v, along lines inclined at an angle θ. Given that at a certain time the ships are distant respectively a and b from the point of intersection of their courses, find the least distance between the ships.

80. If $y^3-3x^2y+x^3=0$, prove $\frac{dy}{dx}=\frac{y}{x}$.

81. If $u=f(x)$ has a maximum value, show that generally $\frac{du}{dx}=0$, and that $\frac{d^2u}{dx^2}$ is finite and negative.

82. Of all triangles on the same base and having the same perimeter, show that the isosceles triangle is the greatest.

83. If r be the radius of curvature at any point of a plane curve, p the corresponding perpendicular on the tangent, show that the radius of curvature at the point where r meets the curve$=\frac{rdr}{dp}$.

In the ellipse the equation from the focus being $p^2=\frac{b^2r}{2a-r}$, where a, b are the semi-axes major and minor, find from the above expression the radius of curvature at the extremity of the axis minor.

84. Find the maxima and mimima values of the function $x^3+\log(x-3)^4$.

85. Prove that the locus of the feet of the perpendiculars from the origin on the tangent to the curve $ay^2=x^3$ is given by the equation $27y^2(x^2+y^2)=4ax^3$; and that of the locus of the feet of the perpendiculars from the origin on the normals is

$$27x^3(x^2+y^2)=4ay^2(2y^2+3x^2).$$

86. Enunciate and prove Taylor's theorem, and explain what is meant by the failure of Taylor's theorem. Deduce Maclaurin's theorem from Taylor's theorem, and apply it to find the coefficient of x^r in the expansion of $e^x\cos x$ in powers of x.

87. If $y^3-xy^2-1=0$, find the first four terms of the :pansion of y in terms of x.

88. Explain any method of finding the asymptotes of a ırve given by a rational algebraic equation in x and y.

Find the asymptotes of the curves

(1) $(y-x)(y^2-4xy+3x^2)=4x^2+5xy+1$.

(2) $(y^2-4xy+3x^2)^2=4x^2+5xy+1$.

89. Define the circle of curvature at any point of a curve, ıd prove that the radius of curvature

$$=\frac{dx}{ds}\left(\frac{dy^2}{ds^2}\right)^{-1}=-\left(\frac{dy}{ds}\right)\left(\frac{d^2x}{ds^2}\right)^{-1}.$$

90. If s be the arc of a curve, measured from the point here it cuts the axis of y, and if y^2-s^2 be constant, prove ıat the radius of curvature varies as y^2.

91. Enunciate and prove Leibnitz's theorem for the dif- rentiation of the product of two functions.

If $y=ax^3+bx^2+cx$, prove that

$$x^3\frac{d^3y}{dx^3}-3x^2\frac{d^2y}{dx^2}+6x\frac{dy}{dx}-6y=0, \text{ and}$$

$$x^3\frac{d^{n+3}y}{dx^{n+3}}+3(n-1)x^2\frac{d^{n+2}y}{dx^{n+2}}+3(n-1)(n-2)x\frac{d^{n+1}y}{dx^{n+1}}$$
$$+(n-1)(n-2)(n-3)\frac{d^ny}{dx^n}=0.$$

92. If $f(a)=0$, and $\phi(a)=0$, show how to find the ulti- ıate value of the fraction $\frac{f(x)}{\phi(x)}$ as x approaches the value a.

Examine the case in which $f(a)$ and $\phi(a)$ are both infinite.

Find the ultimate values of (1) $\frac{(\sec x)^6}{\varepsilon^{\tan x}}$;

(2) $\frac{\cos x-\cos\frac{2x^2}{\pi}}{\cot x-\cot\frac{2x^2}{\pi}}$ when $x=\frac{\pi}{2}$.

93. What is meant by contact of the nth order between two curves?

Prove that the coordinates of the vertex of a parabola having contact of the second order with a curve at the point (x, y), and having its axis parallel to the axis of x, are

$$x+\frac{\frac{dy}{dx}}{2\frac{d^2y}{dx^2}} \text{ and } y+\frac{\left(\frac{dy}{dx}\right)}{\frac{d^2y}{dx^2}}.$$

Find the locus of the vertex when the given curve is the circle $x^2+y^2=a^2$.

INTEGRAL CALCULUS.

CXXVI.

1. Explain the meaning of Integration; and find the value of $\int_a^b x\,dx$, regarding it as the limit of a certain summation.

2. Show how to integrate the fraction $\frac{F(x)}{f(x)}$, when the roots of the equation $f(x)=0$ are real and unequal.

3. Integrate the following functions of x:—

(1) $\sin ax \sin bx$; (2) $x^2 \cos x$; (3) $\frac{x^2}{(x-a)(x-b)}$.

4. Integrate, with respect to x, the functions:—

(1) $\frac{1}{x^2+a^2}$; (2) $\frac{x}{x^4+a^4}$; (3) $x^3 \cos x$.

5. Prove

(1) that $\int_1^0 \frac{1}{x} \log(1-x)\,dx=\frac{\pi^2}{6}$;

(2) if m and n be positive integers,

$$m>n, \int_0^\infty \frac{x^{2m}dx}{1+x^{2m}} = \frac{\pi}{2n \sin \frac{2m+1}{2n}\pi}.$$

6. Find the relation between a and b, that the envelope of $\left(\frac{x}{a}\right)^2+\left(\frac{y}{b}\right)^2=1$ may be $xy=c^2$.

7. Integrate the functions:—

(1) $\int a^{m+x}dx$; (2) $\int \frac{dx}{x\sqrt{a^2+x^2}}$;

(3) $\int x^3(1+x^2)^{-\frac{1}{2}}dx$.

8. Find the whole length of the hypocycloid whose equation is $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$.

9. The equation to the cycloid from the vertex being $\frac{y}{c}=\sqrt{\frac{2a-x}{x}}$, show that its whole length is $8a$.

10. Prove the formula of integration by parts.

11. Find the area of a portion of a common parabola.

12. Integrate—

(1) $\int \frac{x^3dx}{(x-a)(x-b)(x-c)}$; (2) $\int \frac{xdx}{(x-3)^2(x+2)}$.

13. Find the volume of an oblate spheroid, and show that the volume of the sphere described on the minor axis of an ellipse as diameter is a third proportional to the oblate and prolate spheroids generated by the same ellipse.

14. Prove that—

$$\int \frac{dx}{(x^3+a^3)^n} = \frac{1}{3(n-1)a^3}\cdot\frac{x}{(a^3+x^3)^{n-1}} + \frac{3n-4}{3(n-1)a^3}\int \frac{dx}{(x^3+a^3)^{n-1}}$$

15. Evaluate the integrals—

(1) $\int_0^{\frac{\pi}{2}} (\sin x)^n\, dx$; (2) $\int_0^\infty e^{-x^2}\, dx$.

16. Integrate the functions—

$$(1)\int\frac{dx}{x^3(a+bx)};\qquad (2)\int\frac{x^3dx}{\sqrt{1-x^2}};$$

$$(3)\int dx\,(\sin x)^2;\qquad (4)\int\sqrt{a^2-x^2}dx.$$

17. Find the differential of the surface of a solid of revolution, and hence find the surface of a given right cone.

18. Obtain the integral $\int_0^{\pi} r\sin\theta\,d\theta$ in the curve where $r=a\,\theta$.

19. Integrate the functions—

$$(1)\int\frac{dx}{a+b\cos x};\qquad (2)\int\frac{dx}{\sqrt{x^2+a^2}};$$

$$(3)\int\theta\sin\theta\,d\theta;\qquad (4)\int\frac{dx}{1+\tan x}.$$

20. Investigate an expression for the volume of an ellipsoid.

21. Integrate the functions

$$(1)\ \frac{adx}{a^2-x^2};\qquad (2)\ \frac{xdx}{(a^2-x^2)^{\frac{7}{2}}};$$

$$(3)\ \frac{\tan^2 x}{4+\tan^2 x};\qquad (4)\ \frac{1}{x\sqrt{x^2+2x+2}}.$$

22. Integrate, with respect to x—

$$(1)\ \frac{x^2-5x+7}{x^2-5x+6};\qquad (2)\ \frac{x^2+7}{x^4+5x^2+4};$$

$$(3)\ \frac{x^2+2x+4}{x^3+2x^2+4x+8};\quad (4)\ x^3(\log x)^2.$$

Find a formula of reduction for $(a^2+x^2)^{\frac{1}{2}}$.

23. Integrate the functions :—

$$(1)\int x^{\frac{p}{q}}dx;\qquad (2)\int\frac{xdx}{a^2-x^2};$$

$$(3)\int\frac{dx}{x\sqrt{x^2-a^2}}.$$

24. Find the differential coefficient of the surface of a lid of revolution, and apply it to find the surface of a given gment of a sphere.

25. Integrate :—

$$(1)\ \frac{x\,dx}{\sqrt{a^4-x^4}}; \qquad (2)\ \frac{x\,dx}{(x+1)(x+3)};$$

$$(3)\ \frac{d\theta}{\cos^2\theta-\sin^2\theta}; \qquad (4)\ \frac{1+\cos\theta}{\theta+\sin\theta}d\theta.$$

26. Integrate the following functions of x :—

$$(1)\ \sqrt{\frac{a+x}{x}}; \quad (2)\ \frac{x^2-1}{x^2-4}; \quad (3)\ e^{ax}\sin cx.$$

27. Find the area of the loop of the curve,

$$y^2(a^2+x^2)=x^2\ (a^2-x^2).$$

28. Trace the curve $a^2y^4=x^4\ (a^2-x^2)$, and find the area one loop of it.

29. Find the areas of the curves—

$$(1)\ x^2-2xy+10y^2=a^2; \qquad (2)\ \left(y-\frac{x^2}{a}\right)^2=a^2-x^2.$$

30. Find the integrals of

$$(1)\ \frac{dx}{a^2+x^2}; \qquad (2)\ \frac{dx}{\cos x}; \qquad (3)\ \frac{dx}{\sin x}.$$

31. Integrate—

$$(1)\ \frac{du}{dx}=\frac{x^2}{x^2-4}; \qquad (2)\ \frac{du}{dx}=(\sin x)^2.$$

32. Investigate the expression for the differential coefficient of the volume of a solid of revolution.

Find the volume of the frustum of a paraboloid in terms the radii of its extreme circular sections, and of the rtion of the axis intercepted between them.

33. Write down the integrals with regard to x of the pressions

$$(1)\ x^3+a^3; \quad (2)\ \sin x; \quad (3)\ \cot x; \quad ae^{mx}; \quad (a^2-x^2)^{-\frac{1}{2}}.$$

34. Integrate the expressions

(1) $x^4 \log x$;

(2) $(x^2+a^2)^{-1}$; $(x^2+2a^2)^{-1}$; $\dfrac{x^2-1}{x^4+3x^2+1}$.

35. Investigate a form of reduction for the integration of $(x^4+a^4)^{-n}$, n being a positive integer.

36. Evaluate the expressions

(1) $\int_0^{\frac{\pi}{2}} (\sin x)^{2x}\, dx$;

(2) $\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$; (3) $\int_0^{\infty} \dfrac{dx}{(x^2+1)^3}$.

37. Show how to employ the integral calculus in finding the area contained (1) between the curves $y=f(x)$, $y=\phi(x)$, and two ordinates $x=a$, $x=b$; (2) between two curves $r=f(\theta)$, $r=\phi(\theta)$, and two radii $\theta=\alpha$, $\theta=\beta$.

38. Find the area enclosed by the lines

$$ay^2=x^3 \text{ and } y^2=2ax-x^2.$$

39. Write down the integrals with regard to x of

(1) $\cos x$; (2) a^x;
(3) $(x^2+a^2)^{-1}$; (4) $(a^2-x^2)^{-\frac{1}{2}}$.

40. Integrate

(1) $\dfrac{\sin x+\cos x}{\sin x-\cos x}$; (2) $\dfrac{x+3}{(x+1)^2(x+3)}$.

41. Explain the method of integration by parts, and apply it to integrate

(1) $x^3 \sin x$; (2) $e^{cx} \cos ax$.

42. Prove that the area between a curve and two radii from the origin is given by the expression

$\int \dfrac{r^2}{2} d\theta$, taken between proper limits.

Show that such an area is also given by the expression

$$\tfrac{1}{2}\int(x\,dy - y\,dx).$$

43. Find the complete areas enclosed respectively by each of the curves

(1) $r = a \sin 2\theta$; (2) $r = a \sin 3\theta$.

44. Determine the area enclosed by a three-cusped hypocycloid.

45. Find the volumes of the solids generated by the revolution of the curve $r^2 = a^2 \cos 2\theta$.

(1) About the initial line; (2) about one of the tangents at the origin.

46. If r be the distance of any point of an ellipse from one of the foci, and a be the semi-major axis, prove that $\int \sqrt{\frac{2a-r}{r}}\,.\,ds$ along any arc is proportional to the sectional area subtended by that arc at the other focus.

MIXED MATHEMATICS.

FORMULÆ IN STATICS.

R=resultant of P and Q acting at an angle θ,

$R^2=P^2+Q^2+2\,PQ\cos\theta$; if P=Q, R=2P $\cos\frac{1}{2}\theta$.

$\theta=0$, R=P+Q; $\theta=\pi$, R=P−Q.

If P, Q, R are in equilibrium at a point O,

P : Q : R=sin QOR : sin POR : sin POQ.

R=resultant of any number of forces acting upon a body in one plane.

$$R^2=\{\Sigma(X)\}^2+\{\Sigma(Y)\}^2 \;:\; \tan\theta=\frac{\Sigma(Y)}{\Sigma(X)};$$

in case of equilibrium $\Sigma(X)=0$: $\Sigma(Y)=0$. $\Sigma(Yx-Xy)=$

R=resultant of any number of forces acting upon a in any directions,

$$R^2=\{\Sigma(x)\}^2+\{\Sigma(Y)\}^2+\{\Sigma(z)\}^2;$$

$$\cos\theta=\frac{\Sigma(X)}{R} : \cos\phi=\frac{\Sigma(Y)}{R} : \cos\xi=\frac{\Sigma(z)}{R} :$$

in case of equilibrium $\Sigma(X)=0$, $\Sigma(Y)=0$: $\Sigma(z)=0$;

$\Sigma(zy-Yz)=0$; $\Sigma(Xz-zx)=0$; $\Sigma(Yx-Xy)=0$.

If $\bar{x}$, $\bar{y}$, be the coordinates of the centre of gravity of a system of bodies,

$$\bar{x}=\frac{\Sigma(P.x)}{\Sigma(P)} : \bar{y}=\frac{\Sigma(P.y)}{\Sigma(P)}.$$

In the lever,

$$\frac{P}{W}=\frac{\text{perpendicular from fulcrum on W's direction}}{\text{perpendicular from fulcrum on P's direction}}.$$

Pressure on fulcrum $=[P^2+W^2-2PW\cos(\alpha+\beta)]^{\frac{1}{2}}$.

Direction of pressure $=\theta$; $\tan\theta=\dfrac{P\sin\alpha+W\sin\beta}{P\cos\alpha-W\cos\beta}$.

In the wheel and axle $\dfrac{P}{W}=\dfrac{\text{rad. of axle}}{\text{rad. of wheel}}$.

Single moveable pulley, $\dfrac{W}{P}=2\cos\alpha$; 2α=angle between the strings.

System of pulleys each hanging by separate string—

$P=\dfrac{1}{2^n}\{W+(2^n-1)w\}$, n being no. of moveable pulleys.

System of pulleys, same string passing round all the pulleys, $W+B=nP$. B=weight of block.

System of pulleys, when all the strings are attached to the weight, $W=(2^n-1)P+(2^n-n-1)w$; n being no. of strings attached to weight.

Inclined plane, smooth; $\dfrac{P}{\sin\alpha}=\dfrac{W}{\cos\varepsilon}=\dfrac{R}{\cos(\alpha+\varepsilon)}$.

Inclined plane, rough, $P=\dfrac{W\sin\alpha\mp\mu W\cos\alpha}{\cos\varepsilon\mp\mu\sin\varepsilon}$.

Screw, smooth—

$$\frac{P}{W}=\frac{\text{vertical distance between two threads}}{\text{circumference of circle described by P}}.$$

Screw, with friction—

$$\frac{P}{W}=\frac{b(\sin\alpha-\mu\cos\alpha)}{a(\cos\alpha+\mu\sin\alpha)}=\frac{b}{a}\tan(\alpha-\varepsilon),\text{ if }\mu=\tan\varepsilon.$$

In the catenary $y+c=\sqrt{c^2+s^2}$;

$$x=c\log\frac{y+c+\sqrt{y^2+2yc}}{c};\quad y'=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right);$$

$$t=y+c;\quad \sqrt{(\lambda^2+k^2)}=c\left(e^{\frac{k}{2c}}-e^{-\frac{k}{2c}}\right).$$

FORMULÆ IN DYNAMICS.

Motion uniformly accelerated—

$$v=ft;\ s=\tfrac{1}{2}ft^2;\ v^2=2fs;\ u=v\pm ft;$$
$$s=vt\pm\tfrac{1}{2}ft^2;\ u^2=v^2\pm 2fs.$$

When the body moves freely $f=g=32{\cdot}2$: on inclined plane $f=g \sin \alpha$: if P moving on an inclined plane draw Q up an an inclined plane $f=\frac{P \sin \alpha - Q \sin \beta}{P+Q} g$.

Impact: A moving with velocity a, impinges on B whose velocity is b.

Velocity of A after impact $=\frac{Aa \pm Bb - Be(a \mp b)}{A+B}$.

Velocity of B after impact $=\frac{Aa \pm Bb + Ae(a \mp b)}{A+B}$.

Projectiles in vacuo—

Range at end of time $t=vt \cos \alpha$;
height $=vt \sin \alpha - \frac{1}{2} gt^2$.

Equation to path $y=x \tan \alpha - \frac{x^2}{4h \cos^2 \alpha}$ where $h=\frac{v^2}{2g}$.

Time of flight $=\frac{2v \sin \alpha}{g}$: horizontal range $=\frac{v^2 \sin 2\alpha}{g}$.

Greatest height $=\frac{v^2 \sin^2 \alpha}{2g} = h \sin^2 \alpha$.

Range on inclined plane $=\frac{4h \cos \alpha \sin (\alpha - i)}{\cos^2 i}$:

time of flight $=\frac{2v \sin (\alpha - i)}{g \cos i}$.

Simple pendulum: $t=\pi \sqrt{\frac{l}{g}}$.

A body moving in a circle: accelerating force in direction of normal $=\frac{v^2}{r}$; moving force $=\frac{mv^2}{r}$.

Conical pendulum $v^2=\frac{gl \sin^2 \alpha}{\cos \alpha}$; $t=2\pi \sqrt{\frac{A C}{g}}$.

FORMULÆ IN HYDROSTATICS.

Normal pressure on area a^2, the depth of whose centre of gravity below surface of fluid is h=weight of volume $a^2 h$ of the fluid.

Specific gravity of a substance

$$= \frac{\text{weight of any volume of substance}}{\text{weight of same volume of water}}.$$

In a compound body: $G = \frac{v_1\rho + v_2\rho + v_3\rho + \&c.}{v_1 + v_2 + v_3 + \&c.}$; when there is no change of volume.

Density of air in receiver of air pump after n strokes $= \left(\frac{A}{A+B}\right)^n \rho$.

Bramah's press, $\frac{P}{W} = \frac{b\,r^2}{a\,R^2}$.

In diving-bell tension of string = weight of bell $- g\rho\, A x$ where A = area of top of bell, x = length occupied by air, is found from the equation $hb = (h+a)\,x + x^2$, b = length of bell, a = depth of its top. (*Besant.*)

FORMULÆ IN PRACTICAL MECHANICS.

Unit of work = pressure of 1 lb. exerted through a space of 1 foot.

Work done in moving resistance of m lbs. through n feet $= m\,n$ units.

Unit of horse power = 33000 units of work.

$$\text{Modulus of a machine} = \frac{\text{work yielded}}{\text{work expended}}.$$

Work done in moving a body on a horizontal plane $= f \cdot Ws$ where f = coefficient of friction, W = weight of body, s = space described.

Work done in moving a body up an inclined plane = work due to friction + work due to force of gravity $= Wh + f \cdot Wl$: h being the height, l the length of plane, whose inclination is very small: otherwise work done $= Wh + f \cdot W \cos a \times l$.

Work accumulated in body, moving with velocity v, $= \frac{Wv^2}{2g}$.

Work done in upsetting a heavy body = work requisite to

raise the body vertically through the height which its centre of gravity is raised.

Work done in raising material of given form=weight of material in lbs. × number of feet through which C. G. is raised.

Moment of inertia of a system of bodies

$$=m_1r^2{}_1+m_2r_2{}^2+m_3r_3{}^2+\&c.=\text{I}.$$

Radius of gyration

$$=k=\sqrt{\frac{\text{I}}{m}}=\sqrt{\frac{m_1\,r_1{}^2+m_2\,r_2{}^2+m_3\,r_3{}^2+\&c.}{m_1+m_2+m_3+\&c.}}$$

If I=moment of inertia about an axis passing through C. G. I_1=moment about an axis at a distance a from former, $\text{I}_1=\text{I}+a^2m$; and on similar suppositions $k_1{}^2=k^2+a^2 \therefore k_1{}^2-a^2$ is constant.

Radius of gyration of wheel $=\frac{\text{radius of wheel}}{\sqrt{2}}$.

Radius of gyration of rod revolving about its middle point $=\frac{l}{2\sqrt{3}}$.

Radius of gyration of rod revolving about its extremity $=\frac{l}{\sqrt{3}}$.

WOOLWICH PAPERS.

I.

1. Prove the '*Triangle of Forces.*'

2. A B C is an isosceles triangle, A and B the equal angles, C D a perpendicular from the vertex on the base, take G D=$\frac{1}{3}$ C D; then if G A, G B represent two forces acting on a point at G, G C will represent the force that will keep the point G at rest.

3. Three forces in the plane of a triangular board act

upon its sides in directions perpendicular to them: prove that if the forces are in equilibrium, they are proportional to the sides on which they act.

4. Three forces, represented by 3, 4, and 5 pounds respectively, keep a particle in equilibrium: determine the angles at which they act.

5. Forces act on a triangle which are represented in magnitude and direction by the straight lines drawn from the vertices to the points of bisection of the opposite sides; show that they will maintain the triangle in equilibrium.

6. The resultant of two forces is 100 lbs., and the angles which it makes with the directions of the forces are 20° and 30°; find the component forces.

7. A heavy body is drawn up an inclined plane by means of a string, which being parallel to the plane, and passing over a pulley, has another heavy body hanging vertically at the end of it; find the accelerating force. If the plane be inclined at an angle of 30°, and the accelerating force be $\frac{1}{2}g$, find the ratio of the masses of the two bodies.

8. A weight Q is drawn along a smooth horizontal table by a weight P hanging vertically: find (1) the accelerating force on P, (2) the accelerating force on the centre of gravity of P and Q.

9. A body is projected with a velocity v in a direction making an angle of 30° with the horizon; find its velocity and direction of motion when it has reached a height $2g$. What is the horizontal space described in this time?

10. A body is projected horizontally with a velocity $4g$ from a point whose height above the ground is $16g$; find the direction of motion (1) when it has fallen half way to the ground, (2) when half the time of flight has elapsed.

11. Two particles are projected simultaneously from a given point with different velocities and in different directions, but in the same vertical plane; find the path described by their centre of gravity.

12. A lump of metal weighs 59 oz. in water and 61 oz. in alcohol, whose *s. g.* is ·8; find the weight and *s. g.* of the metal.

II.

1. Three forces act upon a particle in a plane; show how to find their resultant.

2. Three forces of 3, 4, 5 lbs. act on a particle in the centre of a square in directions towards three of the angles of the square: find the magnitude and direction of the force which will keep the particle at rest.

3. Can the forces 3, 5, 11 acting on a particle keep it at rest?

4. A force of 40 lbs. acting on a point between two forces of 20 lbs. and $20\sqrt{3}$ lbs. respectively, makes an angle of 60° with the former and 30° with the latter; find the magnitude and direction of the force which will keep the point at rest.

5. Two forces, P and Q, act upon a point; find the greatest and the least forces which can keep the point at rest. How will the directions of the forces in each case be adjusted?

6. At what angle must two forces, P and Q, act so that a single force which keeps them at rest shall be a mean proportional between the greatest and least values, as determined in *Quest.* 5?

7. A ball A impinges directly upon a ball B, in motion: show that the velocity of their centre of gravity is not affected by the impact.

8. Show that the time of a projectile's describing any arc of its path is the same as would be occupied in moving uniformly along the chord with the velocity which it has when its motion is parallel to the chord.

9. Find the direction in which a body must be projected with a given velocity to hit a given mark, and show generally that there are two directions that will satisfy the conditions.

10. A body oscillates once in 2·5″; find the distance between its centres of suspension and oscillation.

11. A cube of wood floating on water descends $\frac{1}{2}$ in. when a weight of 15 oz. is placed upon it; find the size of the cube, supposing a cubic foot of water to weigh 1000 oz.

12. A B C D is a parallelogram whose diagonals A C, B D intersect in E, and A B is in the surface of a fluid; show that when the fluid is homogeneous the pressures on the triangles A E B, B E C, C E D are as 1, 3, 5. Compare the pressures when the parallelogram is a square and the density of the fluid varies as the depth.

III.

1. Show how forces may be represented geometrically by straight lines. Define a moment, and show how it may be represented geometrically, in accordance with the above representation of a force.

2. Deduce the principle of equality of moments from that of the parallelogram of pressures.

3. There is a uniform bar, weighing 10 lbs., and 5 ft. long: weights of 9 lbs. and 5 lbs. are suspended from its extremities: on what point will it balance?

4. A uniform beam A B, whose weight is W, rests with one end on a smooth horizontal plane A C, and the other end B on a smooth plane C B inclined to the horizon at 60°. Find the tension of a string C A=C B which keeps the beam from slipping.

5. Two planes of equal altitude are inclined at angles of 60° and 45° to the horizon; required what weight resting on the latter will balance 20 lbs. on the former, the weights being connected by means of a string passing over the common vertex parallel to the planes.

6. The mouth of a harbour is $\frac{3}{4}$ mi. wide, and the tide is running out of it at the rate of $2\frac{1}{2}$ mi. an hour; in what direction must a man, who can row in still water at the rate

of 5 mi. an hour, point the head of the boat in order to make for a point directly opposite to him ?

7. In *Qu.* 6, find the length of the man's course when he keeps the head of the boat pointed to the opposite shore, and determine the length of time employed in making this course, and compare it with that employed in making a course direct across the harbour.

8. A body has fallen from rest through a feet at the moment when a second body begins to fall, b feet below the point which the first then occupies; find the distance traversed by the second before it is overtaken by the first, the force of gravity being *constant*, but unknown in amount.

9. Find the inclination of a gradient in a railway that a carriage descending down it, by its own weight, may move through $\frac{1}{2}$ a mile in 30″; find the space it will describe in the next 30″.

10. Describe the *force-pump*, and explain its action.

11. The specific gravity of gold being taken as 19·3 and of copper as 8·6, find the specific gravity of an alloy containing 2 oz. of copper to 23 oz. of gold.

12. Find the depth at which a hemisphere floats with its base downwards, when the specific gravities of the fluid and solid are as 16 : 11.

IV.

1. Show that if θ be the angle between the directions of two given forces, their resultant is greatest when $\theta=0$; least when $\theta=\pi$; and intermediate for intermediate values of θ.

2. Find the resultant of two equal forces acting on a point.

3. Three forces, 99, 100, 101 lbs. respectively act upon a point in directions making an angle of 120° with each other, successively; find the magnitude of their resultant, and the angle which it makes with the force 100.

4 An endless band A B C D passes over three pulleys at

A, B, C, and a weight of 100 lbs. is attached to D. Supposing A B, C B each to make angles of 45°, and A D, C D angles of 30°, with the vertical, and A and C to be in the same horizontal line, find the magnitude and direction of the pressures on B, A, and C.

5. Four bodies weighing 1, 2, 4, 7 lbs., respectively, are placed with their centres of gravity in a straight line at the respective distances of 3, 5, 7, 9 ft., from a given point in that line; find the common centre of gravity.

6. Find the conditions of equilibrium of any system of forces acting in one plane. Account for only one of these conditions being necessary, in the case of a lever attached to a fulcrum.

A uniform beam A B, whose weight is W, rests in equilibrium between a vertical wall B C and the horizontal plane A C, both smooth; C E is a string without weight attached to a point E in the beam. If $B A C = \alpha$, and $A C E = \beta$, show that the tension of the string $= \frac{W \cos \alpha}{2 \sin (\alpha - \beta)}$.

7. If a bullet be fired from a level ground in a direction making an angle i with the horizon, and the horizontal range be R, find the time of flight.

8. If a body be projected from A in the direction A B C, and from C any point in that line a vertical line C D be drawn, meeting the curve described by the projectile in D, and if B, the middle point of the finite line A B C, be joined to D, show that B D will be the direction of the motion at D, and that the velocity at D will be to that at A as B D : A B.

9. How long will 10 men be in pumping dry the hold of a ship which contains 30,000 cubic ft. of water, the centre of gravity of the water being 14 ft. below the point of discharge, and each man yielding 1500 units of effective work per minute?

10. A uniform rod of length l and weight w resting vertically upon one extremity on a horizontal plane, is allowed to fall over. On what point must it strike a nail in

the plane, that the whole accumulated work may take effect upon the nail; and how far would it drive the nail, supposing the resistance constant?

11. Explain the construction of the '*Hydrometer*.'

12. A cylindrical pontoon floats with $\frac{3}{4}$ of its diameter out of water, but sinks to the water level when loaded with a weight of 32 cwt.: supposing the diameter of the external surface to be 3 ft., and a cubic foot of water to weigh 1000 oz., find the length of the pontoon.

V.

1. Prove the principle of the parallelogram of forces.

2. The two extremities of a string are fastened to two nails in a vertical wall, and the string is stretched tight by a weight tied to a given point in it: show how the tensions of the two parts of the string may be determined by geometrical construction.

3. If two equal forces act upon a point at an angle of 150°, what will be the magnitude of the resultant?

4. Two men pull, each with a force of 80 lbs., in different directions, making an angle of 60° with one another, at the end of a rope, passing freely round a fixed point. Find the pressure on the point.

5. A B C D is a square. A force of 3 lbs. acts from A to B, a force of 4 lbs. from B to C, a force of 6 lbs. from D to C, and a force of 5 lbs. from A to D. Find the magnitude and position of the resultant force.

6. Explain what is meant by the '*modulus of a machine*.'

7. A tank is 12 feet long, 7 feet wide, 8 feet deep; the surface of water in a well from which it is filled by a pump is 56 ft. below the bottom of the tank, the man working the pump performs 2600 units of work per minute, and fills the tank in 30 hrs.; find the modulus of the pump, a cubic foot of water weighing 1000 oz.

8. State the 'Third Law of Motion,' and show how it is applicable to the direct collision of elastic balls in determining their motions after impact.

9. A ball, $3m$, strikes a ball m at rest, the modulus of elasticity being ·6; compare the velocities of the striking ball before and after impact.

10. A particle being projected with a velocity of 1610 ft. a second and an elevation of 30°, find the *latus-rectum* of its path, and the distance of the focus from the point of projection.

11. When a body floats in water, show that the weight of the body is equal to that of the fluid displaced.

12. A hemisphere whose weight is 5600 grs. floats with its vertex downwards, and $\frac{1}{3}$ of its vertical radius immersed; what weight must be placed on the hemisphere so that $\frac{2}{3}$ of the vertical radius may be immersed?

VI.

1. State fully the principle of the equality of moments. What other condition is necessary to the equilibrium of any number of pressures acting in the same plane?

2. O P, O Q represent two equal forces acting upon O, at right angles to each other, and O R represents the third force which keeps O in equilibrium; show that the triangle formed by the intersection of the lines drawn perpendicular to O P, O Q, O R, through P, Q, and R, is isosceles and right angled.

3. The resultant of two forces is 9 lbs., one of them is 6 lbs., and the other is inclined to the resultant at an angle of 30°; find the other force and the angle between the two.

4. A beam 45 ft. long balances itself on a point $\frac{1}{3}$ of its length from the thicker end, but when a weight of 15 lbs. is suspended from the smaller end, the prop must be moved 3 ft. towards it, to maintain equilibrium. Find the weight of the beam.

5. How is the nature of the equilibrium of a body or of

a system of bodies affected by the circumstance of the centre of gravity being in its highest or its lowest position ?

Examine the nature of equilibrium when P supports W on the common wheel and axle.

6. The mean section of a stream acting on the floats of a water-wheel is $5\frac{1}{3}$ by $1\frac{1}{2}$ ft. ; its mean velocity is 40 ft. a minute, it has a fall of $17\frac{1}{2}$ ft. ; the modulus of the wheel is ·7 ; it is required to raise water by means of the wheel to a height of 300 ft. How many cubic feet will it raise per minute ?

7. Two shots are fired from the same point with the same velocity, the elevation of one being α, that of the other $\alpha+\delta$, where δ is a very small quantity ; find the intersection of their paths.

8. A body is projected upwards over a plane making an angle of 15° with the horizon ; find the direction of projection when the range is a maximum.

9. Two billiard balls in contact are struck by a third, moving in a direction perpendicular to the line joining their centres ; find the velocity of the latter after impact, v being the velocity of the impinging ball before impact, e the modulus of elasticity.

10. What must be the length of a pendulum to beat 9 times in 1′ at a place where the seconds' pendulum is 39·14 inches long ?

11. Show how to compare the specific gravities of two fluids by weighing the same solid in each.

12. The diameters of the tube and bellows being respectively $\frac{5}{8}$ in. and $1\frac{7}{8}$ in. in a hydrostatic bellows, how much will the weight upon the lid be raised above its original position when 5 lbs. of water is poured in through the tube, a cubic foot of water weighing 1000 oz. ?

VII.

1. Prove the '*Polygon of forces.*'

2. Three pressures, of 10 lbs. each, act on a point. The

first two are inclined at an angle of 60° to each other, and the third is inclined at an angle of 30° to the second ; find their resultant.

3. A bar, each foot of which weighs 7 lbs., rests upon a fulcrum distant 3 ft. from one extremity ; what must be its length, that a weight of $71\frac{1}{2}$ lbs. suspended from that extremity may just be balanced by 20 lbs. suspended from the other extremity ?

4. A string with a weight W at one end, and running through a small loop C at the other, passes over two fixed points A, B, at the extremities of a line 4 ft. long and making an angle of 15° with the horizon; find the angles between this line and the oblique portions of the string when there is equilibrium. The length of the portion A C B A of the string being 12 ft.

5. Two forces act at a right angle, their resultant is double the less ; what is the ratio of the forces ?

6. A B C is a right-angled isosceles triangle, C being the right-angle. A force of 4 lbs. acts along A B from A to B; a force of 3 lbs. acts along C B from C to B ; and a force of 2 lbs. acts along A C from A to C. Determine the magnitude and direction of the resultant.

7. If a body be projected up a given smooth inclined plane, show how to determine the space it will describe in a given time.

8. An inclined plane rises 1 ft. vertically for every 100 ft. in length ; with what velocity must a body be projected up the plane that it may describe 183·9 ft. in 10 seconds, gravity being taken as 32·2 feet ?

9. Define the terms, '*work*,' '*labouring force*,' '*unit of work*,' '*horse-power*.'

10. If a pressure of 1 ton is exerted through 20 yds., how many units of work are done ? And if it is done by a steam engine in half a minute, what is the horse-power of the engine ?

11. Describe the '*barometer*;' explain how it is used to determine the height of a mountain.

12. State the conditions of equilibrium of a floating body.

The weight of a cubic foot of water is 1000 oz.; a cone, whose specific gravity is half that of water, floats with its vertex downwards and axis vertical: find the weight of water which it displaces, the height of the cone being 30 in., the diameter of its base 9 inches.

VIII.

1. Find the condition of equilibrium in the system of pulleys where the same string passes round all the pulleys.

2. A rectangular table stands on a rough inclined plane, and has two sides horizontal; if the coefficient of friction between the lowest feet and the plane be μ, and between the highest feet and the plane μ_1, find the inclination of the plane when the table is on the point of moving.

3. Two heavy spheres, the radii of which are 2 and 1·5 in. respectively, are suspended at the opposite extremities of a horizontal straight rod $7\frac{7}{12}$ ft. long, where does the direction of their resultant cut the rod?

4. A uniform heavy string has its ends fixed; determine the form of the string when in equilibrium. If the length of the string be 100 ft., and the two fixed points be in the same horizontal plane and 90 ft. apart, compare approximately the tension at the lowest point with the weight of the string.

5. When an isosceles wedge is urged by a given pressure between two obstacles, show how to find the resistance of the obstacles.

6. An isosceles wedge whose angle is 120°, is urged by a pressure of 2 cwt. between two obstacles, and is in a state bordering on motion forwards; if 30° be the limiting angle of resistance, find the resistance of each obstacle.

7. Prove that the portion of the centrifugal force at the earth's surface varies as the square of the cosine of the latitude.

8. Two bodies are let fall at an interval of 1″, how far will they be apart after an interval of 5″, from the fall of the first?

9. The plane of a circle being vertical, prove that the time of descent down all chords drawn from either extremity of the vertical diameter will be the same. Find the line of quickest descent from a given point to a given circle.

10. A body moving on a rough inclined plane is subject to a constant resistance equal to $\frac{1}{3}$ of its weight: how long will it continue in motion, and how far will it go, if started with a velocity of 30 ft. a second?

11. A weight of 12 oz. draws up another of 6 oz., by means of a string passing round a fixed pulley; determine the pressure on the pulley.

12. A cylindrical tube, 2 ft. long and closed at one end, contains mercury which occupies 16 in. of its length, the tube is then inverted (without any of the air escaping), and 2 in. of its length from the open end introduced into a basin of mercury: determine the space which the air will occupy in this position. The height of the mercury in the standard barometer being 30 inches.

IX.

1. Show that every system of particles has one centre of gravity, and only one whilst in the same position.

2. If in the system of pulleys in which each string is attached to the weight, these strings are fastened to a bar at a distance of 2 inches from each other, this being the radius of each pulley; find from what point the weight must be suspended in order that the bar may be kept horizontal, the weights of the pulleys being neglected.

3. What is meant by the '*limiting angle of friction*?' Show that when a particle is placed on a rough horizontal plane and acted upon by a pressure towards the plane, the condition of its moving or remaining at rest depends upon the direction of the pressure, and not upon its magnitude.

4. A uniform beam, whose length is $2b$ and weight w, rests upon a given rough cylinder, of radius r, in a vertical plane; determine the weight w that must be suspended from one extremity of the beam so that it may just be on the point of sliding off the cylinder, the coefficient of friction being μ.

5. Two equal weights are made to balance on two inclined planes with a common vertex, by means of a string passing freely over a fixed pulley situated above the vertex: compare the angles which the string makes with the planes, the length of one plane being double that of the other.

6. Prove the formulæ $s=ut-\frac{1}{2}ft^2$; $v^2=u^2-2fs$.

7. With what velocity must a body be projected vertically upwards that it may just rise to the height of 1600 ft.? and what time will it take in rising, supposing $g=32$ ft.?

8. A shot is fired with a velocity v towards a tower whose height is h, and whose horizontal distance is one-half of the greatest range attainable with the given velocity. Find between what limits the elevation must lie in order that the shot may hit the tower.

9. A particle describes a circle with uniform velocity, show that the force on the particle is directed towards the centre, and determine its magnitude.

10. Find the time of oscillation of a simple pendulum in a small circular arc.

Taking the force of gravity at a station to be 32·2 ft, find the length of the pendulum which will oscillate in 1·5″.

11. A diving-bell is lowered in water until its mouth is 18 ft. below the surface, and the water 2 ft. high in the bell; supposing the bell when full to contain 1000 lbs. of water, find how much the tension of the rope is diminished when air is forced into the bell until it is free from water.

12. A triangle A B C is immersed in fluid in such a position that the point A is in the surface, and the lines A B, A C equally inclined to it, B C being produced to meet the surface

in E; show that the pressures on the triangles A B C, A C E are as $AB^2 - AC^2 : AC^2$.

X.

1. If two surfaces are in contact, how is the condition of their being (1) smooth, (2) rough, expressed in mechanical problems?

2. If two rough bodies rest on an inclined plane at a distance from each other equal to the length of a string connecting them, and if the coefficients of friction for the two bodies be different, find the greatest inclination the plane may have without their sliding.

3. A frustum of a right cone has a height h, the radii of its ends, r and r_1, $r > r_1$; determine the distance of its centre of gravity from the centre of its smaller end.

4. Find the power which will just support a weight of 100 lbs. by means of a system of 4 pulleys, the strings being parallel and all attached to the weight, each moveable pulley weighing 1 lb.

5. A rod without weight is 2 ft. long; at one end a force of 4 lbs. acts, at the other end a force of 8 lbs., and these forces are parallel. Find the magnitude and point of application of a single force which will keep the rod at rest.

6. At the earth's surface $g = 32{\cdot}2$ ft., and the space through which a body falls in 1″ from rest is 16·1 ft., explain and account for this relation. Find the velocity acquired by a body falling freely 6440 feet.

7. Supposing the force of gravity to vary inversely as the square of the distance from the earth's centre, find the space through which a body would fall in 1 hour at a distance of 240,000 miles from the earth's centre, the radius of the earth being 4,000 miles.

8. Find the line of quickest descent from a given point to a given fixed straight line in the same vertical plane.

9. A ball of 4 lbs. weight moving from left to right with

a velocity of 8 yds. per second, impinges directly upon a ball of 10 lbs. weight moving in the same direction with a velocity of 2 yds. per second: the first ball is reduced to rest by impact, find the modulus of elasticity.

10. In the conical pendulum the length of the string is 6 ft., and the weight describes a circle in ·5″: compare the weight and the tension of the string.

11. The *s. g.* of gold is 19·25, that of copper, 8·9. How many cubic inches of copper must be mixed with 12 cub. in. of gold so that the *s. g.* of the compound may be 18·45?

12. A cubical box filled with 1000 oz. of water, is supported in such a position that one edge is horizontal, and one of the edges, perpendicular to it, inclined at an angle of 45° to the horizon: find the total pressure on the six faces of the cube.

XI.

1. Define the '*centre of gravity*;' and find the centre of gravity of a pyramid.

2. Prove that the centre of gravity of the perimeter of a triangle is the centre of the circle inscribed in the triangle formed by joining the middle points of the given triangle.

3. A string fastened to a nail, and hanging vertically, supports the apex of a uniform isosceles triangle of weight w, which rests with its base upon a smooth horizontal plane; find the tension of the string and the pressure on the plane.

4. A bar of iron 15 inches long, which weighs 12 lbs., and is of uniform thickness, has a weight of 10 lbs. suspended from one extremity; where must a fulcrum be placed that it may just balance upon it?

5. Illustrate the statement, 'The number of units of work yielded by any agent in a given time is a measure of its efficiency.'

6. The diameter of the piston of a steam engine is 80 in., the mean pressure of the steam 24 lbs. per square inch, the length of stroke 10 feet, the number of strokes per minute 11.

How many cubic feet of water will it raise per minute from a depth of 25 fathoms, the modulus of the engine being ·6, and a cubic foot of water weighing 62·5 lbs.?

7. Define the terms, '*centre of oscillation*,' '*centre of percussion*,' '*angular velocity*.'

8. A uniform rod of given length and weight rests by one extremity upon a horizontal plane, and is allowed to fall over (without slipping) from a vertical to a horizontal position. Find the angular velocity which it will acquire.

9. The monument in London is said to be 210 feet high; (1) with what velocity must a shot be fired from its base, vertically, to reach the top in ·75″ ? (2) what will be its velocity on passing the top? (3) how high will it rise?

10. In what time will a shell range 4500 feet at an elevation of 40°, neglecting the resistance of the air?

11. If α be the angle between the two tangents at the extremities of any arc of the parabolic path of a particle acted on by gravity, v and v' the velocities at these extremities, u the velocity at the vertex; show that the time of describing the arc is $\frac{v\,v'\sin\alpha}{g\,u}$.

12. A cylindrical pontoon 12 feet long and 3 feet in diameter, floats with $\frac{2}{3}$ of its diameter immersed; what additional weight in pounds will it bear without sinking, taking 1000 oz. as the weight of 1 cubic foot of water?

13. Describe Smeaton's air-pump, and find the density of the air in the receiver after n ascents of the piston.

XII.

1. The centre of gravity of a body being given, and also that of a portion of it, show how to find the centre of gravity of the remainder.

2. The top of a triangle is cut off by a straight line parallel to its base, at a distance of $\frac{2}{3}$ of the altitude of the triangle; find the centre of gravity of the remaining trapezoid.

3. Find the centre of gravity of a shell bounded by two nonconcentric surfaces whose radii are r and r_1.

4. How many horses, each exerting a traction of 200 lbs., would be required to draw a waggon weighing 5 tons up a hill whose slope is 1 in 18, the traction on the road, if level, being estimated at $\frac{1}{20}$ of the gross load?

5. In an isosceles roof, when there is no tie beam, show how to determine the amount and direction of the pressure of the roof on the side walls which support it.

6. If each rafter be inclined to the horizon at an angle θ, the span remaining the same, show that the pressure is least when $\tan\theta = \frac{1}{\sqrt{2}}$.

7. A and B are two equal elastic spheres, A moving with a velocity v impinges upon B at rest, the direction of A's motion before impact making an angle of 60° with their line of centres; determine the directions and velocities of A and B after impact.

8. There are three equal elastic balls A, B, C; A is let fall from a given point; at the moment that it reaches the horizontal plane, B is let fall from the same point; and at the moment that A in returning meets B, C is let fall. Show that B will meet C for the second time where it first met A.

9. An imperfectly elastic ball A falls down a circular arc through a height h, and at the lowest point impinges upon another imperfectly elastic ball B. Show that if A be reduced to rest by impact it will drive B up a height e^2h, and that when B returns it will drive A up a height e^2h, following it through a height $e^2(1-e)^2h$.

10. A hollow sphere is half filled with mercury, and then filled up with water; compare the pressures on the upper and lower hemispheres.

11. Find the depth of the centre of pressure of a parallelogram immersed in a fluid, one side of the area coinciding with the surface of the fluid.

12. A given cylinder floats vertically with $\frac{1}{m}$ of its axis

mersed in one fluid, and $\frac{1}{n}$ of its axis immersed if placed another; supposing p parts of the first fluid mixed with parts of the second, what portion of the cylinder will be mersed when it floats in the mixture?

XIII.

1. Show how to find the centre of gravity of any number bodies in the same plane.
2. A body cannot be in stable equilibrium upon a horintal plane if it rests on less than three points of support.
3. Two weights, connected by a string passing over the mmon vertex of two inclined planes, balance each other; they are set in motion, show that their centre of gravity ll move in a horizontal straight line.
4. A right cone whose weight is w, is suspended by two rtical cords attached to the extremities of a side which is rizontal; determine the tensions of the cords.
5. Weights of 1, 3, 5, 7 ounces are placed in order in the gles of a square; find the distance of their centre of avity from that of the square, which is without weight d has a side of 10 inches.
6. Find the resultant of four parallel forces of 1, 2, 3, 4 lbs. ting at the angles of a square.
7. A body moves under the action of a uniform force; ow that the spaces described from the commencement of otion are as the squares of the times.
8. A body describes 300 feet in the fifth second from the ginning of motion; if it be acted on by a uniformly celerating force, how many feet will it describe in the ghth second?
9. In what time will a shell range 3000 feet at an elevaon of 30°?
10. If R and R_1 be the ranges of a shot fired at the same rizontal angle and with the same velocity up and down inclined plane, whose angle of inclination is θ, and r the

range on a horizontal plane with the same angle of projection and the same velocity, prove $(R+R_1)\cos\theta=2r$.

11. What assumptions are made in the theory of projectiles which are not true in practice?

12. A pontoon is in the form of a cylinder with hemispherical ends, its extreme length is 22 feet, its diameter $2\frac{2}{3}$ feet, its weight with superstructure 1550 lbs.; what additional weight will be required just to immerse the pontoon in water of which 1 cubic foot weighs 62·5 lbs.?

XIV.

1. Find the ratio of the power to the weight in the screw, with friction.

2. A uniform ladder is placed between a rough horizontal plane and a rough vertical wall, at an angle of 45°, the coefficient between the wall and the ground being $\frac{5}{8}$; a man whose weight is half that of the ladder ascends: find what the coefficient of friction must be between the ladder and the wall, that when the man reaches the top of the ladder it may just begin to slip.

3. Find the least number of turns that may be given to a thread upon a cylinder 4 ft. long and 1 ft. in circumference, in order that the weight w may be supported by friction alone, the coefficient of friction being ·5.

4. A line is drawn cutting off $\frac{1}{4}$ of a square; find the distance of the centre of gravity of the remainder from the centre of gravity of the square, (1) when the line is parallel to a side of the square, (2) when it is parallel to a diagonal.

5. Four bodies, weighing 3, 4, 5, 7 lbs. respectively, are placed with their centres of gravity in a straight line at the respective distances of 2, 4, 6, 8 feet from a fixed point in that line. Find the position of the centre of gravity of the bodies.

6. A stone is let fall from the top of a tower at the same instant that another is projected vertically upwards from the

bottom of the tower; find the velocity of projection if the stones meet half way, the height of the tower being h.

7. Two bodies are projected from the top of a tower with the same velocity, one horizontally, the other at an elevation of 45°; they reach the ground t'' apart: find the height of the tower.

8. How is a cycloid described? Show that if a body oscillate in a cycloid, the time of a vibration is independent of the length of the arc of vibration.

9. A seconds' pendulum is carried to the top of a mountain, and there loses 8″ a day. Determine approximately the height of the mountain.

10. A hollow cone resting on its base is filled with fluid; what is the direction of the pressure at any point of its surface? Find the total pressure on the base; height of cone 12 in., radius of base 3·25 in., $s.g.$ of fluid=2·56.

11. State and investigate the conditions of equilibrium of a floating body.

12. A solid cylinder is of a substance whose density is $\frac{2}{5}$ of that of the fluid in which it floats; how much of the cylinder is immersed when it floats (1) with its axis perpendicular to the surface of the fluid, (2) when its axis is parallel to the surface?

XV.

1. Show that the coefficient of friction between two substances is the tangent of the angle of inclination of a plane formed of one of the substances, when a body formed of the other is about to slide down it.

2. Two bodies resting on two given inclined planes with a common vertex are connected by a cord parallel to the planes: find the relation between the weights of the bodies, and that between the pressures on the planes.

3. A circular arc of 150°, supposed without weight, rests on a horizontal plane with its concavity upwards, and with

weights of 8 lbs. and 12 lbs. suspended from its extremities. Find the position of equilibrium.

4. In the funicular polygon, find the tensions of the several strings, and the ratio between any two suspended weights.

5. A power P sustains a weight W on a system of weightless pulleys in two blocks, each block having 3 sheaves, and the string as it passes from one block to the other being vertical. Find the ratio of P to W.

6. Prove that the principle of virtual velocities is true in the system in *Qu.* 5. Show that this principle here expresses the fact that when a weight is raised by this system of pulleys the useful work yielded is the same as the work applied by the moving power. Is this generally the case with machines in practice? Why?

7. Distinguish between '*moving force*' and '*accelerating force.*' If a body be projected with a velocity u in the direction of the force of gravity (g), and if v be the velocity acquired, s the space described in the time t, show that $t=\frac{v-u}{g}=\frac{2s}{v+u}$.

8. Two bodies are projected, at the same instant, in the direction of gravity, the velocity of one being 5 times that of the other, after 5·5″ their velocities are respectively 277·1 and 677·1 ft. a second; find the initial velocity of each and the force of gravity.

9. A body is projected up an inclined plane whose inclination is 45°, it reaches the top of the plane with $\frac{1}{3}$ of its original velocity, where will it strike the horizontal plane passing through the foot of the inclined plane.

10. Find the radius of gyration of a circular plane 10 ft. in radius round an axis through a point in its circumference perpendicular to its plane.

11. When a body is immersed in a fluid, show that the weight lost by the body is to its whole weight as the *s. g.* of the fluid is to the *s. g.* of the solid.

12. The *s. g.* of gold is 19·25; a vessel filled with water

alone weighs 8·5 oz.; when a nugget of gold is put into the vessel and it is filled up with water, it weighs 44·75 oz.: determine the weight of the nugget.

XVI.

1. Investigate the conditions of equilibrium of a wedge.

2. A B C D is a quadrilateral, and is acted upon by forces represented in magnitude and direction by A B, A D, C B, C D: find their resultant.

3. A weight P when suspended successively at each end of a uniform rod, one point of which is fixed, is balanced by weights Q, R respectively: find the position of the fixed point, and the weight of the rod.

4. A weight just rests on a given inclined plane, and it is found that a force equal to half the weight, if applied in the most favourable direction, is just sufficient to move the weight up the inclined plane; determine the coefficient of friction.

5. A heavy body rests with its spherical end upon a convex sphere, show how to determine whether the equilibrium is *stable* or not, sliding being prevented.

6. If the resting body be a hemisphere, determine the extreme value of its radius within which equilibrium will be possible, the radius of the fixed sphere being 20 ft.

7. If a heavy particle is allowed to fall from the edge down the interior of a smooth hemispherical bowl, prove that its pressure on the surface at the lowest point is 3 times the weight of the particle.

8. If the moving force upon a body varies directly as its distance at any time from a given point towards which it falls, the whole time of the body's falling to that point will be the same whatever may be the distance from which it falls. Apply this principle to prove the principle of the cycloidal pendulum.

9. A railway train running at the rate of 40 miles an hr.

is detached from the engine at the foot of an incline of 1 in 50: how far, and how long would it move up the plane, friction being neglected?

10. Explain the formula $t=2\pi\sqrt{\frac{h}{g}}$ applicable to the motion of a conical pendulum; and apply it to determine the number of revolutions per minute which will cause the suspending bars, each 1·5 ft. long, of the governor of a steam-engine to stick out at an angle of 30° with the vertical.

11. A body projected from a point in the circumference of a circle returns to the point it started from, after two reflections from the circumference, supposing the coefficient of elasticity to be ·9, find the angle between the line along which the body starts, and that in which it returns to the starting-point.

12. A rectangular sluice-gate 12 ft. square is placed vertically in water, and the pressure on the half of it cut off by a horizontal line is $\frac{1}{15}$ greater than on the half bounded by a diagonal: how high has the water risen above the top of the gate?

XVII.

1. State the principle of virtual velocities, and apply it to determine the relation between the power and the weight, in '*the wheel and axle.*'

2. A body weighing 2 cwt. is supported on an inclined plane, making an angle of 70° with the vertical, by means of a force parallel to the base of the plane. Find the force and the pressure on the plane.

3. Supposing that $5\frac{1}{2}$ turns cause the head of a screw to advance $\frac{2}{3}$ of an inch in a straight direction, what power applied at the extremity of an arm 18 inches long will be required to produce a pressure of 1000 lbs. upon the head of the screw?

4. Define the term '*centre of gravity*;' and show how to

find practically the centre of gravity of an irregularly-cut piece of cardboard, proving the proposition on which your method depends.

5. Find the centre of gravity of a triangle.

6. If a piece of uniform wire, bent into the form of a triangle A B C, be suspended freely from the angular point A, prove that it will only rest with the side B C horizontal when the angle A B C is equal to the angle A C B.

7. If a body be projected with a given velocity down a smooth inclined plane, find the space it will describe in a given time.

8. With what velocity must a body be projected down an inclined plane of given length whose inclination is 30°, so that the time down the plane may be equal to the time of a body's falling freely through the vertical height?

9. A given body would just rest on an inclined plane when inclined at an angle of 30° to the horizon: find the velocity which the body would acquire in sliding down the plane for 2″, when it is inclined at an angle of 60° to the horizon. If the body be projected up the plane with a velocity of 100 ft. per second, how far will it rise?

10. A train of 75 tons ascends an incline of 1 in 800, with the uniform speed of 40 miles an hour: what is the horse-power due to friction and gravity, assuming the friction to be 6 lbs. a ton?

11. Show how to compare the specific gravity of a solid body with that of a fluid of greater density.

12. The apparent weight of a piece of marble when weighed in water is 9 oz., when a piece of elm weighing 6 oz. is tied to the marble, the apparent weight of the two is 5 oz.: find the specific gravity of the elm, that of the water being unity.

XVIII.

1. If four couples be represented by the four faces of a triangular pyramid, prove that they will be in equilibrium.

2. The upper hinge of a gate sustains only a horizontal strain, the whole weight being borne by the lower hinge. Show how the direction and magnitude of the thrust on the lower hinge may be determined: and find it in the case of a rectangular gate whose weight is 56 lbs., whose centre of gravity is in the intersection of its diagonals, whose hinges are at the angles, and whose length is 8 ft., height 4 ft.

3. Four forces P, Q, R, S, act along the sides of a rectangle and are proportional to those sides; find the direction of their resultant.

4. Describe the common steelyard, and show how to graduate it.

5. A B C D is a square. A force of 3 lbs. acts from A to B, a force of 4 lbs. from B to C, and a force of 5 lbs. from C to D; find the single force which will preserve equilibrium.

6. If the square in *Qu.* 5 have its centre fixed, what force acting along A D will keep it in equilibrium?

7. Explain how variable velocity is measured. Prove the formula $v = V + ft$.

8. A particle is projected vertically upwards with a velocity of 100 ft. per second, how long will it continue to rise?

9. Explain the principle of '*composition of velocities*;' and describe in general terms how the motion of a body may be determined.

10. A and B are perfectly elastic balls, which move in opposite directions, and impinge directly: the mass of A is double that of B, and B's velocity is treble that of A; determine the motion of each after impact.

11. Find the weight of a hydrometer which sinks as deep in a fluid of specific gravity ·86 as it does in water when loaded with 60 grains.

12. A ring is composed of gold, and has a diamond set in it: the whole weighs 69·5 grains in air, and 64·5 grains in water. The specific gravity of the gold is 19, and that of the diamond $3\frac{1}{2}$. Find the weight of the diamond.

XIX.

1. Determine the conditions of equilibrium in the case of the lever, the forces acting at any angles. Find the magnitude and direction of the pressure on the fulcrum.

2. A and B are the extremities of a uniform straight lever 4 ft. long, B C, D A are each taken equal to 1 ft., from B a weight of 25 ounces is suspended, and from D a weight of 30 oz., the same weight P is suspended both from A and C; find P so that the fulcrum may be at the same point in case of equilibrium, whether the lever has weight or not.

3. A uniform beam rests with its extremities upon two given smooth inclined planes, determine its inclination to the horizon.

4. Two beams, A C, B C, whose weights are w_1, w_2, are connected together at a given angle α, and turn about a horizontal axis at their point of junction: find the position of equilibrium which they will take by the action of their weight.

5. Describe and graduate the Danish steelyard.

6. If a beam A B is supported in a horizontal position at A and B, and the weight be uniformly distributed along A B, find the moment of the strain at any point C of the beam.

About what point will this moment be greatest? determine its magnitude in this case.

7. If two elastic spheres impinge in direct impact with given velocities, show how to find the velocity of each sphere after impact, and explain what assumptions are made in the investigation.

8. Two balls are projected at the same instant from two points in the same horizontal plane, and in opposite directions, so as to describe the same parabola; if the mass of one ball be double that of the other, and if the modulus of elasticity be ·5, show that after impact one ball will descend vertically, and the other return in its original parabola.

9. Explain what is meant by '*centrifugal force*.' A stone weighing 1 lb. is attached to the end of a string 3 ft. long, of which the other end is fixed; the stone describes a horizontal circle revolving 3 times a second: find the inclination of the string to the vertical, and its tension.

10. A seconds' pendulum is lengthened ·01 inch: find how many seconds it will lose in 24 hrs.

11. Define a fluid. Show that the pressure round an internal point in a fluid is the same in all directions.

12. A cylinder partially filled with a fluid revolves round its axis with a uniform velocity, show that the form which the surface of the fluid assumes is a paraboloid.

XX.

1. Investigate the relation of the power to the weight in the case of a straight horizontal lever, taking into account the friction at the cylindrical axis upon which it works.

2. Find the power necessary to sustain 6 cwts. in a system of pulleys 3 in number, each weighing $2\frac{1}{2}$ lbs.; each string being attached to the weight.

3. The sides of a uniform heavy triangle are $3a$, $4a$, $5a$ respectively: find the centre of gravity of the remainder after the inscribed circle is removed.

4. A heavy triangle A B C is suspended successively from the angles A and B, and the two positions of any side are at right-angles to each other; show that $5c^2 = a^2 + b^2$.

5. Find the position of the centre of gravity of 4 heavy particles P, Q, R, S, situated in one plane, with reference to two straight lines, in the same plane, at right-angles to one another.

6. P and 2P, weights, are placed in two adjacent angles of a square, find the weights that must be placed in the other angles so that the common centre of gravity of the 4 weights may be at the centre of the square.

7. State the '*third law of motion*.'

8. A given weight P descends drawing up a weight Q, by means of a string over a fixed smooth pulley; find the accelerating force and the tension of the string. Find the relation of P to Q, so that P may descend from each rest 16·1 feet in 2 seconds.

9. If a body start from the bottom of a given inclined plane with a velocity v, find the space through which it will ascend (1) when the plane is perfectly smooth; (2) when the plane is rough, stating how the friction in the formula obtained is to be estimated.

10. The path of a projectile *in vacuo* is a parabola; its velocity at any point is that due to the height of the direction above the point: prove the latter statement, assuming the former. Find the focus of the parabola when the elevation is α, and the velocity of projection 64·4 ft. a second.

11. Define the '*centre of pressure*,' and find it for any rectangular area one side of which is horizontal.

12. If a square whose side is 20 inches be immersed vertically in a liquid with one side horizontal, find the depth of the centre of the square when the distance between it and the centre of pressure is $1\frac{2}{3}$ inches.

XXI.

1. Show that a single force acting on a point may be replaced by two forces at right-angles to each other, one of which may have any direction with respect to the single force. The single force is 100 lbs., one of the two makes an angle of 60° with it: find the two equivalent forces

2. If 4 forces acting upon a point be represented in magnitude and direction by 4 of the sides of a regular pentagon taken in order, show from the algebraical expression for determining their resultant, that it will be represented by the fifth side.

3. When a door is supported by two hinges, show how to

find the horizontal pressures at those hinges, and prove that the vertical pressures are indeterminate. If the door weigh 100 lbs. and be 4 ft. wide, find the horizontal pressure when the distance between the hinges is 6 ft.

4. A number of penny pieces are cemented together so that each just laps over the one below it by the nth part of its diameter: find how many may be thus piled without falling, when the lowest stands on a horizontal plane.

5. From a circular disc, of radius r, a circular disc of radius r_1 is removed, the distance between the centres being d: determine the centre of gravity of the remainder.

6. Show that the ratio of the relative velocities of two balls before and after impact depends only on the modulus of elasticity.

7. A particle is projected from a point in a horizontal plane, and after describing a parabolic arc impinges again upon the plane which is imperfectly elastic; compare the height (h_1) and the range (r_1) of the second parabolic arc with h and r those of the first; and determine the whole space described before the particle comes to rest.

8. In the conical pendulum determine the inclination of the string to the vertical when its length is 10 ft. and it makes 30 revolutions in one minute.

9. Show how to find the velocity of a cannon ball by observing the angle of recoil of a ballistic pendulum.

10. A body being wholly immersed in a fluid, find the moving force tending to cause it to ascend or descend.

11. A cylinder rests in a fluid A, with $\frac{1}{3}$ of its axis immersed, in another B with $\frac{3}{5}$ of its axis immersed; how deep will it sink when floating in a fluid which is a mixture of equal quantities of A and B?

12. A block of ice, the volume of which is a cubic yard, is observed to float with $\frac{2}{25}$ of its volume above the surface, and a small piece of granite is seen imbedded in the ice; find the volume of the stone, the *s. g.* of ice being ·918 and that of granite 2·65.

XXII.

1. Find the relation between the power and the weight on the screw.

2. Four forces act along four edges A C, A D, B C, B D of a triangular pyramid A B C D, and are proportional to these lines; find their resultant.

3. What is the height from the ground of the centre of gravity of a pyramid formed of 4 8-inch shells touching each other?

4. The third part of a circular hoop, at liberty to roll on a horizontal plane, is kept at rest by two weights of 20 lbs. and 30 lbs. respectively, hanging freely from its extremities; determine the angle subtended at the centre between the point of contact with the plane and either extremity, neglecting the weight of the hoop.

5. Find the ratio of the power to the weight in a system of pulleys in which each pulley hangs from a fixed point by a separate string, and all the strings are parallel, taking the weights of the pulleys into account. If P be the power, W the weight, w the weight of each pulley, n the number of moveable pulleys, show that

$$P - w = \frac{1}{2^n}(W - w).$$

6. When a balloon is half a mile high, and rising at the rate of 16·1 ft. per second, a heavy stone is released from the hand of the aeronaut; when will the stone begin to fall towards the ground, and in what time will it reach it?

7. From the top of a tower 170 ft. high a shot is fired vertically with a velocity of 1000 ft. per second: find the height to which it will rise, and the velocity with which it will reach the ground.

8. Projectiles are discharged *in vacuo* from a given point with a constant elevation, what is the locus of the vertices of the parabolas which they describe?

9. A shot is fired with a given velocity from the point A

up an inclined plane, so as to have a maximum range, and strikes the plane at a point A′: with what velocity and elevation must it be fired back from A′ so as to return to A by the same course?

10. A vessel of given weight, moving with a velocity v, is brought to rest by the tension of a rope of length l: supposing the modulus of elasticity of the rope to be e, find its elongation when the vessel stops.

11. Describe the siphon, and the manner in which it acts.

12. Show that a homogeneous rectangular parallelopiped with a square base, immersed with its base horizontal in any fluid of greater *s. g.* than itself, will float in a position of stable equilibrium therein, if the square of its height $< \frac{2}{3}$ the area of its base.

XXIII.

1. Prove '*the parallelopiped of forces.*'

2. Apply the principle of virtual velocities to determine the relation between the power and the weight in a system of 4 moveable pulleys, with parallel strings, all attached to the weight.

3. A two-wheeled carriage, weighing W, is just capable of being set in motion on a horizontal plane by a force H. The radii of the wheel and axle being r and ρ respectively. Find the coefficient of friction between them.

4. Determine generally the conditions of the stability of a pier of masonry sustaining on its summit a given pressure acting in a given direction.

5. Find the centre of gravity of three uniform beams forming a right-angled isosceles triangle, the greatest side of the triangle being 20 feet.

6. Three weights 4, 6, 8 lbs. are suspended on a straight horizontal lever without weight, and at equal distances of 12 inches from each other, where must the fulcrum be placed for equilibrium?

7. Define the '*unit of work*;' and show that when a

pressure of m lbs. is exerted through n ft., the work done is represented by $m \times n$.

8. Find the horse-power of a fall of 1000 cubic feet of water per minute, to a depth of 20 feet.

9. A train of 200 tons weight is travelling at the rate of 20 miles an hr. How far would it run on a horizontal line before it came to rest, if the steam were thrown off, supposing the friction of the rails to be 8 lbs. a ton?

10. A body which weighs 18 lbs. is put in motion on a horizontal plane so as to gain 5 ft. of additional velocity every second. Supposing the friction of the plane to be 2 lbs.; what must be the amount of pressure acting parallel to the plane?

11. A cubical vessel is filled with fluid, compare the pressures on its sides with that on its base.

12. When the cube in *Qu.* 11 has one of its diagonals vertical, find the pressure on one of its lower faces: edge of cube, a in., *s. g.* of fluid 1·75.

XXIV.

1. Determine the relation between the power and the weight on the inclined plane (1) when the plane is smooth; (2) when it is rough.

2. What force is necessary to support a weight of 50 lbs., on a plane inclined at an angle of 15° to the horizon, the force acting horizontally?

3. A heavy body is drawn along a horizontal plane subject to friction by a force inclined at an angle to the plane. Determine generally the magnitude of the force, and show at what angle it is least.

4. Find (1) what amount of force is requisite to sustain, and (2) what amount of force is just insufficient to push a weight of 250 lbs. up a plane inclined at an angle of 45° to the horizon, the direction of the force being supposed to make

an angle of 30° with the plane, and the coefficient of friction being $\frac{1}{8}$.

5. If a force, acting at an angle of 45° to the horizon, can just support a weight on a smooth plane, inclined at an angle of 15° to the horizon, what is the greatest angle of inclination of a rough plane to the horizon on which the same force acting in the same direction can just support the same weight, the coefficient of friction being $2-\sqrt{3}$.

6. Assuming 32·2 to represent the constant force of gravity upon the foot and second scale, what number will represent the same force upon the mile and minute scale?

7. A heavy body is observed to be 2·25″ in falling down a well; what would be the depth of the well, neglecting the resistance of the air?

8. Prove that 45° is the angle of projection which gives the maximum horizontal range for a projectile *in vacuo*. Show also that the diminution of the range when this angle is increased or diminished by the same quantity, is the same.

9. At what inclination to the horizon must a wooden way be laid in order that a block of iron may just slide down by its own weight, the coefficient of friction being ·625?

10. Find the ratio between the radii of a wheel and axle which shall cause a heavy weight to draw up another, $\frac{1}{3}$ of itself in magnitude, through a given space in the shortest time: neglecting inertia and friction.

11. Two equal weights of 50 lbs. are suspended over a fixed pulley, what weight must be taken from one and added to the other to make it descend through 20 ft. in 2·5″?

12. Investigate the conditions which must be satisfied when a body floats in an incompressible fluid.

XXV.

1. Define '*accumulated work*;' and investigate an expression for the work accumulated in a body (w) moving with a velocity v.

2. Find the work accumulated in a ball weighing w lbs. and describing n revolutions a minute, in a circle of r ft. radius.

3. Explain generally the conditions on which the stability of an arch depends.

4. A gun is made to move in the circumference of a circle traced upon the deck of a ship, show that the common centre of gravity of the ship and gun will thereby be made to describe a circle.

5. Describe the '*Common Balance.*' What is the '*sensibility*' of a balance? Show how it may be increased.

6. A body is projected vertically upwards with a velocity of 80 ft. a second, to what height will it ascend?

7. A body weighing 6 cwts. moves from rest, under a constant pressure, upon a perfectly smooth horizontal surface, and acquires a velocity of 8 ft. per second at the end of 1 minute. Find the pressure.

8. A body is projected obliquely on a smooth inclined plane; determine its path.

9. A weight of 20 lbs. raises another of 112 lbs. by means of a wheel and axle, whose radii are $2\frac{1}{2}$ ft. and 4 in. respectively. Find the space through which the lifting weight descends in 10″: neglecting friction and the inertia of the machine.

10. Define the term '*specific gravity.*' The volumes and specific gravities of two liquids being given, determine the specific gravity of the fluid resulting from their admixture.

11. In a cylinder $\frac{3}{4}$ filled with water, a hydrometer, whose weight w is given, is observed to sink to a certain depth. If the cylinder be now filled with a fluid of 3 times the *s. g.* of water, find the weight with which the hydrometer must be loaded to cause it to sink to the same depth as before.

12. If an air-pump have a single exhausting barrel whose content is ·01 of that of the receiver, compare the pressure of the air in the receiver, after 10 double strokes of the piston, with the pressure of the atmosphere.

XXVI.

1. Find the centre of gravity of a uniform rod bent so as to form a quadrilateral figure.

2. Find the centre of gravity of the surface of a right cone.

3. The lengths of the arms of a balance being unequal, how may it be used so as accurately to ascertain the weights of articles?

4. Divide the base A B of a triangle A B C into segments A D, D B, having to one another the ratio $m : n$; and join C D: prove that if forces be represented in magnitude and direction by m . C A and n . C B their resultant will be represented by $(m+n)$ C D.

5. On a plane whose inclination is α, P, acting in a direction making an angle ϵ with the plane, is just on the point of moving W up the plane; if ϕ be the limiting angle of resistance, prove that $\frac{P}{W} = \frac{\sin(\alpha+\phi)}{\cos(\epsilon-\phi)}$.

6. The measure of the force of gravity being 32·2 when a second is the unit of time, what will be its measure when ten seconds is the unit of time?

7. Two particles are simultaneously projected with velocities u, v at angles of inclination to the vertical α and β: determine the time which will elapse before they are moving in parallel lines.

8. What is meant by the proposition that the centres of oscillation and suspension are reciprocal? Prove that this proposition is true for a system of material particles.

9. Find the centre and time of oscillation of 3 weights 4, 5, 6, placed at distances of 4 in. from each other on a rod 12 in. long without weight, the weight 6 being at the lower end of the rod.

10. Show how to determine the *s. g.* of a substance by means of Nicholson's hydrometer.

11. A substance when put in the upper cup of the

hydrometer requires 110 grs. to be placed with it, and when put in the lower cup, 135 grs. must be put in the upper cup, to sink the instrument to the same point in the stem to which 560 grs. in the upper cup alone would sink it; find the *s. g.* of the substance.

12. By what experiments can it be proved that air has weight? Assuming that 100 cubic inches of air weigh 31 grains, and that a cubic foot of water weighs 1000 oz.; compare the *s. g.* of air with the *s. g.* of water.

XXVII.

1. If any number of forces not in equilibrium act upon a point in a plane, show that they will have a single resultant, and find it.

2. Triangles are inscribed in a circle having equal chords for their bases and a common vertex: show that the locus of the centre of gravity of the triangles is a circle.

3. A uniform beam is placed between two rough inclined planes, the inclinations of which are complementary to each other, find either of the limiting positions of equilibrium: if θ, θ_1, be the greatest and least values of the inclination of the beam to the horizon for the limiting positions of equilibrium, and if the coefficient of friction be tan ϕ, prove

$$\phi = \tfrac{1}{4}(\theta - \theta_1).$$

4. Find the equation of '*the common catenary.*'

5. A rigid body stands upon a horizontal plane upon three supports, how do you determine whether it will stand or fall? If it stands, estimate the pressure upon each point of support.

6. Explain '*the second law of motion.*'

7. A particle is projected with a velocity of 161 feet per second in a direction making an angle of 30° with the horizon: find its velocity and the direction of its motion when it has reached a height of 64·4 ft Find also the horizontal space described in this time.

8. Find the range of a projectile *in vacuo*.

Ex. Gravity=32·2 ft.; velocity of projection= 3220 ft. per second; sine of angle of elevation=·6.

9. Particles are projected from the same point simultaneously with different velocities and in different directions, but all in the same plane: find the path of the common centre of gravity of the particles.

10. If the atmosphere be divided into thin strata of equal thickness, show that their densities decrease in geometrical progression.

11. Show how to determine the difference of the heights of two stations by means of a barometer and a thermometer.

12. A tube closed at one end and filled with air has its open end stopped by a piston capable of sliding into it freely. To what depth must the tube be immersed in a horizontal position in water that the piston may be driven in through $\frac{1}{3}$ of the tube's length, the pressure of the atmosphere being equal to that of a column of water 34 feet high?

XXVIII.

1. Explain how a plane area is said to have a centre of gravity. Find the centre of gravity of two material plane isosceles triangles on the same base, their vertices being on opposite sides of the base at distances h_1, h_2 from it.

2. Distinguish between stable and unstable equilibrium, give an example of neutral equilibrium.

3. A homogeneous elliptical spheroid rests with its smaller end on a concave hemisphere, what must be the radius of the hemisphere that the equilibrium may be stable?

4. A force acting parallel to a rough inclined plane just supports a weight upon it, find the inclination of a smooth plane, such that the same force acting along it shall support the same weight.

5. Determine the amount and the direction of the least

orce which will draw a body weighing W lbs. up a rough lane inclined at $\alpha°$ to the horizon.

6. A smooth rod slides between two fixed rings B, C, so as o be constrained to move in a vertical line. Its lower end rests on a fixed plane inclined at 45° to the horizon. If he rod weighs 4 lbs., and if AC=2 AB, find the pressure on ach ring.

7. Calculate the length of a pendulum beating seconds in London, assuming $g=32{\cdot}19$.

8. A heavy body suspended by a flexible cord revolves in vertical circle: under what circumstances may the strain pon the cord vanish?

9. Determine the motion of a '*conical pendulum*.'

10. Find the velocity with which a cannon ball must mpinge upon a bank of earth that, sustaining a given uniorm resistance, it may enter it a given depth.

11. Describe and explain the common '*suction pump*:' and nvestigate a formula for determining the height to which he water is raised by the first stroke of the piston.

12. A hollow spherical iron shell, of 6 in. radius, floats n water so as to be exactly half immersed: what must be ts thickness, the *s. g.* of iron being 7·78?

XXIX.

1. Two forces P and Q have a resultant R which makes an ngle α with P: if P be increased by R, whilst Q remains nchanged, show that the new resultant makes an angle $\frac{1}{2}\alpha$ with P.

2. A rod without weight is 1 ft. long: at one end a force f 2 lbs. acts, at the other a force of 4 lbs., and at the middle force of 6 lbs.; these forces are all parallel. Find the nagnitude and direction of a single force which will keep he rod at rest.

3. A plane is inclined to the horizon at an angle of 30°: ind the magnitude and direction of the least force which

will prevent a body weighing 100 lbs. from sliding down the plane, the coefficient of friction being ·25.

4. Apply the principle of virtual velocities to find the relation of the power to the weight in the case of a bent lever.

5. A ladder, 10 ft. long, rests with one end against a smooth vertical wall, and the other on the ground, the coefficient of friction being ·5: find how high a man, whose weight is 3 times that of the ladder, may ascend before the ladder begins to slip, the foot of the ladder being 6 ft. from the wall.

6. Prove that a body on a horizontal table will stand or fall as the vertical line through its centre of gravity falls within or without its base. Is the statement true if the plane be rough?

Two books, similar in every respect, each 10 inches long, lie one exactly upon the other on a table, over the edge of which they project 3 in. How much farther may the upper book be pushed out before they fall over?

7. P hanging vertically draws Q along a smooth horizontal table, find the accelerating force, and the space described in t seconds.

Ex. P=2 oz., Q=8 oz., the given time=2·5″.

8. Find the range of a projectile on an inclined plane passing through the point of projection; and find what the direction of projection must be that with a given velocity this range may be the greatest.

Ex. Velocity of projection 1610 ft. per second, angle of projection 45°, inclination of plane 30°.

9. A body is projected vertically upwards with a velocity of 966 ft. a second; find the time in which it will attain a height of 2753·1 ft. from the point of projection; and explain the double result.

10. In what time will the velocity of the body in *Qu.* 9 be reduced one half?

11. Show that the common surface of two fluids which do not mix is a horizontal plane.

12. The volume of the receiver of an air-pump is 20 times that of the barrel, and a piece of bladder is placed over a hole in the top of it. The bladder can bear a pressure of 3 lbs. to the square inch. Determine the number of strokes that can be made without bursting the bladder.

XXX.

1. Explain what is meant by the following terms in the theory of roofs:—'*truss*,' '*tie-beam*,' '*king-post*,' '*queen-post*,' '*strut*.' In the simple isosceles truss-roof, find the tension of the tie-beam.

2. A weight of 1000 lbs. is placed upon a rough inclined plane: the inclination of the plane to the horizon is 45°, and the coefficient of friction $\frac{1}{\sqrt{3}}$; find the least force which will keep the body from sliding down the plane, and the least which will draw it up.

3. Ten cannon balls are piled on a horizontal plane, in the form of a triangular pyramid: find the height of their common centre of gravity.

4. A heavy string is stretched over a smooth plane curve, find the tension at any point, and the pressure on the curve.

5. A chain of given weight is suspended from two equal vertical posts of given height, and the angle which the chain makes with either post is measured: find the moment of the force which tends to overturn one of the posts.

6. Define the terms—'*pressure*,' '*velocity*,' '*accelerating force*,' '*moving force*,' '*impressed*' and '*effective forces*,' '*angular velocity*,' '*moment of inertia*.'

7. A stone is projected vertically upwards with a velocity of 1000 ft. a second, how far will it ascend?

8. A block of stone of given weight and dimensions

stands upon a railway truck which is suddenly stopped. It is prevented from *sliding* upon the truck, it is required to determine the speed of the train so that it may just be overturned: and if it be allowed to slide and the coefficient of friction be μ, what must be the velocity if it slides a feet?

9. A perfectly flexible rope, part of which rests on a table, is dragged off it by the weight of the other part, which hangs over the side; determine a general expression for the velocity when μ is the coefficient of friction.

10. Two rectangular planes, whose lengths are 4 in. and 6 in. respectively, and their breadths 3 in. and 4 in., are immersed horizontally 4 ft. and 2 ft. respectively beneath the surface of a fluid. Compare the pressures upon them.

11. A cubical box is filled half with mercury and half with water. Compare the pressure on the sides with that on the base, the specific gravity of mercury being 13·5 times that of water.

XXXI.

1. Explain the different kinds of levers, giving examples of each. Where must the fulcrum be placed in order that a man who can lift a weight of 120 lbs., may, with a heavy crowbar weighing 30 lbs. and 5 ft. long, just raise 5 cwt.?

2. When a body is placed on a horizontal plane, examine the conditions of its standing or falling.

3. A right cone and a hemisphere are cemented to the same circular base: find the height of the cone, so that if the hemispherical portion of the solid be placed in any position, on a smooth horizontal plane, it will remain at rest; distance of centre of gravity of hemisphere from that of sphere being $\frac{3}{8}$ of the radius.

4. What is the '*key-stone*' of an arch? what the '*intrados*' and the '*extrados*?' Describe the manner in which an arch ordinarily gives way: and explain the principles upon which the point of rupture may be approximately found.

5. If tan β represent the coefficient of friction between weight W and the inclined plane, α being the angle of lination of the plane, ϵ the angle which the force P makes h the plane, show that the limiting values of the ratio W for equilibrium are

$$\frac{\sin(\alpha+\beta)}{\cos(\epsilon-\beta)} \text{ and } \frac{\sin(\alpha-\beta)}{\cos(\epsilon+\beta)}.$$

3. Determine the space described in a given time by a ly which moves in a straight line, and whose motion is formly accelerated or retarded.

7. Find the time in which a body will descend by gravity vn a smooth inclined plane 200 ft. long, the height of the ne being 3 ft.

3. A train weighing 10 tons descends a plane of the above gth and height: find the velocity acquired, friction being lbs. a ton.

). A projectile shot with a velocity of 1600 ft. a second, o strike the top of an object 60 ft. high at a distance of) ft. Find its elevation.

.0. Each additional 100 tons weight depresses a vessel 1. in the water: how far can her magnitude be estimated m these data?

.1. If w_1, w_2, w_3 be the apparent weights of a body when ghed in three fluids whose densities are respectively ρ_2, ρ_3; show that

$$w_1(\rho_3-\rho_2)+w_2(\rho_1-\rho_3)+w_3(\rho_2-\rho_1)=0.$$

.2. Find the height to which the water will rise in a ing-bell when its lower extremity has sunk d feet below surface of the water.

XXXII.

. A horizontal lever is 12 in. long, a weight of 8 oz. igs from one extremity, where must the fulcrum be placed order that a force of $11\frac{3}{7}$ oz. acting at the other end at an le of 150°, may keep the lever in equilibrium?

2. A fourth of the area of a triangle is cut off by a straight line parallel to the base, find the centre of gravity of the remaining trapezoid.

3. Describe the '*compound wheel and axle*' known as the '*differential wheel and axle*:' and investigate the relation between the power and the weight in the case of equilibrium.

4. Define a '*couple*;' and prove that its effect is not altered by turning its arm through any angle in the plane of the forces.

5. If the algebraic sums of the moments of a system of forces, in one plane, about any two points in that plane, be separately zero, what inference can be drawn?

6. Three uniform bars jointed together form an isosceles triangle A B C, the angle C being a right-angle. The frame is hung in a vertical plane with the angle C downwards, and the side A B horizontal, by a string attached to the middle point of A B. Find the inclination to the horizon of the direction of the mutual action of the bars at the joint A or B.

7. Prove the formulæ $v^2=2fs$; $u^2=v^2\pm 2fs$.

8. Find the number of feet described by a falling body between the fourth and seventh seconds.

9. P and Q are two balls at rest on a horizontal table, find in what direction P must impinge on a vertical plane in order that after reflexion it may strike the ball Q.

10. Find the relation between the work necessary to drag a body up an inclined plane, subject to friction; and the work necessary to drag it along the base of the plane and to lift it vertically through its height.

11. Describe the '*Hydrostatic Press*.'

12. A ship sailing into fresh water is observed to sink through a small space, a, and then on being discharged of a weight, w, to rise through a small space, b. Find the weight of the ship, the density of salt water being $\frac{1}{40}$ greater than that of fresh, and the area of the plane of floatation between the limits of the observations being constant.

XXXIII.

1. Determine generally the relation between the power
l weight in the wheel and axle, taking into account the
ction of the axis: (1) when the power and the weight
; in parallel lines; (2) when they are inclined to one
other.

2. Supposing the tenacity of iron wire ·1 in. in diameter
be 41 tons per square inch; how many such wires must
put together to sustain a strain of 3 tons?

3. A cylindrical bar of iron, 2 in. in diameter and 40 ft.
ıg, is subjected to a strain of 12 tons. By how much will
be elongated supposing every additional ton of strain per
in. of the section of such bar to produce an elongation of
a ten-thousandth part?

4. Find the centre of gravity of a trapezoid.

5. If the mass of each one of a system of particles be
ltiplied by the square of its distance from a given point,
sum of the products is least when the given point is the
ıtre of gravity of the system.

6. If equal particles, the mass of each being $3m$, are
ced in the angles of a triangle, whose sides are a, b, c,
ow that the sum of the least products, taken as stated, will
$m(a^2+b^2+c^2)$.

7. Investigate an expression for the work done by the
am on the piston of an engine at every stroke: diameter
piston a in., length of stroke b in., mean pressure of steam
lbs. to the sq. in.

8. A man weighing 120 lbs., pulls himself up in a bucket
ighing 20 lbs. by means of a system of pulleys in which the
ight is to the power as 3 : 1; what force must he exert
raise himself from the ground, neglecting the weight and
ction of the cords and pulleys?

9. A gun being fired at an elevation of 45°, the shot
ıches the ground in 30″ at a distance of 5000 yds. Find

the velocity of projection, and the greatest height to which the shot rose.

10. A body falls without friction, under the action of gravity, down a plane curve, show that the velocity at any point will be the same as if the body had fallen freely through the same vertical distance.

11. Define the '*metu-centre*;' and show how its position may be determined.

12. How much cork of specific gravity ·25 does a man require, whose *s. g.* is $1\frac{1}{2}$ and weight 12 stone, to enable him to float in water? How deep will a sphere of elm-wood of two feet diameter sink in hydrochloric acid of *s. g.* 1·2, that of elm being ·7?

XXXIV.

1. Find the relation of the power to the weight in the system of pulleys where each pulley hangs by a separate string.

2. A weight of 21 cwts. 1 qr. 20 lbs. is to be lifted by means of a crane, of which the axle for receiving the rope is a foot in diameter, and carries a wheel of 50 teeth worked by a pinion of 6 teeth. What force must a man apply to the handle of the winch by which the pinion is worked, and of which the arm is 16 in. long, in order to raise the weight?

3. To determine the pressure which applied in a given direction to the inner edge of the top of a pier of uncemented stones, will be just sufficient to overturn it.

4. A uniform beam A B, whose weight is W, rests with its end A on a smooth horizontal plane A C, and with its end B on a smooth plane B C, inclined 60° to the horizon. Find the tension of a string C A=C B which keeps it from sliding.

5. A cube has its base attached to a horizontal table. What is the greatest portion of it that can be cut off by a plane through one of its upper edges that this portion may

not slide upon the lower one when the coefficient of friction is $\frac{1}{4}$?

6. What is a '*simple pendulum*?' Find the time of oscillation of a simple pendulum in a small circular arc.

7. If a seconds' pendulum loses 10 beats in 24 hrs. by being taken to the top of a mountain, find approximately the height of the mountain: gravity varying inversely as the squares of the distances from the earth's centre, and the radius of the earth being 4000 miles.

8. How is the '*mass*' of a body estimated? Show how to find the acceleration of the motion of a given body by a given pressure.

9. A weight of 8 oz. hanging vertically draws a weight of 12 oz. by means of a string along a smooth horizontal table: find the space described by either body in the third second of their motion, and determine the tension of the string.

10. Distinguish between liquids and gases, between elastic and non-elastic fluids. What is the characteristic property of fluids in respect to their transmission of pressure?

11. The mean section of a stream is 4 ft. by 2 ft., its mean velocity 40 ft. per minute, it has a fall of $17\frac{1}{2}$ ft.; it is required to raise water by means of a water-wheel whose modulus is $\cdot 7$; to what height will it raise 196 cubic ft. per minute?

XXXV.

1. Find the thrust at the ends of the rafters of a simple triangular roof without a king-post, in terms of their weight, their mutual inclination, and the horizontal span.

2. Two forces of 6 and 8 lbs. respectively, act at the ends of a rigid rod, without weight, 10 ft. long: the forces are inclined at angles of 60° and 30° to the rod: find the magnitude and position of the force which will keep the rod in equilibrium.

3. A B C D is a four-sided figure, and the diagonals A C, B D intersect in O; find the distance of the centre of gravity of the figure from O in terms of O A, O B, O C, O D and the angle between the diagonals.

4. On a uniform straight lever weighing 5 lbs., and 5 ft. in length, weights of 1, 2, 3, 4 lbs. are hung at the distances 1, 2, 3, 4 ft. respectively. Find the position of the fulcrum on which the whole will rest.

5. Four parallel forces P, $-3P$, $-5P$, and $6P$ act on a rigid rod at distances from one of its ends proportional to their respective magnitudes. Find the point in which the resultant meets the rod.

6. A weight of 1 oz. hanging over the edge of a smooth horizontal table, draws a weight of 1 lb. along the table by means of a string. Find the tension of the string, and the space described in 5″.

7. Assuming $g=32{\cdot}19$, find the length of a seconds' pendulum: and if a seconds' pendulum loses 10 beats in 24 hrs., find how much it must be shortened to keep correct time.

8. A shot is fired *in vacuo* at an elevation $\sin^{-1}\frac{3}{5}$, with a velocity of 6440 ft. per second: find its height after 5″ have elapsed, and the greatest height it attains.

9. Show that the time of a projectile's describing any arc of its path varies as the difference of the tangents of the angles which the tangent lines at the extremities of the arc make with the horizon.

10. If a heavy particle slide down the convex arc of a vertical circle from its highest point, show where it will quit the circle, and find the *latus-rectum* of the parabola which it afterwards describes.

11. How is the pressure at any point of a fluid estimated? In what direction does the fluid pressure at any point of the surface of a vessel take place? If the atmospheric pressure be 15 lbs. on the sq. in., and the steam in a boiler have an elastic force of 4 atmospheres, with what weight must a

circular safety-valve be loaded whose diameter is 3·5 inches, so as to prevent the escape of any steam?

12. Given that the surface of a sphere is four times the area of its greatest section, and that the centre of gravity of a hemispherical surface bisects the radius, find the pressure on a hemispherical bowl filled with water, the radius of the bowl being 12 inches. Weight of 1 cubic foot of water= 1000 oz.

XXXVI.

1. Two heavy bodies, P and Q, are situated at distances p and q from a plane, on the same side of it; find the distance of their centre of gravity from the plane.

2. The edge of a square is 2 ft., and coincides with the foot of a plane inclined at an angle of 15° to the horizon; how high may a parallelopiped be constructed upon the square, as a base, without toppling over?

3. Two beams 12 ft. and 16 ft. long respectively are connected by a tie-beam 20 ft. long, which rests upon a horizontal surface, and at their junction support a weight of 112 lbs.; find the pressure upon each prop and the strain on the tie-beam.

4. Seven bodies of equal weight are placed so that their centres of gravity coincide with as many angles of a cube, of which the diagonal measures a ft. Find the distance of their common centre of gravity from the unoccupied angle of the cube.

5. Three equal smooth balls of given weight lie in contact on a horizontal table, and a string passes round them in the plane of their centres. If a fourth equal ball be laid upon them so that the four centres are the angular points of a regular tetrahedron, find (t) the tension of the string.

6. State some cases in which *pressures* do no work. Find the work accumulated in a train weighing 80 tons and moving 60 miles an hour.

7. A right pyramid, with a square base of 16 ft. side, has an altitude of 24 ft., and stands on a horizontal plane, find the '*work*' necessary to overturn it round one of its edges, a cubic foot of its material weighing 100 lbs.

8. A train weighing 60 tons has a velocity of 40 miles an hour; if the steam be turned off, how far will it ascend an inclined plane rising 1 ft. in 100, supposing the friction to be 8 lbs. a ton?

9. A cylinder that weighs 100 lbs. turns upon its axis, which is horizontal; if motion be communicated by a weight of 10 lbs. attached to a perfectly flexible string without weight, which is coiled round the surface of the cylinder in a vertical plane, find the space through which the weight will descend in 10″.

10. Show how to determine the *s. g.* of a substance by weighing it in air and in water. A pound weight of gold, *s. g.* 17·5, is coined into $46\frac{2}{10}$ sovereigns; how many grains will a sovereign weigh in water?

11. A hollow cone, whose vertical angle is 2θ, is filled with water, and placed with its base downwards; determine the place where a small orifice must be made in its side so that the issuing fluid may just strike the bottom of the cone.

12. In the vertical side of a vessel is a rectangular orifice having one side horizontal. This aperture is closed by a rectangular board just fitting it, and capable of turning in one direction only round a horizontal axis in its own plane. Determine the position of this axis so that no fluid may escape from the vessel until it has risen a certain height above the aperture.

XXXVII.

1. If two parallel forces act on a straight rod, without weight, tending to twist it in opposite directions, find the

magnitude and point of application of the force which will keep it at rest.

2. If the parallel forces 7 and 5 act at the opposite ends of a rod 12 in. long, making an angle of 30° with the direction of the rod, where and how must a single force act for equilibrium? Find the perpendicular pressure on the rod.

3. The angle at the vertex of an isosceles triangle is 120°, from this angle and the middle points of the equal sides perpendiculars are drawn to the base: supposing the perimeter of the triangle and these perpendiculars to be uniform rods, find the centre of gravity of the framework thus formed.

4. Given the distances of the centres of gravity of any system of heavy bodies from a given plane, to find the distance of their common centre of gravity from the same plane.

5. A B is a rod, without weight, turning about its middle point C; A D is a heavy uniform rod jointed to the former at A, whilst its end D is attached to the point B, of that rod by a string D B, so that D A B is a right angle. Draw a figure exhibiting the position of equilibrium, and mark upon it the direction of the mutual action at the joint A.

6. A body weighing 6 lbs. slides from rest down a rough inclined plane through 20 ft. in 4 seconds. What force acting along the plane would sustain the body in equilibrium?

7. An elastic ball impinges obliquely on an immoveable plane; given the velocity and direction of motion before impact, find the motion after impact.

8. Show that if a perfectly elastic ball be let fall upon an inclined plane, and rebound striking the plane again, the interval between the collisions is independent of the inclination of the plane.

9. If a ball A impinge upon B at rest, and the modulus of elasticity be $\frac{2}{3}$, what must be the ratio of the masses in order that A may be reduced to a state of rest by the impact?

10. The elevation of a projectile being 15°, what must be the velocity of projection in order that the horizontal range may be a mile? And to what height will it rise in its flight?

11. The capacity of the barrel of an air-pump is half that of the receiver; if a barometer that stands at 30 in. be placed under the receiver, after how many strokes of the piston will the barometer fall below 3 inches?

12. Find the quantity of water discharged in a given time through an orifice made in the bottom of a vessel kept constantly full. If a circular orifice be 2 in. in diameter, the head of the water 4 ft., and the discharge $\frac{1}{8}$ of a cubic foot per second, will the discharge agree with the theoretical result? What is meant by the coefficient of velocity? Find it in the preceding example.

XXXVIII.

1. Define '*statical friction*;' and state the laws which apply to it.

2. A uniform beam rests with its ends upon two given inclined planes; find its inclination to the horizon when the planes are rough, the coefficients of friction being μ and μ_1.

3. Find the altitude of the largest cone upon a base of given diameter ($2r$) that will just stand upon a rough plane, inclined at an angle $\alpha°$ to the horizon. Find the highest point in its side to which a weight may be attached without upsetting the cone.

4. If a solid cone whose height is twice the diameter of its base be suspended from a point in the circumference of its base, prove that in the position of equilibrium its axis will be inclined at an angle of 45° to the horizon.

5. If an inch in length of a catenary weighs 2 oz., and the tension at the lowest point is 20 oz., find the tension at a point

of catenary whose vertical height above the lowest point is 6 inches.

6. A particle moves over 14, 22, and 34 ft., respectively, in the 1st, 3rd, and 6th seconds of its motion. Is it subject to the action of a uniform force?

7. A body falls from the top of a tower 330 ft. high. How many feet does it pass over in the last 3″?

8. Two weights of 25 and 6 lbs., respectively, are connected by a cord passing over a pulley. In what time will the former descend through 166 ft. from rest?

9. A ball is projected upwards with a velocity v, and when it has attained $\frac{3}{4}$ of its greatest altitude another ball is projected upwards from the same point with the same velocity; determine when and where they will meet.

10. Determine the velocity with which a particle must be projected so as to strike the top of an object 50 ft. high and 1000 yds. distant, the angle of projection being 45°.

11. At what rate would a locomotive engine working at 60-horse power draw a train up an inclined plane having a rise of 1 in 100, supposing the train and engine to weigh 60 tons, and the friction to be 6 lbs. a ton?

12. The *s. g.* of quartz is 2·62, that of gold 19·35; a nugget of quartz and gold weighs 11·5 oz. and its *s. g.* is 7·43; find the weight of gold in the nugget.

XXXIX.

1. Prove the property of Guldinus for determining the volume of a solid of revolution, and apply it to determine the volume of a ring whose section is an equilateral triangle.

2. A weight of 5 lbs. is suspended freely from a fixed point by a perfectly flexible string; find what horizontal force applied to the string will draw the upper portion of it 30° out of the perpendicular.

3. The ends of a cord whose length is $2a$ are fastened at

the points A and B whose distance is $2c$; a smooth ring running along the cord sustains a weight W: determine the position of equilibrium and the tension of the cord, A B being inclined at $\alpha°$ to the horizon.

4. A bar of iron of uniform thickness, 10 ft. long and weighing $1\frac{1}{2}$ cwt., is supported at its extremities in a horizontal position, and carries a weight of 4 cwt. suspended from a point distant 3 ft. from one extremity. Find the pressures on the supports.

5. Each side of a square A B C D is 18 inches, A C and B D the diagonals intersect in O, the triangle B O C being removed; find the centre of gravity of the remainder, and show that its distance from A D is 7 inches.

6. How are *moving* forces and *uniformly accelerating* forces respectively estimated in dynamics? Give an example of each kind of force, and show how the relation of these forces to each other is expressed.

7. If two unequal weights P and Q, P > Q connected by a string are suspended over a fixed pulley, find the space through which the heavier will descend in t seconds.

8. If two equal weights, each of 50 oz., are suspended over a fixed pulley, what weight must be added to one of them to make it descend 64·4 feet in 5 seconds?

9. If the string in *Qu.* 8 be cut after an interval of n'' from the commencement of motion, how far will the bodies be apart after a second interval of m''?

10. Describe the construction and action of the air-pump.

11. If the mercury in a barometer placed in the receiver of an air-pump stand at 30 in., and if after 12 barrels have been pumped out the mercury sinks 13 in., compare the capacities of the receiver and the barrel.

12. Show how to find the pressure on any plane area immersed in a liquid. If an isosceles triangle be immersed vertically in a liquid with its base horizontal and its vertex coincident with the surface of the liquid, and if the perpendicular from the vertex on the base be bisected by a line

parallel to the base, show that the pressure upon the upper portion of the triangle is to the pressure on the lower as 1 to 7.

XL.

1. Show how to determine the centre of gravity of the frustum of a pyramid, the ends being parallel.

2. A body P suspended from one end of a lever without weight is balanced by a weight of 1 lb. at the other end of the lever; when the fulcrum is removed through half the length of the lever, it requires 10 lbs. to balance P: determine the weight of P.

3. A heavy beam rests upon a peg with one end against a smooth vertical wall: find the position of equilibrium.

4. A uniform lever 4 ft. long weighs 10 lbs., and weights of 30 lbs. and 40 lbs. are appended to its extremities; where must the fulcrum be placed to produce equilibrium?

5. If three parallel forces P, Q, R, act at given distances from each other *perpendicularly* on a straight lever without weight, find where the fulcrum must be placed to maintain equilibrium; would it affect the position of the fulcrum if the word in italics were omitted?

Ex. P=5, Q=7, R=9; distance between P and Q= 3 feet, distance between Q and R=4 feet.

6. Explain why the force of gravity at the earth's surface is represented by 32·2 feet.

7. How is velocity estimated in dynamics when variable? Investigate the formula for descending bodies $s=vt+\frac{1}{2}gt^2$. What change must be made if the body is ascending?

8. A body is projected vertically upwards with a velocity of 644 ft. per second; find the time at which it will be at the height of 2817·5 ft. Explain the double result.

9. How is fluid pressure estimated at any point of the surface of a vessel with which the fluid is in contact?

10. Show how to find the pressure on the whole surface of any vessel of known form containing liquid.

11. A cylinder whose radius is 9 in., and height 24 in., is filled with water; find the pressure on the internal concave surface of the cylinder, the weight of a cubic foot of water being 1000 oz. avoirdupois.

12. How is it shown that the elastic force of the air at a given temperature varies inversely as the space it occupies?

XLI.

1. Find the relation of P to W in the system of 5 pulleys in which each string is attached to the weight. When P=100 lbs., find (1) W; (2) the strain on the fixed pulley, each pulley weighing 5 lbs.

2. The arms of a bent lever are at right-angles to one another, and one of them is twice as long as the other: a weight of 18 lbs. is suspended from the shorter arm, and 12 lbs. from the longer: find the angle which the latter makes with the horizon in the position of equilibrium.

3. An iron wire, $\frac{1}{8}$ of an inch in diameter and 12 ft. long, is securely attached to two fixed points 10·5 ft. apart; what weight suspended in the middle point of the wire will break it, assuming that it will bear a strain of 75000 lbs. to the sq. inch?

4. If a system of forces acting at a point O and keeping it at rest be represented in magnitude and direction by O L, O M, O N, &c., prove that O will be the centre of gravity of a system of particles of equal weights placed at L, M, N, &c.

5. Explain what is meant by '*friction*,' and by the '*limiting angle of resistance*.'

6. A heavy beam rests with one end on the ground, and the other end in contact with a vertical wall. Having given μ, μ_1, the coefficients of friction for the wall and the ground respectively, a and b the distances of the centre of gravity of the beam from its upper and lower ends: find its least possible inclination to the ground.

7. Find the tension of the string in the conical pendulum.

8. A weight of 2 lbs. revolves at the extremity of a string 2 ft. long, and makes two revolutions in a second: find the tension of the string in pounds.

9. Show how to find the centre of oscillation of any number of material particles.

Four particles, whose weights are as 1, 3, 7, 5, are situated in a straight line at distances, measured in the same direction, 1, 2, 3, 4 from a horizontal axis perpendicular to this line; find the centre of oscillation of the weights.

10. If the axis be shifted and made to pass through the centre of oscillation thus found, where will the new centre of oscillation be?

11. In the air-pump the capacity of the barrel is $\frac{1}{10}$ that of the receiver; show that, after 3 strokes of the piston, the air in the receiver will have lost $\frac{1}{2}$ of its density nearly.

12. Three observed readings of an imperfect barometer are h_1, h_2, h_3, are respectively a_1, a_2, a_3 less than they would be were the barometer true; prove

$$\frac{h_2-h_3}{a_1}+\frac{h_3-h_1}{a_2}+\frac{h_1-h_2}{a_3}=0.$$

XLII.

1. Find the magnitude and direction of any number of parallel forces acting upon a body.

2. Equal forces act in parallel directions at the angles of a regular hexagon: determine the line of action of their resultant.

3. A uniform rod rests partly within and partly without a fixed smooth hemispherical bowl: determine the position of equilibrium.

4. Find the centre of gravity of weights 3, 4, 5, placed in the angles of a triangle whose sides are 3, 4, 5: the least weight being placed in the least angle, and the greatest in the greatest angle.

5. Prove that the distance of the centre of gravity of n equal particles from any plane is $\frac{1}{n}$th part of the sum of the distances of the particles from the same plane.

6. Find the distance from the ground of the centre of gravity of a square pyramid formed of 12-inch spherical shot, the number of shot in a side of the base being 15.

7. In how many seconds would a stone fall, *in vacuo*, through a mile? What is the momentum acquired by a 24 lb. shot when it strikes the earth after falling from the top of a tower 100 ft. high? With what velocity must the shot be thrown to reach the ground in 2″?

8. Given the moment of inertia of a body or system of bodies about an axis passing through its centre of gravity, find its moment of inertia about any axis parallel to the first.

9. A shaft 8 ft. in diameter is to be sunk 60 ft. in chalk, whose *s. g.* is 2·315. How many days' work would be expended in raising the chalk to the surface by a man working with a windlass, and yielding 70000 units of work daily?

10. Two weights, P and 2P, are connected by a flexible cord passing over a pulley: find how far 2P will descend in two seconds, and determine the tension of the cord.

11. A cannon ball weighing 68 lbs. after traversing 4 ft. of the bore of a cannon, leaves it with a velocity of 1000 ft. per second; what mean pressure must have been exerted upon it to give it this velocity, neglecting friction and the resistance of the air?

12. The lever of a hydrostatic press is 3 ft. long, and the distance from the fulcrum to the point at which it acts on the smaller piston is 8 in., the diameter of the greater piston is 1 ft., and that of the less $\frac{1}{2}$ inch.; with what pressure does each pound applied to the extremity of the lever tend to raise the greater piston?

XLIII.

1. Prove, that if a body be suspended freely from a fixed point, it will not be in equilibrium, unless the line joining the point of suspension with the centre of gravity be vertical.

2. Show that the principle of virtual velocities holds good in the case of the lever.

3. The arms of a false balance are to one another as 7 to 8, and the weight is put into one scale as often as into the other: what will be the gain or loss per cwt. to the seller?

4. A triangular slab of uniform thickness is supported at its 3 angular points. Show that the pressures on the supports are equal to one another.

5. Find the dimensions of the strongest rectangular beam that can be cut out of a given cylinder.

6. A body, of weight w, is placed on a rough plane which is inclined at an angle α to the horizon. If the coefficient of friction be $\tan\beta$, show that the body cannot be drawn up the plane by a force less than $w \sin(\alpha+\beta)$.

7. A body is projected upwards with a velocity u, and, after passing through a space s, has a velocity v; show that $v^2=u^2-2gs$. If the velocity of projection be $5g$, find the time in which the particle will rise to the height $8g$.

8. A weight of 100 lbs. is drawn along a smooth horizontal table by means of a weight of 20 lbs. hanging vertically: find the space described in 5″.

9. Show that the centre of gravity of the weights in *Qu.* 8 moves in a straight line, and find its velocity at the end of the time t.

10. One perfectly elastic ball strikes another equal ball at rest, the direction of the motion of the moving ball making an angle of 45° with the line joining their centres: find the direction of the motion of each ball after impact.

11. A circular plate has a radius of 4 in., find the moment of inertia of the plate about an axis perpendicular to its plane at a distance of $\frac{1}{2}$ in. from its centre.

12. Describe the diving-bell. If the bell be cylindrical and sustained by a rope in the direction of its axis, find the tension of the rope when the bell has descended to a given depth.

XLIV.

1. Prove the parallelogram of couples.

2. There is a homogeneous cylinder, whose height is 12 in., and radius of base 6 in.; it is placed upon its end on a horizontal plane which is gradually inclined. At what inclination will the cylinder fall over? What is the coefficient friction if the cylinder be on the point of overturning and slipping at the same instant?

3. If a tetrahedron M M′ M″ M‴ be formed by joining the centres of gravity of 4 masses m, m', m'', m''', these masses will be respectively proportional to the tetrahedron, whose common vertex is the centre of gravity of the four masses, and whose bases are respectively the opposite faces of the tetrahedron M M′ M″ M‴.

4. Two uniform beams of unequal lengths are placed in the interior of a smooth spherical surface, with their lower ends in contact: determine the position of equilibrium?

5. Weights of 1, 2, 3, 4, 5, 6 lbs. are placed in order at the angular points of a hexagon: find their common centre of gravity.

6. Show that the principle of virtual velocities holds in the case of a body in equilibrium on a smooth inclined plane.

7. Prove that centrifugal force is directly proportional to the radius of the circle described, and inversely proportional to the periodic time. Also that the earth's attraction is about 289 times the centrifugal force at the equator.

8. A body is projected in a direction making an angle of

15° with the horizon, and a velocity of 50 ft. a second; find its range, greatest altitude, and time of flight.

9. A shell being discharged at an angle of 45°, its explosion, which took place at the moment it reached the ground, was heard at the mortar 3·5″ after the discharge. Required the horizontal range, the velocity of sound being $35g$ ft. a second.

10. A weight is placed against the inner surface of a hollow cylinder which revolves round its axis, which is vertical. The coefficient of friction being μ, determine the angular velocity so that the weight w may, by its centrifugal force, be just prevented from slipping.

11. Describe briefly the '*Hydrostatic Bellows.*'

12. A cylindrical pontoon 9 ft. long and $2\frac{3}{4}$ ft. radius floats with $4\frac{1}{8}$ ft. of its vertical diameter out of the water; what weight will be necessary to sink it to the water level?

XLV.

1. What is the axis of a statical couple? Three couples act upon one body in three coordinate planes at right angles to each other: find the magnitude and position of the resultant couple.

2. If the 4 faces of a triangular pyramid be equilateral triangles, each having a side equal to a, and if h be the distance of the centre of gravity from any angle of the pyramid, prove that $h^2=\frac{3}{8}a^2$.

3. A pyramid is cut from a cube by a plane which passes through the extremities of three edges that meet in a point. Find the distance of the centre of gravity of the remainder from the centre of gravity of the cube.

4. How is work estimated mathematically? In a machine is the work applied always equal to the effective work?

5. A cistern is 20 ft. long, 7 ft. wide, and 8 ft. deep, the height of the bottom of the cistern from the surface of the water in a well is 56 ft.; in what time will a man pump the

cistern full of water, allowing that he performs 2600 units of work per minute, and that the modulus of the pump is ·66 ?

6. Two velocities are impressed simultaneously upon a body ; show how to determine its motion. A ship is sailing at the rate of $10\frac{9}{11}$ miles an hour ; across the deck perpendicular to the direction of the ship's course a ball is bowled with a velocity of 12 ft. a second ; find its path and velocity.

7. A given weight is fastened to one extremity of a string, and whirled round the other end in a horizontal plane with a uniform velocity describing the circumference in a given time ; find the tension of the string.

8. A string 3 ft. long is capable of sustaining a weight of 6 lbs. ; if a weight of 1 lb. be attached to the end of the string, find the greatest number of revolutions it can make in a minute without breaking the string.

9. Define the terms, '*neutral line*,' '*neutral surface*.' Show how to find the point of rupture of a rectangular beam.

10. Three weights, 3, 4, 5 lbs., attached to rigid rods without weight of 1, 2, 3 ft. in length, respectively revolve round a vertical axis in 30″; find the accumulated work in the system, and its radius of gyration.

11. For what surfaces when immersed in a fluid will the fluid pressures have a single resultant? For what surfaces will the directions of the pressures all pass through one point ?

12. If a right-angled triangle, having one of its oblique angles double the other, be immersed vertically in a fluid with its hypothenuse coinciding with the surface of the fluid, show that the pressures on the two triangles formed by joining the right angle with the middle of the hypothenuse will be equal.

XLVI.

1. (1) If four forces act upon a point to keep it at rest, *state* generally how any one of the forces must be related to the resultant of the other three.

(2) If three forces acting on a point are in equilibrium, show that each force is proportional to the sine of the angle contained by the other two.

(3) If OP, OP_1, represent two equal forces acting upon O at right angles to each other, and OQ represent the third force which keeps O in equilibrium, show that the triangle formed by the intersection of the lines, drawn perpendicular to OP, OP_1, OQ, through P, P_1, Q, is isosceles and right-angled.

2. Find the conditions of equilibrium of any system of forces acting in one plane. Account for one of these conditions *only* being necessary in the case of a lever attached to a fulcrum.

3. A uniform beam AB, whose weight is W, rests in equilibrium, between a vertical wall BC and a horizontal plane AC, both smooth. CE is a string without weight, attached to a point E in the beam. If $BAC=\alpha$, $ACE=\beta$, show that the tension of the string $=\frac{W \cos \alpha}{2 \sin (\alpha-\beta)}$.

4. Explain the construction of the Screw, considered as a mechanical power. Find the relation between the power P and the weight W, when the screw is rough and the friction opposed to P.

5. If a beam AB is supported in a horizontal position at A and B, and the weight is uniformly distributed along AB, find the moments of the strain at any point C of the beam. About what point will this moment be greatest?

6. When is the impact of two bodies said to be direct? An imperfectly elastic ball A impinges directly with a given velocity upon another ball B, of the same material, at rest; investigate the velocity of each after impact.

7. If B in *Qu.* 6 impinge upon another ball C at rest, find the velocity of C, and show that it is greatest when B is a mean proportional between A and C.

8. A projectile has a velocity of 1,600 ft. a second, and is projected at an angle of 30° ; find its horizontal and vertical distances from the point of projection after 12″, gravity being taken at 32 ft.

9. Three equal balls are projected at the same instant, from the same point, with the same velocity, 1,600 feet in a second, and at angles 30°, 45°, 60°, respectively. Find the height of the common centre of gravity of the balls above the horizontal plane after 12′.

10. If O be the centre of oscillation, K the centre of gyration, G the centre of gravity, of a body suspended from a horizontal axis passing through C, prove $CG \times CO = CK^2$.

11. Find an expression for the time of a small oscillation of an equilateral triangle, oscillating in its own plane about a horizontal axis, through an angle perpendicular to the plane of the triangle.

12. How is it shown that the elastic force of the air at a given temperature varies inversely as the space it occupies ?

A cylindrical tube 2 ft. in length, and closed at its upper end, is immersed vertically in sea-water, and it is found that the water has risen in the tube $1\frac{1}{2}$ ft. ; find the depth to which the tube has been sunk, supposing 32 ft. of sea-water measures the atmospheric pressure.

XLVII.

1. (1) Define the terms '*Force*,' '*Tension*,' '*Velocity*.' (2) If two men pull the ends of a short cord in opposite directions, each exerting a force of 20 lbs., what is the tension of the cord ? (3) When a cricket-ball is flying in the air, what forces are acting upon it ?

2. A uniform heavy bar, 2 ft. long, can turn about a

fixed point. A weight of 10 lbs., hung at its ends successively, is balanced by the weights $4\frac{1}{2}$ lbs. and 21 lbs. hung at the other end. Find the weight of the bar, and the lengths of the portions on each side of the fixed point.

3. Employ the principle of virtual velocities to find the weight which 1 lb. will support by means of two blocks of pulleys, of which the upper one is fixed, and the lower one, weighing 1 lb., is moveable, each block containing 4 pulleys or sheaves, and the portions of string between the blocks being all vertical.

4. A weight of 1 lb. rests on a rough plane, whose inclination to the horizon is 30°, and a string, attached to this weight, passing along the plane and over its lower edge, sustains a weight of 4 oz. hanging by it. Find the ratio of the friction to the pressure on the plane.

5. Two uniform rods AB, CD, are connected by the lower end B of one being jointed to a given point B of the other, and the points A and D being united by a string, so that ABD is a right angle. The system hangs from the point A, about which it can turn. Find the position of equilibrium, and the action at the joint B; having given that, AB=BC =4, BD=8; and that the weight of AB is double the weight of CD.

6. Three elastic balls A, B, C, have their centres in a straight line, and lie at rest. A has a motion impressed upon it, with which it strikes B directly, and B then strikes C. If after impact, A and B are stationary, prove that the masses of the balls are in geometric progression.

7. A body projected at an elevation of 45° strikes an object 20 yds. high, at a horizontal distance of 400 yards. Find the other angle of elevation at which the body may be projected, with the same velocity, to strike the same object. Compare the times of flight in the two cases.

8. A cylindrical tumbler of 3 in. diameter is filled with water to the height of 2 in. Find the whole pressure on

the cylindrical surface, taking the weight of 1 c.ft. of water to be 1,000 oz.

9. Prove that the resultant pressure of a liquid on a solid floating in it, is equal to the weight of the liquid displaced.

10. What is '*Specific Gravity*?'

11. A body weighs 56 oz. in a vacuum, and 49 oz. when immersed in water; required the weight in pounds of a cubic foot of the substance of which the body is composed.

12. Explain the principle of the diving-bell.

A hemispherical bell is sunk in water until the surface of the water under the bell bisects the vertical radius; find the depth to which the bell is sunk.

XLVIII.

1. Show the propriety of representing statical forces geometrically, and mention any advantages which belong to this mode of representation.

2. What is meant by the '*Coefficient of Friction*?' How is it related to the '*limiting angle of resistance*?'

3. A uniform ladder is just supported by the friction of the ground, resting at an angle of 45°, between a smooth vertical wall and a rough horizontal plane; find the coefficient of friction.

4. Find the centre of gravity of three heavy particles placed in the angles of a right-angled triangle; and show that if the weights of the particles be proportional to the opposite sides of the triangle, their centre of gravity will coincide with the centre of the inscribed circle.

5. Distinguish between '*Stable*' and '*Unstable Equilibrium.*'

Two convex surfaces are spherical at their point of contact, the upper surface is displaced by rolling through a small angle; find the condition that the equilibrium may be stable.

6. A heavy right cone rests with its base on a fixed

rough sphere of given radius, find the limit to the height of the cone consistent with stable equilibrium.

7. If material of given weight and form is raised through a given height, how is the work estimated?

A reservoir 120 ft. long, 60 ft. wide, 22 ft. deep is to be filled from a river whose surface is 50 ft. below the bottom of the reservoir; find the horse-power of an engine that will fill it in 24 hrs. (*Wt. of* 1 *c.f. of water* = 62·5 *lbs.*)

8. Distinguish between '*Accelerating Force*' and '*Moving Force*,' and show how they are respectively estimated in Dynamics.

9. A body is projected in the direction of an accelerating force with a velocity u; after a time t it has acquired a velocity v, prove that s the space described $= \frac{u+v}{2} \times t$.

10. A body projected in the direction of gravity with a velocity of 64 ft. a second, describes 241·1 ft. in the nth second; find n; $g = 32{\cdot}2$ ft.

11. How may the force of gravity be determined by means of a simple pendulum? Assuming the expression for the time of an oscillation; show how the height of a mountain may be determined by means of a pendulum.

A seconds pendulum taken to the top of a mountain loses 10 oscillations in 24 hrs.; find the height of the mountain.

12. A weight of W lbs. is pulled up an inclined plane by a pressure of P lbs. acting parallel to the plane; show that the accelerating force is equal to $\left(\frac{P}{W} - \frac{\sin(\alpha+\phi)}{\cos\phi}\right)g$, in which α is the inclination of the plane, ϕ the limiting angle of resistance.

XLIX.

1. A string ABC hangs vertically from the fixed point A with weights B, C, of 7 and 4 oz. respectively, attached to it at given points. State the forces which act on the upper

weight B and are in equilibrium, and find the tension of the parts of the string AB, BC.

2. If three men keep a body at rest by pulling it by cords with equal forces, prove that the directions of the cords include angles of 120°. If the direction of one of the cords is slightly altered in the plane of the forces, the other two remaining unchanged; prove that the men who pull these two must both alter the force with which they pull if equilibrium is to be maintained.

3. A weight of 100 lbs. is carried by two men on a horizontal hand-barrow, the handles which they hold being 6 ft. and 4 ft. respectively, from the weight. What weight does each man sustain? Is the result altered if the barrow be not horizontal.

4. State the conditions necessary and sufficient for equilibrium when forces act in one plane on a rigid body.

5. AB, CB are uniform rods of given weights, whose upper ends are capable of turning about given points A, C in the same horizontal line. If their lower ends are jointed together at B, find the mutual action between them at that point. Explain the result when the rods are together equal in length to AC.

6. In the case of an isosceles roof supporting a given weight, when there is *no tie beam*, show how to determine the magnitude and direction of the pressure, tending to overturn the side walls that support the roof.

7. Find the centre of gravity of a trapezium in which AB, CD are parallel sides, and AC is perpendicular to them, AB being double AC or CD.

8. State how the velocity of a body regarded as a point is measured.

9. If two bodies of masses m, m', moving in the same straight line, have at any instant the velocities v, v', prove that the velocity of their centre of gravity at the same instant is $\frac{mv+m'v'}{m+m'}$.

10. When a body moves from rest under the action of a constant accelerating force, prove the formula $s=\frac{1}{2}ft^2$.

If a body moving as above describes 50 ft. in 50 seconds, what is the velocity produced in it?

11. Prove that the open surface of a liquid at rest under the action of gravity is a plane.—Why does the upper surface of the mercury in a barometer take a curved form?

12. Explain how a '*siphon*' draws liquid out of a vessel.

L.

1. Three given forces are in equilibrium on a point; show how to determine the angle between the directions of any two of the forces.—The magnitudes of the forces are as 2, $\sqrt{6}$, $\sqrt{3}+1$; determine their inclination to each other.

2. Is the tension of a perfectly flexible string affected by its passing over (1) a smooth surface, (2) a rough one? A and B are given fixed points in a horizontal line, BC is a string of given length carrying a small ring at C, which is without weight; a string ACW fixed at A, passes through the ring C, and hanging vertically supports a weight W; find the position of the ring when there is equilibrium.

3. Explain the term '*Virtual Velocity*.' Show that the principle of virtual velocities holds in the case of two forces in equilibrium on a bent lever.

4. When a horse-power is said to be measured by 33,000, what units are referred to?— What must be the horse-power of a locomotive engine which moves at the rate of 20 miles an hour upon a level rail, the weight of the train being 50 tons, and the resistance of friction 8 lbs. a ton?

5. In the impact, whether direct or oblique, of two smooth balls, show that the motion of the centre of gravity is unaffected by impact.—In the case of direct impact, when is the centre of gravity *at rest* both before and after impact?

6. Find the range and time of flight of a projectile on an inclined plane passing through the point of projection; and show, for a given velocity, how the body must be projected that the range shall be a maximum.

7. Find the inclination of the plane in *Qu.* 6 when the maximum range upon it is $\frac{3}{4}$ of the maximum range on a horizontal plane, with the same velocity.

8. Find the centre of oscillation of a right-angled isosceles triangle, about an axis through its vertex and perpendicular to its plane, in terms of the altitude of the triangle.

9. Explain the construction, action and graduation of a common barometer.

10. The mercurial column stands at 30 inches, the *s.g.* of mercury is 13·568; determine in lbs. the pressure of the atmosphere.

11. If a vertical wall, of given thickness and material, sustains the pressure of water in a reservoir, show how to determine the conditions that the wall shall just not be overturned.

12. If the section of an embankment of brickwork supporting the vertical side of a reservoir be of the form of a trapezoid whose parallel sides are horizontal, the upper side being 5 ft. and the lower 10 ft. and the whole height of the embankment 50 ft.; find the height to which the water must rise against the vertical wall so as just to overturn it, the *s.g.* of the brickwork being 2·17.

LI.

1. Show that the conditions necessary to render a force, acting on a particle, determinate may be represented geometrically.

2. A heavy particle W is placed on a smooth horizontal plane, a given weight P hangs over a smooth peg, at a given height above the plane, by means of a string the other end

of which is fastened to W, find the condition that P may just raise W off the plane: if P be not sufficient to raise W, will W remain at rest?

3. How does the friction of a surface modify the direction of its resistance? At what inclination of a plane will a particle be just in the act of moving down it, when the coefficient of friction is $\frac{1}{2}$?

4. If P be the force necessary to support a heavy particle on a smooth inclined plane, P_1 the force acting when the plane is rough and the particle is in a state bordering on motion *up* the plane, and P_2 the force acting on the particle when it is in a state bordering on motion *down* the plane, prove P an arithmetic mean between P_1 and P_2: P, P_1, and P_2 all acting parallel to the inclined plane.

5. Define the centre of gravity of a heavy body. Show that the centre of gravity of an equilateral triangle must coincide with the centre of the circle inscribed in it.

O is the centre of a given circle, AB a chord, OC bisects AB in C, take $OQ=\frac{2}{3}OC$. Prove Q the centre of a circle, which is the locus of the centres of gravity of all triangles standing on AB as a base, and having their vertices in the circumference of the given circle.

6. Show that two equal and opposite parallel forces acting on a rigid body cannot be kept at rest by a single force.—Define the '*axis of a couple*,' and show that any couple acting on a rigid body may be replaced by three couples in planes at right angles to each other.

7. If a heavy, uniform and flexible string be suspended from two fixed points, determine the tension of the string at any point of the curve which it forms. Hence show that if a uniform chain hangs freely over two fixed points the extremities of the chain will be in the same horizontal line, in the case of equilibrium.

8. A weight P hanging freely over the edge of a horizontal table draws Q along the table; find the accelerating force and the tension of the string as P descends (1) when the

table is *smooth*, (2) when it is *rough*, the friction of the string against the table being neglected. If the table be rough and the coefficient of friction $\frac{1}{3}$, find the space described from rest in $3\frac{1}{2}$ seconds, when the weights of P and Q are 4 and 9 oz. respectively.

9. A weight W attached to the end of a string of length r revolves uniformly in a horizontal plane about the other end of the string; show that if T be the tension of the string and P the time of one revolution $\frac{T}{W}=\frac{4\pi^2 r}{P^2 g}$.—A string 3 ft. long is capable of supporting a weight of 6 lbs.: how many revolutions per minute may be made by a weight of 2 lbs., which is attached to one end, whilst it revolves uniformly round the other end, without breaking the string?

10. Find the centre of oscillation of any number of compound bodies when rigidly connected, the distances of the centre of oscillation and the centre of gravity of each separate body, from the point of suspension being given.

Find the centre of oscillation of 3 particles 8, 10 and 12, situated in the same straight line at distances 3, 5 and 7 ft. respectively from an axis perpendicular to this line.

11. Show how to find the *s.g.* of a solid that will not sink in water.

A piece of cork weighs *in vacuo* 2 oz.; when a piece of lead which weighs 8 oz. in water is attached to the cork, the united mass weighs in water $1\frac{3}{4}$ oz.; find the *s.g.* of the cork.

12. Show how to find an expression for determining the difference of altitude of two stations by means of a barometer, the temperature during the observations being constant. In determining the heights of mountains, what other considerations affect the problem?

LII.

1. Explain the meaning of the terms '*Action*' and '*Reaction*.'

A man, weighing 150 lbs., supports a weight of 50 lbs. by a vertical cord passing over a fixed pulley. What pressure does he exert on the ground ?

2. Define the '*Moment*' of a force about a point.

Assuming the parallelogram of forces, prove that the algebraic sum of the moments of two forces about a point in the space between their directions is the same as the moment of their resultant.

3. A uniform rod weighing 10 lbs. passes through two fixed rings, and is horizontal. The rings are 3 ft. apart, and the lengths of the parts of the rod which project beyond them are 2 ft. and 11 ft. Find the pressure on each ring.

4. A cube of given weight resting on a horizontal table is gradually raised round one edge by a smooth wedge pressed under it. When the cube has been raised through an angle, which is half the angle of the wedge, find the friction between it and the table at the edge on which it turns.

5. Find the ratio between P and W on a single moveable pulley without weight, when the strings are not parallel. Under what circumstances will P be less than W ?

6. When a ball of given elasticity impinges obliquely on a smooth plane, find its subsequent velocity, and show whether this is greater or less than its velocity before impact.

7. Prove that a projectile *in vacuo* will strike the horizontal plane through the point of projection, at an angle equal to that at which it was projected.

8. A body is projected with a velocity of 2,000 ft. per second, at an elevation of 10° ; find its greatest altitude.

9. Find the pressure of a liquid upon a plane exposed to it.

A river-wall is inclined at 75° to the horizon. The depth of water being 6 ft., find the pressure on a rod of length of the wall, taking 1,000 oz. as the weight of a cubic foot of water.

10. When the barometer stands at the height of 30 in., find the ratio of the pressure of the air to its density having given that a cubic inch of mercury has a weight 10,462 times that of a cubic inch of air.

11. If a lighter fluid rests upon a heavier, and their specific gravities be s and s', and a body whose *s.g.* is σ, rests with its volume v in the upper fluid and v′ in the lower. Prove $v : v' :: s' - \sigma : \sigma - s$.

12. If a cylindrical vessel containing liquid be whirled round its vertical axis with a given angular velocity, show that the form of the surface of the fluid in a state of equilibrium is a paraboloid.

If the cylinder, open at the top, be filled, find the angular velocity so that half the liquid may be thrown out when the form of equilibrium is attained.

LIII.

1. Two forces, each of given magnitude, P, act perpendicularly on the rod ACB, at the points A, B. Replace them by two equivalent forces, one of which, of given magnitude, Q shall act perpendicularly on the rod at the point C, BC being double AC.

2. Find the position of the centre of gravity of a plane triangle.

Deduce, by Guldin's properties, the volume of a right circular cone.

3. Define a '*Vertical*' line. If a body is capable of swinging freely about a fixed point; prove that the positions in which it rests are those in which its centre of gravity is in a vertical line with the fixed point.

4. A uniform smooth bar, ABC, of weight W, is supported

horizontally at the ends A, C. Another uniform bar DB, of weight w′, can turn about its upper end D, which is vertically above C, and presses with its lower end B on the former bar. Given that AB=$\frac{1}{3}$AC; find the pressures on the points of support A, C, D.

5. If a body describes a straight line with a uniformly accelerated motion, prove that its motion resolved in the direction of any other straight line is also uniformly accelerated. Is any one of the three laws of motion required in the proof?

6. A body shot vertically upwards is 240 ft. above the point of projection at the end of 4″. How high does the body rise (g=32)?

7. Two unequal weights are connected by a perfectly flexible string without weight, passing over a smooth fixed pulley, so that one descending draws up the other. Prove that the tension of the string remains the same throughout the motion.

8. If a body slides from rest down a smooth plane, inclined to the horizon at the angle 23°, in what time will it gain a velocity of 150 ft. per second (g=32·2)?

9. A body is projected *in vacuo* with a velocity of 1,610 ft. in a second, at an angle of 45°; find the range, greatest height, and area of the parabola described.

10. A particle descends the arc of a quadrant of a circle, the radius of which is 16 ft.; find its velocity at the lowest point.

11. A straight tube 4 in. in diameter has its lower end hemispherical. If it be held in a vertical position, and water be poured into it to stand at a height of 6 in. above its lowest point; find the resultant pressure on the hemispherical end (1 *c.ft. of water weighing* 1,000 *oz.*).

12. A uniform cylinder floating vertically in water has 6 in. of its length immersed. How far will it sink in a fluid whose *s.g.* is ·8?

LIV.

1. Explain why a single force acting on a point may be resolved into two others, in any directions at right angles to one another.

Find the resultant of three or more forces acting on a point in a plane.

2. ABC is a triangle, bisect AC in E, bisect AE, CE in G, and F respectively; if BA, BG, BE, BF, BC, represent, in magnitude and direction, forces acting on the point B; prove that their resultant is equal to 5BE.

3. When any number of forces are in equilibrium on a rigid body in one plane, three conditions are generally necessary for equilibrium; state them, and explain the kind of motion to which each condition applies.

In the case of forces acting in one plane on a lever, account for one condition being sufficient for equilibrium.

4. A heavy equilateral triangle is hung upon a smooth peg by a string, the ends of which are attached to two of its angular points; if the distance of the centre of gravity of the triangle from the peg is equal to a side of the triangle, show that the triangle has a side either in a horizontal or a vertical position.

5. If a given pressure act on a given wall, in a vertical plane and in a given direction, how is the stability of the wall determined?

A wall of brickwork 3 ft. thick and 35 ft. high, sustains on the inner edge of its summit a pressure on every foot of its length, the direction of the pressure making an angle of 60° with the horizon: find its amount when it will just not overthrow the wall, the weight of 1 c.ft. of brickwork being 112 lbs.

6. Find the centre of gravity of a sector of a circle.

7. A ball impinges obliquely on a fixed smooth plane; *find* the motion of the ball after impact, the elasticity *between* the ball and the plane being given.

A ball, whose modulus of elasticity is $\frac{1}{3}$, is let fall from a given height h above an inclined plane whose inclination is 30°; after striking the plane the ball rebounds and strikes it again: show that the range on the plane between the two points of impact is $\frac{8h}{9}$.

8. If a heavy body be dragged along an inclined plane, show that the units of work expended will be equal to the numbers that would be expended in dragging it along the base, supposed equally rough, and in lifting it up the perpendicular height.

What distance will an engine of $92\frac{2}{11}$ horse-power drag a train in 10 minutes with a uniform velocity up an incline of 1 in 200, the weight of the train being 80 tons and the friction 7 lbs. per ton?

9. What are the '*Centres of Gyration*' and '*Oscillation*' of a material body?

A heavy particle is placed at a distance of 36 in. from the centre of suspension; at what distance must another heavy particle of double the weight be placed, in order that the two when connected may vibrate seconds?

10. If a plane surface be exposed to the pressure of any fluid, what is understood by the '*Centre of Pressure*?'

A rectangle is immersed vertically in a fluid, one side coinciding with the surface of the fluid; find the depth of the centre of pressure.

11. Describe '*Nicholson's Hydrometer*.'

The hydrometer weighs 250 grs. and requires 72 grs. to sink it to the required depth in water and 9 grs. in alcohol; required the *s.g.* of alcohol.

12. Describe the construction and action of the '*Forcing Pump*:'

Find the force necessary to overcome the resistance when the piston is in any position during its descent, and the water stands at a given height in the ascending tube.

LV.

1. Six given forces act on a point in directions of the successive sides of a regular hexagon given in position. Find the magnitude and direction of their resultant.

2. Find the relation between two weights which balance one another on a given wheel and axle.

Prove that the principle of Virtual Velocities is true in this instance.

3. A body weighing 10 lbs. lies on a rough plane between which and itself the coefficient of friction is $\frac{1}{n}$. The plane is inclined at an angle of 5° to the horizon. What pressure, acting along the plane, will the body bear before it begins to slide upwards?

4. Three smooth discs of equal size lie in contact upon a table, and are kept in position by a fixed ring, which circumscribes them all, so that they exert no pressure on it or on one another. If one of them is now acted upon by a given force P in the line joining its centre and the centre of the ring, find the pressures produced between the other discs and their pressures on the ring.

5. If a plane area revolves about a given axis, show that the content of the solid generated is equal to the area that revolves multiplied by the length of the path described by its centre of gravity.

Apply this property to find the content of the cone generated by the revolution of a given right-angled triangle.

6. A bar, 4 ft. long, of 6 lbs. weight, has weights of 3 and 6 lbs. hung from its ends. Find the position of a fulcrum on which the bar will rest in equilibrium.

7. A plane figure is formed by an isosceles right-angled triangle, and by the squares described upon the three sides of the triangle; find the centre of gravity of the whole plane area.

8. A ball impinges with a given velocity on a plane,

between which and itself the coefficient of elasticity is $\frac{1}{3}$, and it is reflected in a direction perpendicular to its direction at incidence. Find the velocity with which it is reflected.

If the velocity of projection be 500 yds. per second, in what second of its flight will the body have a direction inclined at 45° to the horizon? Show that when the body attains this position it will have travelled horizontally twice as far as it has fallen vertically.

9. Find the time in which a body will descend by gravity down a smooth inclined plane 200 ft. long, the height of the plane being 3 ft.

A train of 10 tons descends a plane of the above length and height; find the velocity acquired when the friction is 8 lbs. a ton.

10. Find the pressure on a given plane area under water, taking account of the pressure of the atmosphere on the surface of the water.

When a cube is held under water, state whether the resultant pressure of the water on it will change or not as the cube takes different positions.

11. A straight vertical tube closed at both ends is divided into two portions by a thin moveable disc without weight. The upper, 6 in. long, is full of water, of which a cubic foot weighs 1,000 oz. The lower part is full of air of such pressure as to keep the disc in equilibrium. Compare the density of the air in the tube with that of atmospheric air exerting a pressure of 15 lbs. on the square inch.

12. Draw figures exhibiting, without description in words, the action of (1) the forcing pump, (2) the air pump, (3) the condenser.

LVI.

1. When a weight hangs by a string, what is meant by the statement that the tension of the string is the same throughout? Is the tension the same if the string has weight?

A and B are two points in the same horizontal line, AC and BC are strings without weight, tied at the point C to a weight W, AB and AC are equal to each other, and the angle BAC is 36°. If T and T_1 are the tensions of BC and AC, prove $T - T_1 = \frac{W}{2 \cos 18°}$.

2. What is meant by the coefficient of friction between two substances? If the coefficient of friction between a rough horizontal plane and a moveable particle be $\frac{1}{\sqrt{3}}$, within what angle may a pressure be applied to the particle without causing it to move?

A rough hemispherical bowl, with the coefficient of friction $\frac{1}{\sqrt{3}}$, has a radius of 12 in.; find, approximately, the extreme height above the lowest point of the bowl, at which a particle may be placed on it so as to remain at rest.

3. If a body of any form be placed on a smooth horizontal plane; find the conditions that it may stand or fall.

4. If 2α be the angle of a right cone, standing on its base on an inclined plane whose inclination to the horizon is ϕ; show that when $4 \tan \alpha = \tan \phi$, the cone is on the point of turning over.

5. In the case of an isosceles roof, explain how the thrust on the tie-beam acts.

Two rafters, AB and AC, are each 20 ft. long; the tie-beam is 35 ft.; a weight of 1 ton is suspended from A; determine the strain on the tie-beam, neglecting the weight of the rafters.

6. Show how to find the position of the centre of gravity of any solid of revolution.

If a and b are the radii of the ends of the frustum of a paraboloid, nearer to and further from the vertex respectively, h the height of the frustum, find the distance of the centre of gravity of the frustum from the less end.

7. A heavy body W is drawn up an inclined plane by means of a string attached to a weight P that hangs over

the top of the plane; find the accelerating force, (1) when the plane is smooth, (2) when it is rough.

If the inclination of the plane is 30°, P = 25 lbs., W = 15 lbs., find the space described by P in 7·5″ when the plane is rough, the limiting angle of resistance being 30°.

8. What is a '*Simple Pendulum*?' Show that the time of a small oscillation in a circle whose radius is l will be equal to $\pi \sqrt{\frac{l}{g}}$.

9. A pendulum whose length is L makes m oscillations in one day; its length is diminished by a small quantity, and it is found to make $m+n$ oscillations in a day; show that the diminution of its length is $\frac{2n}{m}$ L nearly.

10. What is meant by the '*work accumulated*' in a moving body?

A train moving at the rate of 15 mi. an hour, comes to the foot of an incline of 1 in 300, the resistances being 8 lbs. a ton: how far will the train move up the incline before stopping?

11. Find the internal pressure on a sphere filled with water, the radius of the sphere being 6 inches and the weight of 1 c.f. of water 62·5 lbs.

12. Give reasons for the statement that the mercury in the barometer tube is supported by the pressure of the air on the surface of the mercury in the basin.

The mercury in a barometer which stands at 30 in. is placed under the receiver of an air pump, and after 12 turns the mercury sinks to 15 inches. Compare the capacities of the receiver and the barrel of the pump.

LVII.

1. The directions of two forces, represented by 3 lbs. and 5 lbs. respectively, include an angle of 60°. Find the

magnitude of their resultant, and the tangent of the angle which its direction makes with the less force.

2. If three forces P, Q, R, applied to the same point O are in equilibrium, prove that

P : Q : R :: sin QOR : sin ROP : sin POQ.

Three forces are represented in direction by the perpendiculars drawn from the vertices of a triangle to the opposite sides, and are inversely proportional to those perpendiculars; prove that they are in equilibrium.

3. Find the magnitude and direction of the resultant of two parallel forces acting on a rigid body.

Explain the result when the forces act in opposite directions.

4. A rigid body is acted on by any number of forces in one plane; find the conditions of equilibrium.

Prove that a force and a couple in the same plane cannot be in equilibrium.

5. ABC is a uniform heavy lever of which B is the fulcrum, and of which the arms BA, BC include an angle of 120°. If, when the lever is left to itself in a vertical plane, the arm BA is horizontal, determine the ratio of BA to BC.

6. Determine the centre of gravity of a homogeneous pyramid having a triangular base. Show that it coincides with the centre of gravity of four equal weights placed at the corners of the pyramid.

7. State Newton's third Law of Motion, and give examples in illustration of it.

8. Prove that a body in falling from an upper point to a lower acquires just that velocity which would be requisite to carry it back from the lower point to the upper one.

9. If a circle be placed in a vertical plane, show that the time of descent down all chords of the circle, terminating at the lowest point of the circle, is the same.

Down which of these chords must a body fall in order to acquire at the bottom the greatest horizontal velocity?

10. Two equal weights P and Q are connected by a thread; P is placed at the bottom of a smooth plane, inclined at an angle of 30° to the horizon: the thread is passed over a pulley at the top of the plane, and Q hangs freely at its extremity. Compare the time in which P will be drawn to the top of the inclined plane, with the time in which, if separated from Q, it would fall down the plane, friction being neglected in both cases.

11. If two imperfectly elastic balls impinge directly, determine the subsequent motion of each.

A ball of given mass moving with a velocity of 1 ft. per second impinges on an equal ball at rest; if the common elasticity of the two balls is ·5, find the amount of *vis viva* lost in the impact.

12. A cone floats in water with its vertex downwards; and the centre of gravity of the cone is level with the surface of the fluid: what is the specific gravity of the cone?

LVIII.

1. Show that any system of forces acting on a rigid body can be reduced to a force and a couple.

2. Investigate a formula for finding the centre of gravity of any solid of revolution, and apply the formula to find the centre of gravity of the hemisphere.

3. A heavy string, of uniform density and thickness, is suspended from two given points; find the equation to the curve in which the string hangs when it is in equilibrium.

If the two points of support are nearly in a horizontal line, and the distance between them nearly equal to the length of the string, determine approximately the tension at the lowest point.

4. A system of bodies is under the action of no forces except their weights, mutual pressures, and pressures upon smooth fixed surfaces; show that the system will be in

equilibrium if placed so that the centre of gravity is in the highest or lowest possible position.

Two equal uniform heavy beams are connected by a hinge, and placed over two smooth pegs in the same horizontal line, the hinge being uppermost; determine the position of equilibrium.

5. Prove that the path of a projectile *in vacuo* is a parabola.

What must be the angle of projection in order that the horizontal range may be m times the greatest height?

6. In any machine moving uniformly and without friction, the mechanical work expended by the power which moves the machine is equal to the mechanical work done by the machine. Explain this principle, and apply it to the Screw and to the Hydrostatic Press.

7. A particle is constrained to move on a smooth vertical curve; find the velocity of the particle and the pressure on the curve.

8. Explain D'Alembert's principle.

A heavy body is capable of motion round a horizontal axis; find the time of a small oscillation.

Ex. Suppose the body to be a cube, and the axis coincident with an edge of the cube.

9. Show how to determine the centre of pressure of a plane surface immersed in a fluid.

Ex. A triangle with one side in the surface.

10. Investigate an approximate formula for finding the height of a mountain by means of a barometer.

11. Explain what is meant, in fluid motion, by the hypothesis of '*Steady Motion*,' and by the hypothesis of '*Parallel Sections*.'

12. What is meant by the '*Vena Contracta*;' and to what extent does it modify theoretical results deduced without regard to it?

LIX.

1. Assuming the theorem of the parallelogram of forces for two equal forces, prove it for two unequal forces.

2. A uniform beam is supported by a cord, which is fastened to both ends of the beam, and passes over a small fixed pulley-wheel: show that the beam can remain at rest only in either a horizontal or a vertical position. Examine whether in the former position the equilibrium is stable or unstable: and supposing the cord to be twice the length of the beam, compare the tension of the cord, and the horizontal strain on the beam, with the weight of the beam.

3. How would you ascertain, without actual measurement, whether the arms of a balance are equal or unequal; and if they are unequal, in what ratio they are unequal?

4. Two inclined planes, of which one is smooth and the other has $\frac{1}{2}$ for its coefficient of friction, have a common altitude and are placed back to back. If the smooth plane is inclined at an angle of 30° to the horizon, determine the inclination of the other plane, so that two equal weights placed upon the inclined planes, and connected by a cord passing over a pulley-wheel at the top, may just remain at rest.

5. Determine the centre of gravity of a body.

Ex. Find the centre of gravity of the plane figure bounded by two circles of given radii, one of which touches the other internally.

6. Define '*Velocity*,' '*Angular Velocity*,' '*Momentum*,' '*Vis Viva*,' '*Mechanical Work*.' What are the units in terms of which each of these quantities is measured?

7. 'The accelerating force of gravity at the sea level is 32·2 nearly.' Explain this statement, and show how it is proved to be true.

8. If a heavy body projected in any direction *in vacuo* crosses twice the same horizontal plane, it crosses it each time with the same velocity, and in directions equally inclined to the plane.

9. Two weights, of 80 lbs. each, are suspended by a cord 80 ft. long, which passes over a fixed pulley, and to the extremities of which they are attached. If, when the weights are at rest in the same horizontal plane, a weight of 1 lb. is added to one of them, in what time and with what velocity will the other reach the plane?

10. A, B, C, are equal spheres whose common elasticity is $\frac{1}{2}$, and whose centres are in a straight line. B and C are contiguous and at rest: A, moving with a velocity of 1 ft. a second, impinges upon B: what will be the positions and velocities of the spheres one second after impact?

11. Show that the direction of gravity is everywhere perpendicular to the surface of a fluid at rest.

12. A cubical vessel is filled with fluid: compare the whole pressure on the inner surface of the vessel with the weight of the fluid.

LX.

1. Define '*Force*' generally with respect to both Statics and Dynamics; and explain the necessity, practically, of expressing specifically the measure of forces with reference to the nature of the problems, statical or dynamical, into which they may enter.

2. Define the properties of bodies designated as '*Solid*,' '*Viscous*,' or '*Fluid*,' with reference to the mutual mechanical action of the component particles on each other, when acted on by extraneous forces. What is the distinction, mechanically considered, between a flexible and an elastic body? How is the elasticity measured (1) when acting statically, and (2) when acting dynamically?

3. Define a '*Couple*,' and point out the necessity of introducing the idea of a couple, or some equivalent idea, into the investigation of many mechanical problems. If 1 lb. be assumed as the unit of statical force, and 1 ft. as that of length, what will be the unit of couple?

4. A body, which cannot roll, is sustained on an inclined plane by a string: find the direction of the string, in which the least tension will support the body, there being friction between the body and the plane, of which the coefficient is μ, and the inclination of the plane to the horizon being α.

5. Two equal uniform beams are placed with their lower ends on two equal piers, and their upper extremities lean against each other like two rafters: if a heavy weight w be placed symmetrically on their upper ends, what will be the horizontal stress on each pier?

6. A chain of length $2b$ and weight w. is suspended from two fixed points in a horizontal plane, and distant $2a$ from each other: find the tension at any point.

7. If the chain in *Qu.* 6, instead of being fastened to two fixed points of support, pass freely over small pulleys in the same positions, the two ends thus hanging vertically, and supporting, by their weight, the middle portion of the chain, what must be the length of the vertical portions of the chain, not to disturb the equilibrium of the *catenary* between the pulleys? What would be the consequence of having these vertical parts of the chain too long or too short?

8. Two inclined planes are so placed that the upper boundary of one coincides with the lower boundary of the other along a horizontal line; the inclination of the higher plane is α, and greater than β, that of the lower one; also, the upper plane is smooth, whilst there is friction, of which the coefficient is μ, on the lower one. A body *slides* from rest at a point, on the upper plane, distant b from the common line of junction of the planes, by the action of gravity: find the velocity of the body at any point on the lower plane, assuming it, on account of its form, to be incapable of *rolling* down the lower plane, and omitting any possible *impulsive* effect of friction at the instant when the descending body first comes in contact with the lower plane.

9. A U-shaped tube of uniform bore, of which the curved part is a semicircle 12 inches in length, is placed in an upright position, and contains as much mercury as fills 15 in. of its length, and as much water above the mercury on one side as fills 41 in. more. If the difference of level between the two mercurial surfaces is 3 in. nearly, what is the *s.g.* of mercury, and what is approximately the height of a column of water, which will produce a pressure equal to that at the bottom of the tube?

10. Prove that, when a body floats in a fluid, the weight of the fluid it displaces is equal to the weight of the body.

11. Show that the fluid pressure on any surface immersed in a uniform fluid at rest under the action of gravity, will be equal to that on a plane surface of equal area immersed in the same fluid, in a horizontal position, and at the same depth as the centre of gravity of the proposed surface.

12. A hollow cube, with four of its sides vertical, is filled with water. If one of the vertical sides were to become entirely detached from the base and the two adjoining sides, find the magnitude, direction, and point of application of the single force which would preserve the unattached side in its position.

ANSWERS.

I.

(3) 1. £7652194 0*s.* 9*d.* (4) 1. $2\frac{31}{38}$; 2. $\frac{64}{105}$; 3. 14645·61; 4. 7·8125.
(5) £144 11*s.* 3*d.* (6) £767. (7) £184 14*s.* 8*d.*
(8) £4502 1*s.* $9\frac{1}{4}$*d.* (9) $2\frac{1}{4}$. (10) £36 13*s.* 1·44*d.*
(11) 8064. (12) $36159\frac{3}{4}$ hrs.

II.

(2) 1. £9897 5*s.* 10*d.*; 2. £1238 3*s.* $11\frac{3}{4}$*d.*
(3) 1. 506722; 2. 1358880. (4) 1. $\frac{20}{21}$; 2. $1\frac{17}{60}$.
(5) 1. 7782·82521; 2. 3011·5026; 3. 5424·8324832; 4. 397543.
(6) £51 1*s.* $8\frac{1}{2}$*d.* (7) £7144 7*s.* 6*d.*
(8) 1. £179 17*s.* $10\frac{3}{4}$*d.*; 2. £570 8*s.* $5\frac{3}{4}$*d.*
(9) £38 2*s.* (10) £214 3*s.* 4*d.* (11) £4125. (12) $106\frac{2}{3}$ yds.

III.

(2) 1. £1053462 17*s.* $7\frac{1}{2}$*d.*; 2. £2328 6*s.* $4\frac{1}{4}$*d.*
(3) 1. $37\frac{1}{2}$ lbs.; 2. 3055709″. (4) $\frac{267}{592}$.
(5) 2. 2·6991408. (6) 2*s.* $2\frac{1}{2}$*d.* (7) £7.
(8) £343 12*s.* $4\frac{3}{4}$*d.* (9) £17 10*s.* (10) 45 mi. an hr.
(11) 3*s.* 6*d.* (12) 20 hrs.

IV.

(2) 2. 340674. (3) £2282 11*s.* $6\frac{3}{4}$*d.*
(4) 1. £180892 15*s.* $0\frac{3}{4}$*d.*; 2. £862287 5*s.* $7\frac{1}{2}$*d.*
(5) 1. £12204 14*s.* $2\frac{1}{2}$*d.*; 2. £78 4*s.* $11\frac{3}{4}$*d.*
(6) £414 13*s.* $0\frac{1}{2}$*d.* (7) 1. 673328; 2. 192669.
(8) 2 cwt. 3 qrs. 14 lbs. (9) $4\frac{27}{32}$ oz.
(10) 1. £1060 6*s.* $7\frac{1}{4}$*d.*; 2. £419 3*s.* $4\frac{1}{2}$*d.*
(11) £293 8*s.* 5·4*d.* (12) 4.

V.

(2) 2. £558308 19*s.* $0\frac{3}{4}$*d.* (3) £87 13*s.* $8\frac{3}{4}$*d.*
(4) 1. 3 yds. 20 ft. 1365 in.; 2. 46 yds. 5 ft. $127\frac{1}{2}$ in.
(5) 1. $16\frac{4}{5}$; 2. 26. (6) $\frac{16}{27}$.
(7) 32. (8) 1. £343 12*s.* $4\frac{3}{4}$*d.*; 2. £1281 19*s.* $0\frac{3}{4}$*d.*
(9) 1. ·015625; 2. ·42552380. (10) £·46553.
(11) £94 3*s.* 4*d.* (12) £750.

VI.

(1) 1. $10\frac{1}{25}$; 2. £3 10*s.* (2) 8203·6; 3·12834; 13400; ·00134.
(3) 1. ·10416; 2. $2\frac{89}{2000}$; 3. $\frac{1}{1600}$; 4. $\frac{13}{50}$; 5. $\frac{8546}{24975}$; 6. ·09375; 1·21.
(4) $45\frac{1}{2}$. (5) 10. (6) 18 lbs. 8 oz.
(7) £269 19*s.* $8\frac{1}{4}$*d.* (8) £1411 9*s.* 2*d.* (9) $4\frac{1}{2}$ yrs.
(10) 10. (11) £40 8*s.* (12) £16206 15*s.*

VII.

(1) $\frac{8}{5}$. (2) 1. $\frac{11}{2400}$; 2. $22\frac{2}{11}$; 3. £5 2*s.* 5*d.*
(3) 19801·12156. (4) 1. ·28182; 2. ·10556·4673.
(5) 17526. (6) 24 ft. 9 in. (7) 30 oz.
(8) $36\frac{2}{5}$ lbs. (9) £50 2*s.* 3*d.* (10) £260.
(11) 8 yrs. (12) £173 17*s.* 6*d.*

VIII.

(1) 2894 f. 3 c. (2) £8 11*s.* (3) £223 11*s.* 3*d.*
(4) £19127. (5) £188 10*s.* (6) 9 hrs.
(7) 12800. (8) £100. (9) 2340 lbs.
(10) £15727 16*s.* 8*d.* (11) 1. $\frac{7}{4000}$; 2. $\frac{36}{275}$. (12) $46\frac{29}{40}$.

IX.

(1) 1. 78256; 2. 29 mi. 1 fur. 63 yds. 10 in.; 3. 2 yds. 26 ft. 57 in.
(2) £36 10*s.* $2\frac{1}{4}$*d.* (3) 1. $1\frac{7}{8}$; 2. £3 12*s.* 2*d.*
(4) ·078125; ·2142857; $23\frac{61}{1600}$; $\frac{11}{6000}$. (5) ·148809523…; 3.
(6) $\frac{67}{495}$; $\frac{17}{1875}$. (7) 17*s.* 6*d.* (8) 45 days.
(9) £36 18*s.* 1*d.* (10) $4\frac{1}{4}$. (11) 140 yds.
(12) 1. 729; 2. $5\frac{2}{3}$.

X.

(1) 1. $\frac{7}{8}$; 2. $\frac{46}{235}$; 3. $11\frac{4}{5}$*d.* (2) 1. ·7625; ·07392; 2. 32·9415; 500
(3) 101. (4) 2 hr. $3\frac{3}{7}$ mi. (5) $22\frac{6}{7}$.
(6) 1. £370703 2*s.* 6*d.*; 2. £31907 9*s.* 11*d.* (7) £438 8*s.* ·165*d.*
(8) £12. (9) £270 2*s.* 5·28*d.* (10) 10*s.* 6*d.*; $8\frac{24}{127}$.
(11) 327; 28·349. (12) Ar. p. 1.

XI.

(1) 2. 36430031911. (2) 85001997; 885768804.
(4) 2. £351154 5*s.* $10\frac{1}{2}$*d.* (5) 60. (6) 112640.
(7) £74 7*s.* 6*d.* (8) 1. £132 2*s.* $3\frac{171}{320}$*d.*; 2. £625 19*s.* $0\frac{3}{8}$*d.*
(9) £7 15*s.* $2\frac{2}{5}$*d.* (10) $3\frac{113}{225}$.
(11) 1. £4 2*s.* $1\frac{33}{35}$*d.*; 2. 970·57005897. (12) 40000.

XII.

(1) 1. £1199679 6*s.* 3*d.*; 2. 45 mi. 7 f. 25 p. 2 yds.
(2) $47\frac{2}{3}$ ft. (3) 14 mo. (4) £12 14*s.* $9\frac{11}{12}$*d*
(5) $\frac{1017}{1955}$. (6) £2338 3*s.* (7) ·06.
(8) 6*s.* 11·04*d.*; 21 da. 4 hr. 25 m. 40·8 sec.
(9) 1. ·107151; 2. 23·456. (10) £500.
(11) £35049 12*s.*; £81782 8*s.*; £105148 16*s.*; £128515 4*s.*
(12) 2256 dol. 96 c.

XIII.

(1) £6 5s. $8\frac{1}{2}$d. (2) 141 tons 16 cwt. 8 oz.
(3) £10. (4) $13\frac{1}{5}$ mi. (5) Ar. p. p. 38, 45.
(6) 1. $3\frac{13}{30}$; 2. $3\frac{4}{57}$; 3. $33\frac{1}{4}$. (7) 21 times. (9) 15 days.
(10) $3\frac{3}{4}$ wks. (11) £601 11s. $10\frac{1}{2}$d. (12) 232 yds.

XIV.

(1) 2. 125550; Remainder = 79. (2) £10656 5s. (3) 23.
(4) 2 ro. 27 po. (5) 1. $12\frac{11}{20}$; 2. 2; 3. $\frac{79}{82}$; 4. $\frac{13}{77} > \frac{14\frac{1}{2}}{88}$ by $\frac{5}{1232}$.
(6) 1. $\frac{1}{2}$; 2. $\frac{3}{10}$. (7) 1. £1156 0s. $7\frac{1}{2}$d.; 2. £30 19s. 4d.
(8) £513. (9) 282 cwt. 2 qrs. (10) 115 ft.
(11) £40 10s.; £46 10s.; £67 10s. (12) 2907.

XV.

(1) 5112 tons 18 cwt. 2 qrs. (2) 14 da. 1 h. 4 m. 19 sec.
(3) 4200 cr. = 10500 fl. = 84000 3d. pieces.
(4) £1 5s. 3d.; and £309 6s. 3d. (5) 960.
(6) Ar. p. p. 38, 39, 40. (7) 1. $11\frac{365}{432}$; 2. $\frac{1}{8}$; 3. 18; 4. $\frac{3}{10}$
(8) £46 3s. 10d. (9) £543 18s; £207 11s. $10\frac{1}{2}$d.
(10) 4s. $0\frac{3}{4}$d. (11) 150 ft. (12) 96.

XVI.

(1) 1907920; 105536. (2) £548 3s. $4\frac{1}{2}$d. (3) £5 14s. 5d.
(4) £1 12s. $9\frac{1}{3}$d. (5) 10710. (6) $\frac{16}{17}$; $\frac{1}{5}$.
(7) 1. $21\frac{31}{36}$; 2. $2\frac{53}{180}$. (8) $48\frac{15}{44}$. (9) £34 12s. $10\frac{3}{16}$d.
(10) £12 2s. $3\frac{193}{320}$d. (11) 90 miles. (12) $47\frac{237}{625}$.

XVII.

(1) 67168. (2) 51964; 1643. (3) 75 tons 5 cwt. 1 qr. 25 lbs.
(4) £2 0s. 10d. (5) 2331; 19 m. 3 fur. 193 yds. 1 ft. 7 in.
(6) 36. (7) 1. $\frac{8}{11}$; 2. $3\frac{13}{36}$; 3. 5; 4. $\frac{29}{78}$.
(8) $\frac{94}{301129}$; 14s. $9\frac{3}{4}$d. (9) £119 0s. $7\frac{7}{20}$d.
(10) £6 8s. 4d. (11) 27 days. (12) 10800 feet.

XVIII.

(1) 154096 oz.; 396 ells. (2) £3 3s. $3\frac{3}{4}$d
(3) £8 6s. 8d.; £6 6s. 8d.; £5 6s. 8d. (4) 119; 18480.
(5) Ar. p. 41.; $\frac{13}{11}$. (6) 1. $10\frac{97}{120}$; 2. 80; 3. $31\frac{26}{35}$.
(7) 10s. $1\frac{1}{4}$d. = £$\frac{97}{193}$. (8) 1. £1246 16s. $6\frac{3}{4}$d.; 2. £15 5s. $4\frac{1}{2}$d.
(9) £4 7s. 6d.; 7d. per lb. (10) 7s. 6d.
(11) 27 days. (12) 6 ac. 293 yds. 3 ft.

XIX.

(1) $238\frac{3}{11}$; 3 f. 24 p. 3 yds. 0 ft. 4 in. (2) 128.
(3) 1. $15\frac{5}{13}$; 2. $1\frac{1}{23}$; 3. 45 : 48 : 40. (4) $\frac{1}{6}$; $\frac{4}{5}$.
(5) £499 19s. 3d.; £32 19s. 4d. (6) £595 6s. 11 $\frac{63}{80}$d.
(7) 2·8; 420; 1000. (8) 10s. 6d.; ·1696428751.
(9) £16 8s. $8\frac{1}{4}$d. (10) 8. (11) 1 yd. 2 ft. (12) $3\frac{1}{4}$.

XX.

(1) 6 ac. 1 ro. 26 po. 23 yds. 4 ft. 110 in. (2) 1. $\frac{8}{105}$; 2. $3\frac{553}{561}$.
(3) £21 17*s.* $2\frac{36}{137}d.$; $\frac{13}{55}$. (4) £100. (5) 23·544 oz.
(6) ·001875; 2040000; 76·923076. (7) ·7317; ·84259.
(8) £27. (9) $6\frac{3}{4}$. (10) £494; $3\frac{21}{23}$.
(11) Ar. p. 99. £245; £1012 7*s.* 3·264*d.*
(12) 15 ft. 7 in.; £18 5*s.* $2\frac{13}{16}d.$

XXI.

(1) 1. 757 tons 3 qrs.; 2. £884818 6*s.* $7\frac{1}{2}d.$; 3. 9 m. 4 f. 39 p. $2\frac{23}{38}$ ft.
(2) 1. 31622400; 2. £2 5*s.* 7*d.*; £4 11*s.* 2*d.*; 3. $562243\frac{33}{37}$.
(3) 1. £25 11*s.* 4*d.*; 2. £47 16*s.* 3*d.*; 3. £21 11*s.* 3*d.*; 8*d.*
(4) $\frac{1}{3}$. (5) $\frac{33}{38}$. (6) $\frac{63}{160}$. (7) ·025592. (8) 2040000.
(9) ·36875. (10) 80; $10\frac{10}{11}$. (11) $9\frac{72}{91}$. (12) 64.

XXII.

(1) 94760000 miles.
(2) 1. 8 mi. 4 fur. 10 po. 4 yds. 1 ft. 1 in.; 2. 363 lbs. 11 oz. 8 dwts. 6 grs.
(3) 2 qrs. 1 lb. (4) 11 lbs. 7 oz. 16 dwts.
(5) £188 10*s.* (6) £524 13*s.* 9*d.* (7) 18 days.
(8) 5 dwts. $3\frac{513}{1869}$ grs. (9) 1. $\frac{13}{15}$; 2. $2\frac{31}{45}$; 3. 2.
(10) 1. ·012016; 2. 286·5; 3. 100·032. (11) $52\frac{1}{2}$ sq. ft. (12) $12\frac{66}{89}d.$

XXIII.

(2) $187870\frac{9}{14}$. (3) $1215\frac{15}{19}$.
(4) 1. 207 c. yds. 17 c. ft. 409 c. in.; 2. £357 10*s.* $7\frac{1}{2}d.$
(5) $8\frac{1}{4}$ ft. (6) £129 15*s.* 10*d.* (7) $3\frac{57}{80}$ days.
(8) $2\frac{2}{5}$ days. (9) 1. $\frac{71}{1568}$; 2. 3; 3. $\frac{119}{180}$.
(10) 1. 1·6; 2. ·5136; 3. 8*s.* 9*d.* (11) $26\frac{3}{5}$.
(12) £55. (13) £41 11*s.* $5\frac{3}{5}d.$

XXIV.

(1) 306495278. (2) 1364. (3) £104 5*s.*
(4) 10*s.* $4\frac{3}{4}d.$ (5) 11. (6) 4217199.
(7) 1. £53 2*s.* $0\frac{7}{8}d.$; 2. 4698. (8) $7\frac{1}{5}$ days.
(9) 1. $\frac{11}{20}$; 2. $1\frac{1}{8}$; 3. $\frac{1}{196}$.
(10) 1. ·00002; 628000; 2. 1·7395; 3. ·821875.
(11) £12 $13\frac{3}{14}$*s.* (12) 140,170,190. (13) $15\frac{685}{1403}$.

XXV.

(1) 563714964. (2) £16 5*s.* and £4 15*s.*
(3) 1. £194 11*s.* 8*d.*; 2. 103085 lbs. (4) 5000; $9\frac{3}{4}d.$
(5) £2 3*s.* 4*d.* (6) 23 lbs. 3 oz. 12 dwts. (7) 15*s.*
(8) 8 cwt. 14 lbs. (9) 1. $1\frac{3}{5}$; 2. $\frac{3}{7}$; 3. £14 1*s.* $10\frac{9}{7}d.$
(10) 1. 58·7378; 2. ·10875; 3. 1·03571428. (11) 8*s.* 4*d.*
(12) $\frac{53}{83}$. (13) 16.

XXVI.

) tons 9 cwt. 11 lbs. 6 oz. (2) £12621 7*s.* 6*d.*
tons 13 cwt. 3 qrs. 15 lbs. $12\frac{9}{11}$ oz. (4) £3 10*s.* $11\frac{1}{2}d.$ gain.
6884 5*s.* (6) £2594 15*s.* $5\frac{1}{2}d.$ (7) £5 19*s.* $4\frac{8}{15}d.$
21 7*s.* $1\frac{1}{2}d.$ (9) 1. 13125; 2. $\frac{7}{10}$; 3. $\frac{31}{3643}$; 4. $51\frac{3}{4}$.
6·300009; 2. ·167433; 3. ·00910776.
$12\frac{1}{2}$ ft. (12) $7\frac{2}{7}$. (13) $\frac{23}{24}$.

XXVII.

ton 18 cwt. 1 qr. 11 lbs.; 335 tons 10 cwt. 3 qrs. 21 lbs.
3. (3) 1846. (4) £224, £240, £350.
3. (6) £23 12*s.* 6*d.* (7) £18 13*s.* $6\frac{72}{365}d.$
$\frac{7}{12}$; £304 6*s.* 8*d.* (9) 1. $1\frac{67}{70}$; 2. 2*s.* 7*d.*; ·5740.
1507 10*s.* (11) $1\frac{3}{4}$.
$\frac{9}{400}$; $1\frac{86}{2475}$; 2. 1200. (13) £1 15*s.* 3·008*d.*

XXVIII.

$1\frac{13}{20}$. (2) $7\frac{331}{390}$. (3) 92. (4) $\frac{42}{155}$.
39·58519. (6) 102·910604. (7) 1·6086255. (8) ·455.
10*d.* (10) 42256. (11) $43\frac{1}{4}$. (12) £185 14*s.* 9*d.*
$1\frac{79}{144}$. (14) $25\frac{251}{500}$. (15) $84\frac{3}{4}$. (16) $1\frac{1}{2}$.
$\frac{1}{2}d.$ (18) ·27 pt. (19) ·03374679. (20) 1240.
238095. (22) 40 lb. 8 oz. 7 grs. (23) £3 15*s.*
945. (25) $3\frac{3}{4}$ yrs. (26) £4 9*s.* $0\frac{3}{4}d.$ (27) £17 14*s.*

XXIX.

$\frac{1}{11}$. (2) $1\frac{23}{55}$. (3) $\frac{3}{8}$. (4) $\frac{45}{69}$.
353·66458. (6) 302·4005. (7) 3·158316. (8) ·177.
10 4*s.* 2*d.* (10) 56 miles 3 fur. 39 po. 4 yds. 1 ft. 11 in.
210*s.* $3\frac{3}{4}d.$ (12) £2601 9*s.* (13) $9\frac{49}{108}$. (14) $2\frac{51}{143}$.
(16) $3\frac{5}{65}$. (17) 9 qts.
hr. 23 mi. 15 sec. (19) ·32304328. (20) 36500.
48828125. (22) 9 ac. 29 po. 26 yds. 6 ft. 112 in.
3. 2*d.* (24) 9*s.* $8\frac{1}{2}d.$ (25) 7. (26) £7 12*s.* $4\frac{3}{4}d.$
) days.

XXX.

$\frac{47}{144}$. (2) $3\frac{91}{110}$. (3) $41\frac{1}{3}$. (4) $4\frac{6}{103}$.
332·38699. (6) 180·81815. (7) ·4487775. (8) ·25.
1·065. (10) 6792. (11) 13 cwt. 3 qrs. (12) £418.
$\frac{121}{300}$. (14) $5\frac{697}{770}$. (15) $\frac{1}{4}$. (16) $1\frac{34}{333}$.
36·5. (18) 24·63 oz. (19) 2·4142003. (20) 440.
27. (22) 51 miles 16 po. 4 yds. 1 ft. 4 in.
60. (24) 12 hrs. $17\frac{1}{2}$ mi. (25) $4\frac{1}{2}$.
) 5*s.* 3*d.* (27) 13*s.* 4*d.*

XXXI.

(1) $3\frac{91}{100}$. (2) $17\frac{7}{8}$. (3) 1. (4) $12\frac{19}{17}$.
(5) 296·09218. (6) 9·8202. (7) ·3421575. (8) 148·279.
(9) £2 1*s*. 8*d*. (10) 149328. (11) £11 8*s*. $6\frac{1}{2}d$. (12) £155 10*s*. 4·8*d*.
(13) $9\frac{5}{24}$. (14) $4\frac{343}{360}$. (15) $2\frac{2}{3}$. (16) $\frac{41}{48}$.
(17) 3795 mi. (18) 3 tons 7 cwt. 1 qr. 14·56 lb. (19) 36·1875.
(20) ·0196. (21) 74·8. (22) 120 yds. 8 ft. 32 in.
(23) £1449. (24) $2\frac{1}{2}$. (25) £113 4*s*. $7\frac{1}{2}d$.
(26) 4.

XXXII.

(1) $6\frac{59}{114}$. (2) $\frac{17}{42}$. (3) $34\frac{2}{3}$. (4) $\frac{9}{16}$.
(5) 1572·5723. (6) 3·6997. (7) 315·88568. (8) 11·4.
(9) 50·4. (10) 700. (11) $202\frac{1}{4}$. (12) £247 7*s*. 2·4*d*.
(13) $16\frac{17}{70}$. (14) $2\frac{29}{48}$. (15) $88\frac{4}{5}$. (16) $1\frac{3}{4}$.
(17) 5956 lbs. 8·816 oz. (18) 14*s*. 6*d*. (19) ·32010468.
(20) ·6804. (21) 121·3. (22) 83·2. (23) ·415.

XXXIII.

(1) $8\frac{119}{198}$. (2) $4\frac{2}{5}$. (3) 9. (4) $1\frac{1}{2}$.
(5) 534·96309. (6) 343·780696. (7) ·1783815. (8) ·064.
(9) 1281. (10) 13056. (11) 13*s*. 4*d*. (12) £396 1*s*. 6·6*d*.
(13) $17\frac{19}{100}$. (14) $6\frac{349}{432}$. (15) $1\frac{17}{72}$. (16) $2\frac{1}{2}$.
(17) 2*s*. 11·55*d*. (18) 65·875 po. (19) 15·90855. (20) ·1798.
(21) 434. (22) ·00728. (23) ·053571428.
(24) 1804 cwt. 3 qr. 18 lb. 7 oz. (25) £9. (26) 19 ac.

XXXIV.

(1) $1\frac{115}{252}$. (2) $12\frac{7}{15}$. (3) $3\frac{16}{21}$. (4) $\frac{39}{50}$.
(5) 773·65738. (6) 618·5246. (7) ·062068. (8) ·0000589.
(9) ·067. (10) $7\frac{73}{300}$. (11) $4\frac{28}{33}$. (12) $1\frac{3}{11}$.
(13) $\frac{41}{111}$. (14) 7489·59 mi. (15) 4994. (16) ·056536.
(17) 168·9604. (18) ·04875. (19) 7427. (20) 1*s*. $10\frac{1}{2}d$.
(21) £5946 15*s*. $3\frac{3}{4}d$. (22) £594.

XXXV.

(1) $10\frac{11}{30}$. (2) $5\frac{2}{3}$. (3) 21. (4) $4\frac{1}{16}$.
(5) 181·649. (6) 80·678. (7) 21806·056. (8) 41·06.
(9) 981·75. (10) 1248. (11) 152. (12) £84.
(13) $8\frac{23}{42}$. (14) $2\frac{24}{35}$. (15) $8\frac{1}{4}$. (16) $\frac{21}{55}$.
(17) 27690 lbs. 4·672 oz. (18) 88*s*. 7·56*d*.
(19) ·02759168. (20) ·0001802. (21) ·00004093. (22) 370·6.
(23) ·0015625. (24) £40 18*s*. $7\frac{1}{2}d$. (25) 5. (26) $3\frac{1}{6}$.

XXXVI.

(1) $3\frac{17}{30}$. (2) $6\frac{17}{18}$. (3) $\frac{3}{8}$
(4) $3\frac{7}{18}$. (5) 73·547045. (6) 292·1468.

·89918. (8) 34·114. (9) £10 4*s.* 9*d.*
; wks. 11 hrs. 26 min. 24 sec. (11) £2 19*s.* $9\frac{3}{4}$*d.*
:190 6*s.* 4·8*d.* (13) $7\frac{9}{100}$. (14) $4\frac{15}{16}$.
$\frac{7}{16}$. (16) $\frac{15}{25}$. (17) 90·16 yds.
2·94 hrs. (19) ·319792. (20) ·0242. (21) ·16.
74 cub. yds. $22\frac{17}{54}$ cub. ft. (23) £5062 10*s.*
3 yrs. (25) £1621 9*s.* 10*d.* (26) £30.

XXXVII.

4; 40. (2) 34. (3) $(x-a)(x-b)(x-c)$.
$2x)^2$. (6) $x-2a$. (7) $x^3+3x+3x^{-1}+x^{-3}$.
. $x=4\frac{1}{2}$. 2. $x=\pm 2$ or 4. 3. $x=a$, $y=b$, $z=c$.
$\frac{1}{3}$ hr. (10) 2·2798950; 5·2801131; 1905·207.
2, 5, −6, −4, −2, −5, 6, 4, (12) ·374408.
.3 cub. ft. (14) 5 ft. (15) 3466145 cub. yds.

XXXVIII.

$^3+x^2y+xy^2+y^3$. (3) $a^{\frac{1}{2}}+2a^{\frac{1}{4}}b^{\frac{1}{4}}+2b^{\frac{1}{2}}$. (4) $\frac{2y^2(x-y)^3}{(x+y)(x^2+y^2)^2}$.
$3x^2-5xy+3y^2$. (7) 1. $x=9$; 2. $x=-2$ or $\frac{1}{3}$, $y=3$ or $\frac{2}{3}$.
54. (12) $x=2$; ·5740313; $\bar{1}$·7501225; ·6573238; 6.
$7\frac{8}{9}$; 1458·04. (14) 189 in. (15) £17 6*s.* 6*d.*
£6 1*s.* 11*d.*

XXXIX.

. $\frac{x^3-x+1}{x^3+x+1}$; 2. $\frac{x+2y+6}{8x(6+y)}$; 3. $\frac{x-3}{x^4-1}$ (3) x^2-6x+5.
1. $x^2+\frac{xy}{2}-\frac{3}{4}$; 2. $\frac{\sqrt{7}-1}{\sqrt{2}}$; 3. x^2+2x-4.
-4; $\frac{1}{\sqrt{2}}\{\sqrt{1+a+a^2}+\sqrt{1-a+a^2}\}$.
1. $x=14$; 2. $x=5$ or 5·4; 3. $x=\pm 3$ or $\pm 3\sqrt{-1}$; $y=\pm 2$ or $\pm 2\sqrt{-1}$. 4. $x=3\frac{3}{8}$; $y=2\frac{1}{4}$ or $\frac{3}{5}$.
$\frac{6\sqrt{91^2}}{91}$; $\frac{8\sqrt{91^2}}{91}$. (10) £24 10*s.* (11) 11352 oz.
25 ch. (13) 15 ac. (15) $\bar{2}$·7958800; 1·1583625.
3·9325462; $\bar{3}$·9325462.

XL.

12, 6, 8. (3) $(a+b+c)(a+b-c)(a-b+c)(-a+b+c)$.
$(x-1)^2$ (7) 1. $x=\frac{a^2b}{a^2-b^2}$; 2. $x=(a+b)^2$, $y=(a-b)^2$;
3. $x=1$ or $2^{-\frac{3}{5}}$. (8) $5\frac{5}{11}$ mi. and $38\frac{2}{11}$ mi. past 4.

(10) 1·8573325; 2·1303338; $\bar{1}$·7406162. (11) $\frac{1}{1520640}$.

(12) 27 in.; 46·764 in.

(13) 2 mi. A. 15 or 16. B. $n=15$, $r=6$. C ·4771213; 6989700. D. 4·54.

XLI.

(1) 10611. (α) $9(x+a)(x-a)$; (β) $(x-a)(x-b)$; $\gamma\,(x-7)(x-11)$.

(2) (α) $x^3+3x^2+15x-11$; (β) $x^2+y^2+z^2-xy-xz-yz$;
(γ) $x^2-3xy+y^2$.

(3) $x-3$; $(x-3)(x^2-3x+21)(x^2-4x+21)$; $r=4$ or 5.

(5) $98-40\sqrt{3}-8\sqrt{15}+6\sqrt{5}$; $5\sqrt{5}-3\sqrt{3}$; 16·4249.

(7) (α) $x=11$; (β) $x=4$; (γ) $x=39$ or 49; (δ) $x=\pm\sqrt{21}$ or $\pm\sqrt{77}$;
(ϵ) $x=11$ or 10, $y=10$ or 11. (8) 87. (9) 60.

(10) 4; 12; 4; -12; 1·118802. (11) 595; 84966.

(12) 11·514; 8·5942. (13) 44196$\frac{3}{7}$. (14) 5026·56; 4398·24.

XLII.

(1) $\frac{1}{2}$; 4.

(2) (α) $2(2b-3c)$; (β) $a^6-a^4b^2+a^2b^4-b^6$; (γ) $4x^2-12ax+9a^2$;
(δ) $a^2+ab\sqrt{3}+b^2$; (ϵ) $1-2\sqrt{x}+3x$.

(3) $(1-x)^2(1+x)^3$; $\frac{3}{2}\cdot\frac{x^3-7ax+12a^2}{x^3+7ax+12a^2}$. (4) $\frac{1}{2\sqrt{3}}$.

(5) (α) $x=15$; (β) $x=8$, $y=12$, $z=16$. (6) 8.

(7) (α) $x=\pm 7$; (β) $x=5$ or $\frac{7}{31}$; (γ) $x=2$, 6, -4.

(8) 598 sq. ft. 128 sq. in.

(9) 3, 9, 27, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$; $\sqrt{10}=3{\cdot}162277$; 8286·972.

(10) 38400. (11) 12470·4 cub. ft. (12) 60 ft.; 103·92 ft.

XLIII.

(1) 1. $(3x-2)(2x+3)$; 2. $\{(a+1)x-a+1)\}\{(a-1)x+a\}$.

(2) $x+6$. (3) 1. $\frac{a+b+c}{a-b-c}$; 2. $\frac{a+y}{a^2-y}$; 3. $\frac{x^3+x+1}{(1-x)^2(1+x+x^2+x^3)}$.

(5) 1. $\frac{3}{2}+2x-7x^2$; 2. $3\sqrt{7}-2\sqrt{6}$; 3. $\sqrt{1-x}$.

(6) Both real when $p^2>q$; both positive when p is negative and $>\sqrt{p^2-q}$; $4p^2-3q$; $6pq-8p^3$.

(7) 1. $x=5$ or 5·4; 2. $x=9$ or 3·6; 3. $x=5$ or 1, $y=1$ or 5; 4. $x=4$ or 10, $y=15$ or 6. (9) 78 yds. and 58 yds. (10) £29 14s.

(11) 11·46; 9; 6·93. (12) 3·968625 ac. (13) 151·2.

(14) $\bar{1}$·969897; $\bar{3}$·30103; $\bar{1}$·860206; 3·7958800.

(16) 3·9005386; ·07953152.

XLIV.

(1) $12x^2-23ax+14bx-10a^2-21ab-10b^2$; $x\,(1+\sqrt{x}+x)$;
$2x-3\sqrt[3]{a^3-x^2}\{\sqrt[3]{a+x}-\sqrt[3]{a-x}\}$. (2) $\sqrt{3}-\sqrt{2}$. (3) 13.

$x+1$. (5) 1. $x=25$; 2. $x=1\frac{10}{11}$ or 3; 3. $x=9$ or 4, $y=4$ or 9.
3, 5, 7. (7) $\frac{121}{305}$. (10) 2·52. (11) 9360.
1728; 823·2192 cub. in. (13) £2 15*s*. (14) 15·15 lbs.

XLV.

$(a-b)(a-c)(b-c)$. (2) 1. $\frac{2x-3}{2x+3}$; 2. n; 3. $\frac{1}{x(1+x)^2(1+x+x^2)}$.
$x-2$; x^4-5x^2+4. (5) 1. $3a-5b-\frac{b^2}{2}$; 2. $3\sqrt{7}-2\sqrt{6}$.
1. $x=6$; 2. $x=1$ or 3; 3. $x=\pm\sqrt{2}-1$, $y=\mp\sqrt{2}-1$.
£5 9*s*. $1\frac{1}{2}d$. (9) 1. 7·45 in.; 2. 3·39 in. (10) 2513·28 c. ft.
·577 in.; ·816 in. (12) $6\frac{3}{4}$ ac.
$\frac{mx}{n}$; 8; -4; $\bar{1}\cdot4$; -8. (15) 1·565065.

XLVI.

B = 41° 41′ 28″ or 138° 18′ 32″.
100° 47′ 0·9″; 14° 12′ 59·1″.

XLVII.

$\sin A=\frac{12}{13}$; $\cot A=-\frac{5}{12}$.
C = 147° 28′ 37″; B = 1° 0′ 23″; $a=4389{\cdot}717$.
171·0836 yds.

XLVIII.

42° 0′ 14″; 55° 56′ 46″; 82° 3′.
Height = 19·73 yds.; distance = 103·918 yds. (14) ·1512 c. ft.

XLIX.

$\frac{2ab}{a^2+b^2}$; $\frac{4ab(a^2-b^2)}{(a^2+b^2)^2}$.
B = 51° 18′ 21″ or 128° 41′ 39″; C = 88° 41′ 39″ or 11° 18′ 21″.
C = 60° 6′ 2·3″; B = 49° 53′ 57·7″.

L.

$\frac{4\pi}{5}$. 6. -2. 11. 60° 52′ 29″. 12. 255·536.
55° 47′ 40″.

LI.

$\tan\frac{A}{2}=\frac{b}{a}$. 13. 89·646 yds. 14. 74° 32′ 44″; 48° 59′ 16″.

LII.

(8) $C = 53° 48'$ or $126° 12'$; $A = 91° 12'$ or $18° 48'$; $BC = 81{\cdot}4687$ or $252{\cdot}744$.

(9) $1 : 16 \sin^2 \frac{A}{2} \sin^2 \frac{B}{2} \sin^2 \frac{C}{2}$. 10. $\theta = (n + \frac{1}{4})\pi$.

LIII.

(8) $\sin x = \pm \frac{\sqrt{2ab+1} \pm \sqrt{1+4ab-4b^2}}{2(a^2+1)}$. (9) 3 and 6.

(10) $A = 32° 51' 54''$.

LIV.

(1) 156572 qs.	(2) £3 19s. $10\frac{1}{2}d$.	(3) £17 14s. $4\frac{1}{2}d$.
(4) £6.	(5) $6\frac{1}{30}$.	(6) $1\frac{5}{55}$.
(7) 18.	(8) $\frac{16}{45}$.	(9) 999·0101643.
(10) 34·83145.	(11) 120897·2.	(12) ·2160.
(13) 15 ro. 14 po.	(14) 321 qrs. 5 bu. 3 pks. 1 qt. 1 pt.	
(15) 17.	(16) £511 3s. $8\frac{3}{4}d$.	(17) £9584 6s. 6d.
(18) $5\frac{13}{21}$.	(19) $\frac{24}{119}$.	(20) 2.
(21) 14.	(22) 634·898367.	(23) 26·31772.
(24) ·2163294.	(25) 6·056.	(26) ·0185.
(27) £2.	(28) 2122.	(29) £136762 10s.
(30) 32s. $0\frac{3}{4}d$.	(31) $2^6 \times 3^4 \times 5 \times 7^2 \times 11 \times 13^2$	
(32) 144·4 lbs.		

LV.

(5) 1. $x^2 - 5x + 3$; 2. $a^{\frac{1}{2}} - a^{\frac{1}{4}}b^{\frac{1}{4}} + b^{\frac{1}{2}}$. (6) 1. $\frac{1}{(x-a)(x-3a)}$; 2. $\sqrt{\frac{a}{b}}$.

(7) $(x+4)(x+3)(x-2)(x+1)$. (8) 1. $x = -1\frac{3}{4}$;

2. $x = \pm 3$ or $\pm 2\frac{1}{2}$; $y = \pm 2$ or $\frac{\sqrt{2}}{2}$;

3 $x = -5$ or 2 or $\frac{-3 \pm \sqrt{241}}{2}$. (9) 1. -2863311530; 2. 2; 3. 100.

(10) $1 + (2n+1)x + n(2n+1)x^2 + \frac{n(4n^2-1)}{3} \cdot x^3 + \ldots$

$\frac{\lfloor 2n+1}{\lfloor n \lfloor n+1} \cdot x^{n+1}$, $\frac{\lfloor 2n+1}{\lfloor n \lfloor n+1} \cdot x^{n+2}$. (11) $\sqrt{3}$.

(12) 3. $2 - \sqrt{3}$; $3\sqrt{6} + \sqrt{3} - 5\sqrt{2} - 2$. (13) $\frac{\sqrt{3}-1}{2\sqrt{2}}$.

(14) 73·2 yds. (16) $78° 28' 13''$.

LVI.

8695 oz. (2) $414\frac{76}{113}$ ft. (3) £444 16*s.* 8*d.*
3 8*s.* (5) $10\frac{3}{56}$. (6) $3\frac{3}{4}$.
$\frac{7}{8}$. (8) $2\frac{13}{16}$. (9) 7975·027604.
3·34305. (11) 162592·5. (12) ·0484.
19·86 yds. (14) 44 mi. 7 fur. 13 po. 2 yds. 6 in.
$\frac{7}{5}$ days. (16) £1390 17*s.* $8\frac{1}{2}d.$ (17) £5359 11*s.* 5*d.*
$\frac{127}{40}$. (19) $6\frac{13}{56}$. (20) $\frac{5}{12}$.
$\frac{3}{5}$. (22) 689·948969. (23) 381·68388.
10592. (25) 24100. (26) ·02875.
125·146; 2. $7\frac{3}{5}$; (28) $2\frac{1}{2}$.
250; £500; £400; £480; £640. (30) 1*s.* 3*d.*
). (32) In $112\frac{1}{2}$ days. $48\frac{3}{4}$ min. past 7; $18\frac{3}{4}$ min. past 8 p.m.

LVII.

). (5) $(x+y)^2-(a+b)(x+y)+(a+b)^2$.

$\frac{bx-ay}{bx+ay}$; 2. $\frac{b-2\sqrt{c}}{b+2\sqrt{c}}$.

$x=7$; 2. $x=b-a$ or $c-a$; 3. $x=0$, -5, $\frac{1}{3}$, or $-5\frac{1}{3}$.

) and 30. (9) 1. $\frac{n(n+1)(a+nb)}{2}$; 2. 1; 3. 8·02526.

3. $\frac{\pi}{4}$ or $\tan^{-1}3$. (14) 30776·835. (15) 25° 21′ 44″.

LVIII.

3 tons 4 cwt. 3 qrs. 16 lbs. 7 oz. (2) 6.
4567 14*s.* $9\frac{3}{4}d.$ (4) £43 19*s.* $4\frac{1}{2}d.$ (5) $5\frac{122}{125}$.
. (7) $\frac{18}{35}$. (8) $\frac{3}{67}$.
331·557604. (10) 465·91874. (11) 2·475727.
74·3170. (13) ·00247933888429. (14) $23787\frac{1}{3}$ yds.
1 2*s.* 9*d.* (16) £9 7*s.* 5*d.* (17) £4104 18*s.* 10*d.*
$2\frac{1}{72}$. (19) $4\frac{3}{4}$. (20) $2\frac{2}{7}$.
$\frac{1}{3}$. (22) 10125·6218832. (23) 8256·21935.
1144·896. (25) 70·4. (26) 5 oz. 10 dwt. 3·552 grs.
10·043; 2. 234. (28) £584 6*s.* 8*d.* (29) $\frac{5}{8}$.
8 days. (31) Tuesday and Monday.

LIX.

264. (6) 1. x^2-2x+2; 2. $x^{\frac{m}{2}}+x^{-\frac{m}{2}}$. (7) 1. $x=0$ or

$\frac{a(m+n-2p)}{p(m+n)-2mn}$; 2. $x=\left\{\frac{a-2\pm\sqrt{a^2-4a}}{2}\right\}^n$; 3. $x=0, y=b, z=c$.

$\frac{\cdot 3\cdot 5\ldots(2n-1)}{\lfloor n}2^n$. (12) -4; $\bar{1}\cdot 7722417$.

$=70°\ 1'\ 44{\cdot}8''$; $C=46°\ 58'\ 15{\cdot}2''$. (18) $\frac{a\sqrt{bc}}{\sqrt{s(s-a)}}$.

LX.

(1) $5\frac{23}{45}$. (2) $\frac{7}{18}$. (3) $2\frac{3}{5}$. (4) 22.
(5) 137·9451. (6) 250·9993. (7) 105·372905. (8) 134·2.
(9) ·03125. (10) $16\frac{37}{80}$. (11) $\frac{1}{44}$. (12) $1\frac{4}{5}$.
(13) $4\frac{1}{5}$. (14) 2938·402. (15) 1·7479401. (16) ·000018.
(17) ·625. (18) 372 mi. 1 ro. 27 po. 20 yds. 1ft, $74\frac{1}{24}$ in.
(19) £25600. (20) £42081. (21) £585 4*s*. 10·512*d*.
(22) 60. (23) £2118 15*s*. 7*d*. (24) 3·13; ·8366.
(25) ·3107278. (26) £2803 5*s*. 2*d*.

LXI.

(1) 71874 in. (2) £32 5*s*. $0\frac{1}{2}d$. (3) 175 dwts.
(4) 192. (5) £1 0*s*. $10\frac{1}{4}d$. (6) 156 boys, 297 girls.
(7) 11·9934 yds. (8) $2\frac{1}{3}$.
(10) ·510914. (11) £19 4*s*. $11\frac{1}{4}d$.; £1 13*s*. $3\frac{1}{3}d$.

LXII.

(1) $8\frac{11}{18}$. (2) $1\frac{52}{105}$. (3) 15. (4) $\frac{39}{98}$.
(5) 29. (6) 74·292. (7) 12·1067415. (8) 59·049.
(9) ·00012. (10) $16\frac{2369}{3740}$. (11) 1. (12) 3.
(13) $\frac{4}{5}$. (14) 2 mi. 8 fur. (15) ·199539504.
(16) ·3183. (17) ·295$\dot{4}$. (18) 350 ac. 1 ro. 11 po. 29 yds. 6 ft. 136 in.
(19) £1 11*s*. 4*d*. (20) £36404 17*s*. $9\frac{3}{4}d$. (21) £398 13*s*. 10·2*d*.
(22) 15 mi. 6 fur. 10 po. 2 yds. 2 ft. 4 in. (23) 58 days.
(24) £103 12*s*. (25) 253; 1·4142; ·6324. (26) $50\frac{1}{2}$ bu.
(27) 1165·695. (28) 38 yrs.

LXIII.

(1) $14\frac{3}{16}$. (2) $9\frac{79}{95}$. (3) $3\frac{5}{8}$. (4) $\frac{160}{297}$.
(5) 295·67. (6) 14·98756. (7) 469·625. (8) 32·114.
(9) ·43625. (10) $99\frac{17}{45}$. (11) $5\frac{55}{96}$. (12) $335\frac{5}{17}$.
(13) $\frac{162}{210}$. (14) 22·29583. (15) 288·975. (16) 700310.
(17) $\frac{1}{44}$ = ·0227. (18) 5 tons 16 cwts. 2 qrs. 11 lbs.
(19) £1 9*s*. 6*d*. (20) £921506 5*s*. 6*d*. (21) £41 18*s*. $2\frac{3}{4}d$.
(22) 15 days. (23) 4 yrs. (24) 30507; 1·42.
(25) 165·9463. (26) £5066 14*s*.

LXIV.

(1) $9\frac{25}{172}$. (2) $3\frac{259}{361}$. (3) $37\frac{4}{5}$. (4) 2.
(5) 80·4227034. (6) 312·225446. (7) ·6423745. (8) ·0000126.
(9) ·51. (10) $6\frac{13}{30}$. (11) $15\frac{79}{81}$. (12) $2\frac{2}{9}$.
(13) $2\frac{7}{10}$. (14) 317·856. (15) 340·4. (16) ·040303662.
(17) 93·457. (18) ·0044583. (19) 43392. (20) 9*s*. $7\frac{5}{7}d$.
(21) £3968 5*s*. $10\frac{3}{4}d$. (22) £488 18*s*. $1\frac{1}{2}d$.
(23) 152550. (24) $52\frac{1}{4}$. (25) £752 6*s*. $10\frac{1}{2}d$.
(26) £7524 9*s*. $1\frac{1}{2}d$. (27) 131·2425.
(28) $\bar{2}$·3344538; $\bar{1}$·9148063. (29) 62573; $9\frac{5}{28}$. (30) £2202 10*s*.
(31) £993 4*s*. 6*d*.

* * * * *

C.

(1) £188 12*s*. (2) $1\frac{13}{35}$. (3) $2\frac{2222}{3375}$ per cent. gain.
(4) 98 yds. 1 ft. 2 in. (5) $145081\frac{1}{8}$ sq. in. (6) £2591 14*s*. $2\frac{60}{169}d.$; $9\frac{63}{193}$.
(7) £50 16*s*. $1\frac{19}{23}d.$ (8) 3·539142857. (9) $\frac{1111317}{4186625}$.
(10) $4\frac{1}{2}$ ft. (11) £7500; £35 5*s*.
(12) $2^2.3.5.11.13^2$; $2^5.3^3.7^3$; $2^{11}.3^3.7$; G.C.M. $= 2^2 \times 3$;
L.C.M. $= 2^{11}.3^3.5.7^3.11.13^2$. (13) $\frac{11}{12}$ in.
(14) £1024; £1280. (15) 53·43. (16) 1061.
(17) 55 ft. 7 in. 62′. (18) 55 ft. $7\frac{31}{144}$ in. (19) £2700.
(20) 25 p. c. increase. (21) 3*d*., $2\frac{1}{4}d.$, $1\frac{3}{4}d.$ (22) £90.
(24) 8 ft. 9 in. (25) £1810. (26) $164\frac{44}{45}$ yds. (27) ·0071875.
(28) 170 c. ft. 551 c. in. 192 c. pts. (29) £1300; £4 loss.
(30) $\frac{2177720}{5747761}$. (31) £430. (32) £4·4*s*. per cwt.
(33) $2\frac{6}{17}$ mi. past 8 p.m., Jan. 16; $46\frac{4}{17}$ mi. past 7; $19\frac{4}{17}$ mi. past 8.
(34) $56\frac{2}{3}$. (35) $7\frac{1}{2}$ ft. (36) £270.
(37) 1. $1\frac{13}{24}$; 2. 12 tons 13 cwt. 3 qrs. $17\frac{1}{2}$ lbs. (38) $1171\frac{7}{8}$ lbs.
(39) £6; £19. (40) 45. (41) £30. (42) $112\frac{23}{29}$.
(43) 500. (44) £28,000,000. (45) 1971. (46) $176\frac{41}{44}$ tons.
(47) 6 ac. 3 ro. 30 po. $22\frac{1}{2}$ yds.; 278 yds. nearly. (48) 17*s*. $0\frac{1}{4}d.$
(49) 6·26*d*. (50) 56·641 oz. (51) £1 1*s*. $9\frac{1}{3}d.$

CI.

(2) $x+2$.

(3) 1. x^3-7x+5; 2. $x^3-\frac{3x^2}{5}-2x+\frac{1}{8}$; 3. $7a^2-3ab+2b^3$; 4. $x^{\frac{1}{2}}+2y^{\frac{1}{2}}$;
5. x^4-8x^2+16; 6. $x^6+x^5-x^3+x+1$.

(5) $(5x+4)(5x-4)$; $(x-2)(x^2+2x+4)$; $(x+3y)(x^2-3xy+9y^2)$
$(ax-4b)(a^2x^2+4abx+16b^2)$.

(6) 1. hx^2-2x+k; 2. mx^3+nx^2+nx+m; 3. $(a+b+c)(b-c)$.

(7) $\frac{a}{16}-\frac{\sqrt{ab}}{4}+\frac{3\sqrt{ab^{-1}}}{16}+\frac{9b^{-1}}{64}$

(9) 1. $x+y$; 2. $x^3-\frac{1}{x^3}$; 3. $a-b-c+d$; 4. $1+5x+15x^2+75x^2+\ldots$

(10) 2. $(x^2-y^2)^2+z^2(x^2-y^2)+z^4$; 3. $4bc$; 4. $x+c$.

(11) 1. $4(a+b)(2a-2b-c)$; 2. $9(x-y)(x^2-xy+y^2)$;
3. $(x-a+1)(2x+a)$; 4. $(x+pq-1)(x-pq+1)$;
5. $(x+a)(2x-3a)$; 6. $(x+3a)(3x+a)$;
7. $(px+qy)(qx-py)$; 8. $(ab+cd)(c^2+d^2)$;
9. $(x-2)(2x+3)(3x-2)$; 10. $(a+b)(b+c)(a+c)$.
11. $(x^6-x^4y^4+y^8)(x^4-x^2y^2+y^4)(x^2-xy+y^2)(x^2+xy+y^2)$;
12. $(x-a+1)(x-a-1)$.

CII.

(1) 1. $2x+5$; 2. $x-3$; 3. $a-8$; 4. $2x-3y$; 5. $3x^2-2$; 6. x^2-1;
7. $x-2$.

(2) 1. $84x^5y^3z^3$; 2. $(x^2-7x+10)(x^2-3x-70)$;
3. $6x^4-7x^3+x$; 4. $a^4-7a^3b^2+26ab^3-40b^4$;
5. $(x^2-y^2)(x^2-4y^2)$; 6. $2(x-2)(16x^4-1)$;
7. $(x^2-1)(x^2-4)$; 8. a^2-x^2;
9. $(x+1)(x-1)^3$; 10. $(x-1)^3(x^2+1)(x+1)$;
11. $(x^2-a^2)(x-1)(x-2)(x+3)$ 12. $(4x^2-9y^2)(x^2-y^2)(x^2-1)$.

CIII.

(1) 1. $\frac{5x+2}{3x+2}$; 2. $\frac{x+4}{x+2}$; 3. $\frac{(x-2)(3x-4)}{x(3x+4)}$;
4. $\frac{a-b-c}{a+b-c}$; 5. $\frac{x^2-3}{x^2-2}$; 6. $\frac{x^2-y^2}{2(x^2+y^2)}$;
7. $\frac{a}{3(a-c)}$; 8. $\frac{x^2-2x+3}{x^2+2x-3}$; 9. $\frac{x^2-2x+3}{x^2+2x+3}$;
10. $\frac{a^2+b^2}{2a(2a^2-b^2)}$; 11. $\frac{ax-by}{ax+by}$;
12. $\frac{x^3+x^2+x-3}{(3x^4-3x^3+3x^2-1)7x+2}$; 13. $\frac{x^2-3y^2}{3x^2+y^2}$;
14. $(x+y)^2$; 15. $\frac{4x^2+3y^2}{2x+3y}$; 16. $\frac{a-bx}{a+bx}$;
17. $\frac{(y-a)(y-b)}{3y-a-2b}$

(2) 1. $\frac{a}{b}$; 2. $\frac{4a^2x^2}{x^4-a^4}$; 3. $\frac{x-a}{x+a}$;
4. $\frac{1}{x(x-a)(x-b)}$; 5. $\frac{2a^3}{(a-x)(a^3+x^3)}$; 6. $\frac{x^2+xy+y^2}{x^2-xy+y^2}$;
7. $\frac{a(yz+n)}{xyz+nx+mz}$; 8. $\frac{x^2+144}{x^2+64}$; 9. 1.
10. $\frac{1-x+x^2}{1-x^2}$; 11. $\frac{a^2-b^2}{2}$; 12. $1-16x^2$;
13. $\frac{x^3+2x^2+2x-2}{3}$; 14. $x-y$; 15. m; 16. $\frac{a+b+c}{a-b-c}$;
17. 1. 18. 1. 19. $\frac{1-x^2}{1+x^2}$; 20. 0. 21. $\frac{ax+by}{ax-by}$;
22. 0. 23. 1. 24. $\frac{1}{abc}$; 25. b^2; 26. 4.
27. $(r+qx)(s-tx)$.

CIV.

(1) 1. $a-b$; 2. 1·3838 . . . (2) 1. $2(a+\sqrt{a^2+b^2})$; 2. $b-\sqrt{-5}$.

(3) 1. $\frac{\sqrt{195}}{5}$; 2. $a^{\frac{2}{n^2-1}}$; 3. $110\sqrt{5}$; 4. 36; 5. $x+y$.

(4) 1. $\sqrt{2}(\sqrt{5}-1)$; 2. $\frac{9-\sqrt{3}}{8}$;

(5) $4(\sqrt[3]{25}+\sqrt[3]{20}+2\sqrt[3]{2})$; $x+\sqrt{x^2+x}$; $7-2\sqrt{3}$.

(6) $\frac{6}{11}(\sqrt[3]{36}-\sqrt[3]{30}+\sqrt[3]{25}$; $\frac{2a+b+2\sqrt{a^2+ab}}{b}$; $3-2\sqrt{-1}$.

(7) $c^{\frac{1}{3}}$. (8) 1. $\left(\frac{b}{a}\right)^{\frac{2}{3}}$; 2. 3.

(9) 1. $5a^2b\sqrt{3ab}$; 2. $3\sqrt[3]{5}$; 3. $a^{\frac{1}{q^2-p^2}}$; 4. $\sqrt{7}+\sqrt{2}$; 5. $2\sqrt{3}-\sqrt{-2}$.

(10) $46-12\sqrt{14}$; $3\sqrt{5}-2\sqrt{3}$.

(11) 1. $5+2\sqrt{6}$; 2. 0; 3. $\frac{7\sqrt{2}}{4}$; 4. $\frac{7+\sqrt{5}}{2}$; 5. $\frac{4+3\sqrt{2}-2\sqrt{3}-\sqrt{6}}{4}$

6. $3\sqrt{7}-2\sqrt{3}$; 7. $6\sqrt{2}-4$; 8. $\sqrt[5]{4000}$.

(12) 2. 4; 4. $xy^{-1}-\frac{1}{2\sqrt{-1}}-x^{-1}y$. 6. $x^{\frac{3}{5}}-x^{\frac{2}{5}}y^{\frac{1}{5}}+y^{\frac{3}{5}}-x^{\frac{1}{5}}(x^{\frac{1}{5}}+y^{\frac{1}{5}}-x^{\frac{1}{5}})$.

(13) 1. $\frac{x^3(x^2+a^2)}{a^4}$; 2. $\frac{2a(a^2-3b^2)}{(a^2+b^2)^2}$. (17). $\frac{1}{7}$. (18) $x^{\frac{3}{4}}+3x^{\frac{1}{2}}-1$.

(19) 1. $x^{\frac{3}{4}}-x^{\frac{1}{2}}y^{-\frac{1}{4}}-y^{\frac{1}{2}}$; 2. $\frac{2+\sqrt{-1}}{10}$.

CV.

(1) $x=\frac{ab(a+3b)}{a^2+ab+2b^2}$. (2) $x=3$. (3) $x=\frac{ab}{a+b}$.

(4) $x=15$. (5) $x=6$. (6) $x=\frac{49}{67}$.

(7) $x=\frac{q+r}{c-b}$. (8) $x=4$. (9) $\sqrt{x}=-2$.

(10) $x=\frac{\sqrt{a}}{\sqrt{a}+2}$. (11) $x=\frac{ab}{a+b(m-n)}$. (12) $x=1\frac{1}{4}$.

(13) $x=\frac{ab}{a+b}$. (14) $\sqrt{x}=-5\sqrt{3}$.

(15) $x=\sqrt{\left\{a^2-\left(\frac{b-2a}{3\sqrt[3]{b}}\right)^2\right\}}$. (16) $x=4\frac{1}{2}$.

(17) $x=\frac{3}{16}$. (18) $x=-\frac{bc}{a}$.

CVI.

(1) $x=12, y=15$. (2) $x=125, y=81$.

(3) $x=\frac{ac+bd}{a^2+b^2}$, $y=\frac{bc-ad}{a^2+b^2}$. (4) $x=\frac{ac(b-d)}{bc-ad}$; $y=\frac{bd(a-c)}{bc-ad}$.

(5) $x=\frac{a^2-b^2}{ap-bq}$, $y=\frac{a^2-b^2}{aq-bp}$. (6) $x=4$, $y=9$.

(7) $x=1\frac{5}{7}$, $y=2\frac{2}{5}$, $z=-12$. (8) $x=4$, $y=5$, $z=6$.

(9) $x=2\frac{1}{4}$, $y=3\frac{3}{8}$. (10) $x=\frac{am-2(b^2+c^2)}{a+4c}$, $y=\frac{2cm+b^2+c^2}{a+4c}$.

(11) $x=11$, $y=7$. (12) $x=20$, $y=12$.

(13) $x=12$, $y=4$, $z=24$. (14) $x=4$, $y=3$, $z=5$.

(15) $x=12$, $y=20$, $z=30$. (16) $x=32$, $y=15$, $z=-7\frac{1}{2}$.

(17) $x=\frac{1}{3}, \frac{1}{4}$; $y=\frac{1}{4}, \frac{1}{5}$; $z=\frac{1}{6}$.

CVII.

(1) $x=\frac{1}{4}$ or $\frac{1}{9}$. (2) $x=4$. (3) $x=\frac{1}{4}$ or -1. (4) $x=9$ or $\frac{1}{9}$.

(5) $x=2$ or $\sqrt[5]{-35}$. (6) $x=\frac{\sqrt{(534\mp150\sqrt{13})(-1)}}{2}$.

(7) $x=4$ or $\frac{1}{2}(21\pm\sqrt{41})$. (8) $x=2$ or $-3\frac{1}{13}$.

(9) $x=\frac{ma}{n}$ or $\frac{mn}{a}$. (10) $x=81$.

(11) $x=1\frac{1}{2}$, $-2\frac{1}{2}$, $\frac{1\pm2\sqrt{3}}{2}$. (12) $x=-2$. (13) $x=2$ or $-\frac{1}{7}$.

(14) $x=2$, -1, $-1\pm\sqrt{-3}$, $\frac{1\pm\sqrt{-3}}{2}$. (15) $x=1$ or -2.

(16) $x=20$ or -4. (17) $x=5$ or $1\frac{1}{3}$. (18) $x=7$ or 2.

(19) $x=\pm\frac{5}{3}$. (20) $x=\frac{ac}{b}$.

(21) $x=0$ or $a\left(1\pm2\sqrt{\frac{3}{c}}\right)$. (22) $x=-7$ or -11.

(23) $x=\frac{1}{2}\left\{-1\pm\sqrt{13\pm4\sqrt{7}}\right\}$. (24) $x=7$ or 5.

(25) $x=21$ or $2\frac{4}{5}$. (26) $x=5$. (27) $x=1{\cdot}5$.

(28) $x=36$. (29) $x=2$, -5, $\frac{-3\pm\sqrt{241}}{2}$.

(30) $x=7$ or $-\frac{7}{4}$. (31) $x=3$, $\frac{1}{3}$, -2, $-\frac{1}{2}$.

(32) $x=\pm2a$ or $\pm a\sqrt{-6}$.

(33) $11x^2-52x+61-10\sqrt{11x^2-52x+61}+25=x^2-12x+36$;
$x=3$, 2 or $\frac{1}{2}\left\{3\pm\sqrt{33}\right\}$.

(34) $x=-13$ or -11. (35) $x=2-\sqrt{3}$ or $2(\sqrt{3}-2)$.

(36) $x=\frac{c^2-ab}{a+b-2c}$. (37) $x=1$ or $\frac{-3\mp\sqrt{5}}{2}$.

(38) $x=\frac{1}{2}\left\{1-\sqrt{3}\mp\sqrt{2(4+\sqrt{3})}\right\}$ or $\frac{1}{2}\left\{1+\sqrt{3}\pm\sqrt{2(4-\sqrt{3})}\right\}$.

(39) $x=6$. (40) $x=\frac{a}{b}$ or $\frac{b}{a}$.

(41) $x=\frac{43\pm7\sqrt{37}}{2}$ or $\frac{11\mp\sqrt{21}}{2}$.

(42) $x=\pm(1+\sqrt{-1})$ or $\pm(1+\sqrt{3})$. (43) $x=0$ or a.

(44) $x=\frac{17\pm\sqrt{269}}{2}$ or $\frac{17\pm\sqrt{282}}{2}$.

(45) $\left\{\frac{x}{\sqrt{a+x}}\right\}^2+\frac{ax}{\sqrt{a+x}}+\frac{a^2}{4}=\frac{a^2+4b}{4}$.

CVIII.

(1) $x=y=8$. (2) $x=4$ or 3, $y=3$ or 4.

(3) $x=2$ or 16, $y=2$ or $\frac{1}{2}$.

(4) $x=1$, $\frac{-1\pm\sqrt{-3}}{2}$, 2, $1\pm\sqrt{-3}$; $y=2$, 1, &c.

(5) $x=7$, 3, $-\frac{1}{3}(31\mp\sqrt{205})$; $y=3$, 7, $-\frac{1}{3}(31\pm\sqrt{205})$.

(6) $x=\left\{\frac{a^2-b^2}{am-bn}\right\}^{\frac{1}{2}}$, $y=\left\{\frac{a^2-b^2}{an-bm}\right\}^{\frac{1}{2}}$.

(7) $x=\frac{a}{\sqrt{a+b}}$, $y=\frac{b}{\sqrt{a+b}}$. (8) $x=a$, $y=z=b$.

(9) $x=\pm\frac{3}{2}$, $y=\pm\frac{1}{2}$. (10) $x=5$ or -6, $y=3$ or $-4\frac{1}{2}$.

(11) $x=2\frac{1}{2}$ or $\frac{1}{2}$, $y=\frac{1}{2}$ or $2\frac{1}{2}$. (12) $x=\pm6$, $y=\pm8$, $z=\pm12$.

(13) $x=10$ or -2, $y=5$ or -1.

(14) $x=\pm3$ or $\pm\frac{5\sqrt{2}}{2}$, $y=\pm2$ or $\pm\frac{\sqrt{2}}{2}$.

(15) $x=\pm3$, $y=\pm2$. (16) $x=5$ or $-3\frac{3}{4}$, $y=20$ or -15.

(17) $x=\pm\frac{1}{3}$, $y=\pm3$ or $\pm\frac{1}{3}$. (18) $x=2$, $y=3$, $z=-5$.

(19) $x=\frac{ac}{a+b}$, $y=\frac{bc}{a+b}$. (20) $x=5$, $y=2$.

(21) $x=\frac{1}{4}(1\mp\sqrt{2})$, $y=-\frac{1}{4}$.

(22) $x=\frac{1}{2}\left\{\sqrt{2b-a}\pm\sqrt{\frac{a}{\sqrt{2b-a}}}\right\}$,

$y=\frac{1}{2}\left\{\sqrt{2b-a}\mp\sqrt{\frac{a}{\sqrt{2b-a}}}\right\}$.

(23) $x=\left\{\frac{bq}{ap}\right\}^{\frac{mp}{mq-np}}$, $y=\left\{\frac{am}{bp}\right\}^{\frac{nq}{mq-np}}$.

(24) $x=\frac{ac}{a+b}$, $y=\frac{bc}{a+b}$. (25) $x=\pm5$ or ±3, $y=\pm3$ or ±5.

(26) $x=2$ or 4, $y=4$ or 2.
(27) $x=16$ or $6\pm3\sqrt{13}$, $y=4$ or $-6\pm3\sqrt{13}$.
(28) $x=\pm\sqrt{\frac{a}{2b}}$, $y=\pm\sqrt{\frac{b}{2a}}$. (29) $x=8$ or 2, $y=64$ or 4.
(30) $x=243$ or 32, $y=64$ or 729.
(31) $x=3$ or -9, $y=5$ or 4, $z=4$ or 5.
(32) $x=\pm3$ or $\mp\sqrt{-1}$, $y=\pm1$ or $\pm3\sqrt{-1}$.
(33) $x=16$ or 146·41, $y=81$ or ·81.
(34) $x=a\pm b$, $y=a\mp b$. (35) $x=4$ or 1, $y=1$ or 4.
(36) $x=\pm\frac{10\sqrt{21}}{7}$, $y=\pm\frac{2\sqrt{21}}{7}$. (37) $x=a$, $y=b$, $z=-c$
(38) $x=\frac{a^4\pm2a^2b^2+2b^4}{4(a^2\pm b^2)}$, $y=\frac{ab}{2}\sqrt{\frac{2b^2\pm a^2}{a^2\pm b^2}}$.
(39) $x=8$, $y=2$ or $\frac{1}{2}$. (40) $x=a$ or $\frac{a+b}{2}$, $y=b$ or $\frac{a+b}{2}$.
(41) $x=4$, $y=9$, $z=16$. (42) $x=\frac{5}{2}$ or $-\frac{3}{2}$, $y=3$ or -5.
(43) $y=\frac{1\pm3}{2}$, $\frac{1\pm\sqrt{-47}}{2}$, $\frac{1\pm3\sqrt{5}}{2}$, $\frac{1\pm\sqrt{-11}}{2}$; $x=\sqrt{y+7}$.
(44) $x=6$ or 4, $y=4$ or 6, $z=5$.
(45) $x=\pm3$ or $\mp5\frac{1}{2}$, $y=\pm1$ or ∓16.

CIX.

(2) 4. $\frac{b^2-ac}{a^2}$; $\frac{3abc-b^3}{a^3}$; $\frac{b^4-4ab^2c+3a^2c^2}{a^4}$; $\frac{b^2-2ac}{c^2}$
(7) $4x^3-8x^2-x+2=0$.
(11) 1. $x=3$, -1, -2; 2. $x=3$, $-\frac{1}{3}$, -4.
(12) $x^2+2x-2=0$. (13) $y^2-4y+1=0$.
(14) $x=1\pm\sqrt{2}$ or ±1. (15) $x=6$.
(19) $x^3-qx^2-prx-r^2=0$. (20) $x=3$, 1, $-2\pm\sqrt{-3}$.
(21) $x=\pm2$. (22) $x=\pm\sqrt{-3}$ or $3-\sqrt{-1}$.
(23) 99. (24) 2·0945. (25) 2·714417; $x=2$·8934.
(26) $x=4$ or -2. (27) $y=0$, 4, or 2.
(29) $x=1$, 1, 1, or -7. (32) $x=1$·769.

CX.

(1) 25 and 35 years. (2) £3 10*s*. (3) 9000.
(4) $\frac{4}{15}$. (5) 27. (6) 100 miles. (7) $\frac{4}{7}$.
(8) Distance = 60 miles; rate, 6 miles an hour. (9) 371; 283.
(10) 7200. (11) 8 and 3. (12) £1 13*s*.
(13) 209 boys, 191 girls, 200 infants. (14) £5000.
(15) 600 E., 400 I., 200 S. (16) 12 at 12*s*., 24 at 9*s*.
(17) £1250. (18) 974. (19) $\frac{5}{8}$. (20) 120; 24.
(21) 4 da. (22) 10 hours and 6 hours. (23) 150 miles.

(24) 3 ft. (25) 28·3613; 33·3613. (26) 61.
(27) 126. (28) 2 ft. × 1 ft., and 2 ft. 6 in. × 1 ft. 4 in.
(29) 4000. (30) 8000. (31) 13. (32) 55; 53.
(33) 4000. (34) 260. (35) 20. (36) 17s.
(37) 8 minutes. (38) 78 yds. (39) Rate, 20 mi.; distance, 140 mi.
(40) 5 yds. and 4 yds (41) £800; £1200. (42) 17.
(43) 55; $18\frac{1}{5}$; $8\frac{1}{3}$. (44) 11; 2. (45) 100 wks.; 24,750 lbs.
(46) 1200; 360 English and Scotch, 480 Irish.
(47) £540; 17d. in the £. (48) 12 and 9. (49) 1000.
(50) 2 hrs. 21 mi. 40 sec. (51) 1880. (52) 48^2 and 36^2.
(53) 4 sov., 59 shil., 55 sixp. (54) 82, 55, 6.

CXI.

(2) 20. (3) 1. 0; 2. $\frac{211\sqrt{6}}{48}$; 3. $\frac{x^n-1}{(x-1)^2}+\frac{nx^n}{x-1}$.

(4) 1. 0; 2. $\frac{2}{3}(2+11\sqrt{3})$; 3. 25. (6) $r=\frac{1}{2}$.

(7) 1. $\frac{4a+b}{5}$, $\frac{3a+2b}{5}\cdot\frac{2a+3b}{5}$, $\frac{a+4b}{5}$; 2. 81; 3. $14\frac{37}{32}$; 4. $\frac{3}{2}(\sqrt{2}+\sqrt{6})$.

(8) B will pass A at the end of the 36th mile, and A will pass B at the end of the 120th mile.

(9) 1. $-366\frac{2}{3}$; 2. $518\frac{1}{3}$. (10) 1. $77\frac{1}{2}$; 2. $1\frac{17}{648}$; 3. $\frac{23}{55}$. (11) $3\frac{1}{6}$.

(12) $\frac{2a+b}{3}$, $\frac{a+2b}{3}$; $\sqrt[4]{a^3b}$, $\sqrt{ab}$, $\sqrt[4]{ab^3}$. (13) 1. -10^2; 2. $6\frac{1}{3}$; 3. 2.

(14) 185. (15) $\frac{7}{3}$.

(16) 1. -7; 2. $\frac{5(3^n-2^n)}{2\cdot 3^n}$; 3. $\frac{n-1}{2}$; 4. 342; 5. $1\frac{1}{5}$.

(17) 1. 252; 2. $\frac{2x}{y}$; 3. $39\frac{3}{5}$. (18) $-2, \frac{1}{0}, 2, 1, \frac{3}{5}$.

(19) 1. 2; 2. $\frac{5}{6}$. (20) 1. $112\frac{1}{2}$; 2. $12\frac{1}{2}$; 3. $\frac{n}{4}\{a(n+1)-b(n-3)\}$.

(21) $\frac{a^2-b^2}{a}$. (23) 2.

CXII.

(20) 21·673 years. (21) $-\frac{78}{25}$. (22) 20 yrs. (24) 6.

(26) $-\frac{5}{2^{11}x^{\frac{21}{2}}}$. (27) $\left(\frac{1+x}{1-x}\right)^n=\left(\frac{1-x}{1+x}\right)^{-n}=\left(1-\frac{2x}{1+x}\right)^{-n}$.

(28) 12 days; 780 miles. (31) 11s. $9\frac{1}{2}d$.

(34) $\frac{\lfloor 11}{\{\lfloor 2\}^3}$. (35) $\frac{\lfloor 36}{\lfloor 5\rfloor 31}$; $\frac{\lfloor 36}{\lfloor 4\rfloor 31}$.

(36) $x=1{\cdot}4743$. (38) $\frac{1}{2}, \frac{1}{18}$. (40) 2.
(41) 24; 6·666. (42) £181 16s. $4\frac{4}{11}d$.

(46) £1014 13*s.* 3·86*d.* (49) 10080.
(50) 6; 7775; 1296. (51) $7+\frac{1}{14}+\frac{1}{14}+\frac{1}{14}$, &c.
(53) $a=7$, $b=9$. (54) 8*d.* (56) 64.

(59) $\frac{2n}{5}$, $\frac{3n}{10}$, $\frac{n}{5}$, $\frac{n}{10}$ (60) 3·4641016. (61) $\frac{1}{218}$.

(62) $x=y+\frac{y^2}{\underline{2}}+\frac{y^3}{\underline{3}}+\ldots$ (63) $r=\left(\frac{s_2}{s_1}\right)^{\frac{1}{n}}$; $a=\frac{{s_1}^2}{s_1+s_2}\left(\frac{s_1}{s_2}\right)^{\frac{n-1}{n}}$.

(64) $n^2+4n-3+3^{n+1}$. (66) 20.
(67) 1867405. (68) 1, 3, 5, 7, 9.
(69) $a^{-4}+2a^{-6}x^2+3a^{-8}x^4+4a^{-10}x^6+5a^{-12}x^8+6a^{-14}x^{10}+\ldots$

(71) 20. (73) 600. (74) $\frac{a^2+b^2}{2}$. (76) 336. (77) 15.

(79) $3+\frac{1}{3}+\frac{1}{6}+\frac{1}{3}+\frac{1}{6}+\ldots$

CXIII.

(1) ·009170328. (2) 1·078774. (3) ·61419.
(4) ·08502884. (5) 1782·888. (6) 31·09557.
(7) 2875733. (8) 5·894009. (9) ·4401446.
(10) ·0001343781. (11) ·00007473765. (12) ·29287.
(13) 1·70279. (14) 1·230509. (15) ·4721987.
(16) ·955753. (17) ·0001403648. (18) 20·95046.
(19) 22·16533. (20) 8·0562. (21) 39·989.
(22) 1120768. (23) 1·815868. (24) 1297·12.
(25) ·00001233072. (26) ·1940783. (27) ·04.
(28) 675. (29) 139·8287. (30) ·096.
(31) ·124546. (32) ·0001128889. (33) 3251·986.
(34) ·9942207. (35) ·000002211518. (36) ·000547.
(37) 2·56. (38) 5·54634. (39) ·000045498.
(40) 1536·582. (41) 19·88159. (42) £3732 11*s.* 9*d.*

CXIV.

(5) 1; 2; −3; −1·5. (6) 1. $\bar{3}$·8750613; 2. 1·4983106; 3. 2·1072100; 4. 2·0969100; 5. 3·3979400.
(9) 1. 1·7781513; 2. $\bar{2}$·4771213; 3. ·0211893.
(10) 1·2435556; 1·5. (11) 1·591872. (12) 2·8.

(13) 125. (14) $\log_{22}8=\frac{3\log 2}{2(1-\log 2)}$.

(15) 12·946395.

(16) ·00001; 8; $\bar{3}$·5440680; $10^{\frac{3}{2}}=31{\cdot}62277$. (17) $10^{\frac{4}{3}}=21{\cdot}54434$.

(18) 1. $x=\frac{\log\frac{1}{2}(cd^{-1}+d)}{\log a}$; $y=\frac{\log\frac{1}{2}(cd^{-1}-d)}{\log a}$; 2. $x=\frac{9\log 2}{3\log 3-4\log 2}$;
3. $x=$·9492328; 4. $x=3{\cdot}773705$; 5. $x=\frac{\log 2}{\log 3}$; $y=\frac{\log 3}{\log 2}$;
6. $x=5{\cdot}435128$. (19) 17.

* * * * * *

CXVI.

(3) $\frac{\sqrt{3}}{2}$, $\frac{1}{\sqrt{2}}$. (7) $\frac{\sqrt{3}}{2}$, $\frac{1}{2}$, $2+\sqrt{3}$. (12) $2-\sqrt{3}$.

(26) $\pm\frac{1}{2}$. (28) 5426·9 and 1122·3. (31) 73° 10′ 26″.

(32) $x=\sqrt{3}$. (35) 6383·73. (36) 89° 54′ 4″.

(40) 45°, 225°, 405°, 590°. (42) ·96696.

(43) 36° 52′ 11·63″, 53° 7′ 48·37″, 90°.

(46) $\sin\theta=\sqrt{\frac{m^2-n^2}{1-n^2}}$; $\cos\theta=\sqrt{\frac{1-m^2}{1-n^2}}$; $\sin\phi=\frac{1}{m}\sqrt{\frac{m^2-n^2}{1-n^2}}$;

$\cos\phi=\frac{n}{m}\sqrt{\frac{1-m^2}{1-n^2}}$.

(47) 13330. (48) B = 18° 21′ 20″, C = 33° 34′ 40″.

(50) 68° 22′ 58″; 55° 24′ 2″. (51) 90° 6′.

(52) $\sin 5A=\frac{\sqrt{3}}{2}$; $A=\left\{\frac{n}{5}+\frac{(-1)^n}{15}\right\}\pi$. (53) 101° 32′ 13″.

(57) B = 144° 34′ 43″; C = 27° 5′ 42″; b = 31802·55.

(58) $x=\left\{n+\frac{(-1)^n}{3}\right\}\pi$.

(65) 91° 52′ 43·6″. (66) 13·3°; 14 g. 81′ 81″; $\frac{2\pi}{27}$

(68) A = 60°; B = 30°. (69) $\frac{\pi}{4}$; $\frac{\sqrt{3}-1}{2\sqrt{2}}$.

(73) ·422. (78) $2+\sqrt{2}$; $2(1+\sqrt{2})$. (82) 60°.

(84) 15°. (87) 65° 7′ 45″; 57° 26′ 7″.

(98) $a=\frac{s\sin A}{2\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$.

CXVII.

(1) $\sqrt{3}$: 1. (2) $15(3+\sqrt{3})$. (3) 351.

(5) $\sqrt{3}$ miles; $1\frac{1}{2}$ miles. (6) 108·6409 yds.

(7) 36·155 yds. per minute. (8) $12\sqrt{3}$; 24. (9) 40·365 ft.

(10) 95·636 yds. (11) $4\sqrt{3}$ yds. (13) 160·85 ft.

(14) $14\sqrt{2}$; $7(\sqrt{6}+\sqrt{2})$. (15) 787·84 yds.

(16) A = C = 65° 38′ 32·5″. (17) 175·9 ft.

(18) 5773·081 yds. (19) 76·545 miles.

(20) PA = 598·664; PB = 720·0018. (21) 318·12 ft.

(22) 27° 23′ 50″. (23) 57·402 miles.

(24) $40(\sqrt{2}+\sqrt{14})$ ft., and $4\sqrt{6}$ ft. (25) 1085·13.

(26) 6236·569 ft. (27) 1095·47 ft. (28) 72 ft.

(29) 1·923497 miles.

(30) $PA=24\sqrt{5}$; $PB=8\sqrt{10}$; $PC=12\sqrt{5}$; $PD=24\sqrt{10}$.
(31) 1·7919 miles. (32) $\angle DBA=85°\ 13'\ 24''$.
(33) $1250\ (2-\sqrt{3})$ yds. (34) 13·72947 mi.
(35) 169·4391 yds. (36) 1194·856 yds.
(37) 273·72 ft.; 158·03 ft. (38) 700.
(39) 400 ft., nearly. (40) 1514·396; 4163·745 yds.
(41) 1122·243 yds. (42) 5·258 mi.
(43) 135° 2′ 25″. (44) $\cos^{-1}(\tan\beta\cot\alpha)$ W. of N.
(45) $34\frac{1}{4}$, nearly. (47) 2·165 yds.
(48) 993·8727 yds. (49) 107·2377 ft.
(50) 178 ft., nearly. (51) $\frac{a^2+c^2}{a-c}$. (52) 269·4.
(53) 204·2 ft. (54) 87·55287. (55) $4\sqrt{3}$; 8.
(56) 1·4149 mi. (57) 4·257169 mi. (58) 80·584.
(59) $SA=777·47$; $SB=502·16$; $SC=790·114$.
(60) 392·9252 yds. (62) 629·83 yds.

CXVIII.

(1) 92267. (2) $\frac{2000\sqrt{2}}{3}$.
(3) $AD=20\sqrt{6}=48·88978$, area = 644·9489.
(4) Radius = 13·361 ft., volume = 1447·23 cub. ft.
(5) 150°; $8\sqrt{29+10\sqrt{3}}$. (6) $14\sqrt{3}+12\sqrt{14}$.
(7) $\frac{427\pi}{3}$. (8) 1164·93 cub. ft., 744 sq. ft.
(9) $\frac{625\pi}{3}$; 1125π. (10) 3593·55 ft.; 85° 48′. (11) 7·331 hrs.
(12) 119·38; $h=2·409$. (13) 7·0735 in. (14) 38·775 cub. ft.
(15) £903 11s. $10\frac{1}{2}d$. (16) $\frac{\pi a^2\sqrt{3}}{2}$. (17) 11, 5, 2.
(18) 1570·8. (20) $\frac{125\pi}{9}$. (21) $\frac{29^2\pi}{12^2}$ cub. ft.
(22) 1207·13. (24) $\frac{200}{3\pi}$. (25) 15 in.
(26) $\sqrt{15}=3·872983$. (27) $750\sqrt{3}$. (28) 1707.
(29) $21000\sqrt{3}$ cub. ft. (30) 4·1887902 cub. ft.
(31) 8 yds. 13 ft. 1153 in. (32) 15268·176 lbs.
(33) $2700\sqrt{3}$ cub. ft. (35) 379·8168. (36) 7·1365 ft.
(37) 1678·792 in. (38) 138 yds. (39) $\frac{19\pi}{6}$.
(40) 44. (41) 4214. (42) 10·1932 in.
(44) $486+250\sqrt{2}$. (45) 121793·4. (46) 45360.
(47) 562600; 4·768 ft. (48) 30·044 ft.

CXIX.

(1) 130° 3′ 10″.
(8) B = 59° 15′ 57″ or 120° 44′ 3″; C = 131° 29′ 46″ or 24° 37′ 28″.
(5) 77·57037 sq. miles.

CXX.

(3) $x=\frac{1}{3}$, $y=\frac{1}{7}$. (6) $\frac{15\sqrt{58}}{58}$. (11) 2·4.

(3) $y-3x-2=0$, $y+\frac{x}{3}-1=0$.

(4) The lines $y-\frac{2x}{3}+\frac{1}{3}=0$, and $y+\frac{5x}{7}-\frac{2}{7}=0$.

(5) $y=\frac{x}{\sqrt{3}}$, $x=0$. (17) $\tan^{-1}(-3)$.

(20) $\left(-\frac{15}{17}, \frac{20}{17}\right)$; $\frac{72\sqrt{5}}{85}$.

CXXI.

(1) Centre (3, 5); radius = 7; $5y-3x=0$.
(6) Equation to common chord $y-4x+15=0$.
(8) Centre $(\frac{1}{2}, 2)$; radius = 3; $y=4x$.
(9) Centre $(-2, 3)$; radius = 4. (12) $xx'+yy'=r^2$.
(13) $3y+3x-2=0$.

CXXII.

(2) 168 ft. (4) $(15a, 2a\sqrt{15})$.

CXXIII.

(7) $\left(-\frac{a}{99}, -\frac{70a}{99}\right)$. (10) 6·4; 20π.

CXXIV.

(3) $x-\frac{x^3}{\lfloor 3}+\frac{x^5}{\lfloor 5}-\ldots.$

(5) 1. $\frac{(x-1)^{\frac{1}{3}}(x^2-56x+135)}{15(x-5)^{\frac{4}{3}}(x-2)^{\frac{1}{3}}}$; 2. $\frac{4}{5+3\cos x}$; 3. $-\frac{1}{(1-x^2)^{\frac{1}{2}}}$;
4. $x^{x^n+n-1}(n\log x+1)$.

(6) 1. $x^x(1+\log x)$, $x^x\{(1+\log x)^2+x^{-1}\}$;
2. $e^{\tan x}\{1+x\sec^2 x\}$, $e^{\tan x}.\sec^2 x\{2+x(\sec^2 x+\tan x)\}$;
3. $\frac{1}{1+x^2}$, $\frac{-2x}{(1+x^2)^2}$.

(8) $-\frac{5}{2}$.

(9) Subtangent $= a$; asymptote parallel to axis at distance $= a$.

(10) $\frac{2}{1+x^2}$, $\frac{1}{1-x^4}$. (11) $x-\frac{x^3}{3}+\frac{x}{5}-\frac{x^7}{7}+\ldots$

(12) $\frac{200\sqrt{3}}{3}$.

(18) 1. $\frac{2a}{x^2-a^2}$ 2. $-\operatorname{cosec}^2 x$;

3. $(\tan x)\cot^{-1}x\left\{2\operatorname{cosec} 2x\cot^{-1}x-\frac{\log\tan x}{1+x^2}\right\}$;

4. $-\sin x\sin(\cos x)\sin\cos(\cos x)$.

(19) $x=a$ gives a min.; $x=\frac{a}{5}$ a max.

(20) $\rho=\frac{a}{\sqrt{2}}$; centre $\left(\frac{3a}{4}, \frac{3a}{4}\right)$.

(21) 1. $\log a+\frac{x}{a}-\frac{1}{2}\cdot\frac{x^2}{3a^2}+\frac{1.3}{2.4}\cdot\frac{x^3}{5a^3}$ &c.;

2. $2^n\left\{1+\frac{nx^2}{\lfloor 2}+n(3n-2)\frac{x}{\lfloor 4}\ldots\right\}$.

(23) $\frac{2(1-x^2)}{(1+x^2)^2}$; $e^{\sin x}\cdot\cos x$; $\frac{\log_a e}{\sin^{-1}x\sqrt{1-x^2}}$.

(24) $x=5$, a min.; $x=3\frac{1}{3}$, a max. (25) $h=r$.

(27) $x=1$, a min.

(28) 1. $2(-1)^{n-1}\frac{\lfloor n-3}{x^{n-2}}$;

2. $e^x\{x^3+3nx^2+3n(n-1)x+n(n-1)(n-2)\}$.

(29) $3x^2+2x+1$.

(30) $\frac{1}{2}\left(\frac{\pi}{2}+a\right)$, a max.; $\frac{1}{2}\left(\frac{3\pi}{2}+a\right)$, a min. (31) [illegible]

(33) $1+x-\frac{2x^3}{\lfloor 3}-\frac{4x^4}{\lfloor 4}\ldots$

(35) $e^{\cos x}\left\{1-\frac{x^2}{\lfloor 2}+\frac{4x^4}{\lfloor 4}-\frac{31x^6}{\lfloor 6}\ldots\right\}$.

(38) 1. $-\frac{x\sin\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$; 2. $\frac{x\cos\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}$; 3. $\frac{x}{\sqrt{x^2+a^2}}$.

(39) Ratio of sides, $2n : m+n$.

(41) $x=\frac{a(5+\sqrt{13})}{6}$, a max.; $x=\frac{a(5-\sqrt{13})}{6}$, a min.

(43) $x+\frac{x^2}{\underline{2}}+\frac{2x^3}{\underline{3}}+\frac{9x^5}{\underline{5}}+\dots$ (44) $\frac{a}{b}$.

(46) $\frac{1}{2}$.

(47) 1. Subtangent $=\frac{x(2a-x)}{3a-x}$, subnormal $=\frac{x^2(3a-x)}{(2a-x)^2}$;

2. subtangent $=a$, subnormal $=\frac{y^2}{a}$;

3. subtangent $=\frac{cy}{\sqrt{y^2-c^2}}$, subnormal $=\frac{c}{4}\left(e^{\frac{2x}{c}}-e^{-\frac{2x}{c}}\right)$.

(49) $\frac{a^2}{b}$. (55) Ratio of sides, 2 : 1.

(56) $x^4+\frac{4x^6}{3}+\frac{6x^8}{5}\dots$ (57) $1-x+2x^3-\frac{7x^4}{4}\dots$

(59) $\frac{x'}{x^{\frac{1}{3}}}+\frac{y'}{y^{\frac{1}{3}}}=a^{\frac{2}{3}}$; a. (61) 1. $\frac{1}{2(1-x^2)}$.

(62) Radius $=\frac{hr}{2(h-r)}$, h being the height, r the radius of base, of cone.

(63) $\rho=\frac{(a^2-e^2x^2)^{\frac{3}{2}}}{ab}=\frac{r_1r_2^{\frac{3}{2}}}{ab}$.

(64) 1. $\frac{1}{\sqrt{a^2+x^2}}$; 2. $\frac{2a(a^2-2x^2)}{(x^2-ax+a^2)}2$; 3. $-\frac{2a}{(x-a)^2}\cos\frac{x+a}{x-a}$; 4. $\frac{2}{1+x^2}$;

5. $e^{\sin x}(\cos^2x-\sin x)$; 6. $\sin^2x\cos^2x(3\cos^2x-4\sin^2x)$.

(66) $h=2\sqrt[3]{\frac{3V}{\pi}}$. (67) $y'-b=-\frac{b}{a}(x'-a)$.

(68) $\frac{a^2}{2}$.

(69) $\sin^{-1}x+\frac{h}{(1-x^2)^{\frac{1}{2}}}+\frac{x}{(1-x^2)^{\frac{1}{2}}}\cdot\frac{h^2}{\underline{3}}+\frac{3x(3+2x^2)}{(1-x^2)^{\frac{1}{2}}}\cdot\frac{h^4}{\underline{4}}\dots$

(71) 1. $1+x+\frac{x^2}{2}-\frac{x^3}{6}\dots$ 2. (72) $\rho=\frac{1}{36}$.

(74) 1. $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\dots$ $x-\frac{x}{2}+\frac{x^2}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\dots$

(75) 1. $-3x\sqrt{a^2-x^2}$; 2. $\frac{2x(2-x^2)}{\sqrt{x^2-1}(x^4-x^2+1)}$; 3. $\frac{m\cos(m-1)x}{(\cos x)}$;

4. $\frac{1}{1+x^2}$.

(76) Height = radius of circle × 3. (77) $\rho=4a\sqrt{2}$.

(79) $\frac{(av-bu)\sin\theta}{(u^2+v^2-2uv\cos\theta)}$ (84) $x=2$, a max.; $x=1$, a min.

CXXV.

(3) 1. $-\frac{1}{2}\left\{\frac{\sin(a+b)x}{a+b}-\frac{\sin(a-b)x}{a-b}\right\}$; 2. $x^2\sin x+2(x\cos x-\sin x)$;

3. $x+\frac{1}{a-b}\left\{a^2\log(x-a)-b^2\log(x-b)\right\}$.

(4) 1. $\frac{1}{a}.\tan^{-1}\left(\frac{x}{a}\right)$; 2. $\frac{1}{2a^2}.\tan^{-1}\left(\frac{x^2}{a^2}\right)$;

3. $x^3\sin x+3x^2\cos x-6x\sin x-6\cos x$.

(6) $ab=2c^2$.

(7) 1. $\frac{a^{m+x}}{\log a}$; 2. $\frac{1}{a}.\log\frac{x}{\sqrt{x^2+a^2}+a}$; 3. $\frac{1}{3}(1+x^2)^{\frac{1}{2}}(x^2-2)$.

(8) $6a$.

(12) 1. $x+\frac{a^3\log(x-a)}{(a-b)(a-c)}+\frac{b^3\log(x-b)}{(a-b)(b-c)}+\frac{c^3\log(x-c)}{(a-c)(b-c)}$;

2. $\frac{2}{25}\log\frac{x-3}{x+2}-\frac{3}{5(x-3)}$.

(15) 1. n, even, $\frac{(n-1)(n-3)(n-5)\ldots1}{n(n-2)(n-4)\ldots\ldots2}.\frac{\pi}{2}$;

n, odd, $\frac{(n-2)(n-4)(n-6)\ldots2}{(n-1)(n-3)(n-5)\ldots3}$; 2. $\frac{\sqrt{\pi}}{2}$.

(16) 1. $\frac{2bx-a}{2a^2x^2}-\frac{b^2}{a^3}\log\frac{(a+bx)}{x}$; 2. $-\frac{1}{5}(1-x^2)^{\frac{1}{2}}(x^2+2)$;

3. $\frac{1}{2}(x-\sin x\cos x)$; 4. $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$.

(18) πa.

(19) 1. $\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left\{\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right\}$; 2. $\log\{x+\sqrt{x^2+a^2}\}$;

3. $\sin\theta-\theta\cos\theta$. 4.

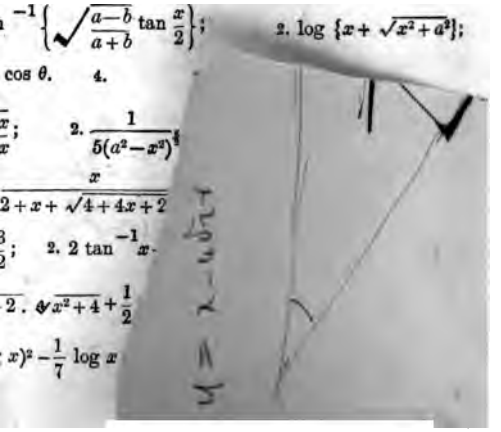
(21) 1. $\log\sqrt{\frac{a+x}{a-x}}$; 2. $\frac{1}{5(a^2-x^2)^{\frac{5}{2}}}$

4. $\frac{1}{\sqrt{2}}\log\frac{x}{2+x+\sqrt{4+4x+2}}$

(22) 1. $x+\log\frac{x-3}{x-2}$; 2. $2\tan^{-1}x$.

3. $\log\sqrt{x+2}.\sqrt{x^2+4}+\frac{1}{2}$

4. $\frac{x^4}{4}\left\{(\log x)^2-\frac{1}{7}\log x\right.$



(23) 1. $\frac{q}{p+q}x^{\frac{p+q}{q}}$; 2. $-\frac{1}{2}\log(a^2-x^2)$; 3. $\frac{1}{a}\sec^{-1}\frac{x}{a}$.

(25) 1. $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{a^2}\right)$; 2. $\log\sqrt{\frac{(x+3)^5}{x+1}}$; 3. $\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)$;

4. $(\log\theta+\sin\theta)$.

(26) 1. $\sqrt{ax+x^2}+\frac{a}{2}\log\left\{\sqrt{x^2+ax}+x+\frac{a}{2}\right\}$; 2. $x+\frac{3}{4}\log\frac{x-2}{x+2}$;

3. $\frac{e^{ax}(a\sin cx-c\cos cx)}{a^2+c^2}$.

(27) $\frac{(\pi-2)a}{2}$. (28) $\frac{4a^2}{5}$.

(29) 2 $a^2\left(\frac{1}{3}+\frac{\pi}{4}\right)$.

(30) 1. $\frac{1}{a}\tan^{-1}\frac{x}{a}$; 2. $\log\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$; 3. $\log\tan x$.

(31) 1. $x+\log\frac{x-2}{x+2}$; 2. $\frac{1}{2}\left(x-\frac{1}{2}\sin 2x\right)$.

(33) 1. $\frac{x^4}{4}$; 2. $-\cos x$; 3. $\log\tan x$; $\log\{x+\sqrt{x^2-a^2}\}$.

(34) 1. $\frac{x^5}{5}\left(\log x-\frac{1}{5}\right)$; 2. $\frac{1}{a}\tan^{-1}\frac{x}{a}$; $\frac{1}{a\sqrt{2}}\tan^{-1}\frac{x}{a\sqrt{2}}$

$\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}$.

(36) 2. $\frac{\pi}{2}\log\frac{1}{2}$. 3. $\frac{3\pi}{16}$.

(38) $2\left(\frac{\pi a^2}{4}-\frac{2a^2}{5}\right)$.

(39) 1. $\sin x$; 2. $a^x\log_a e$; 3. $\tan^{-1}\frac{x}{a}$; 4. $\sin^{-1}\frac{x}{a}$.

(40) 1. $\log\sqrt{\frac{\tan\left(\frac{\pi}{4}+x\right)}{\cos 2x}}$. 2. $-(1+x)^{-1}$.

(41) 1. $-x^2\cos x+3x^2\sin x+6x\cos x-6\sin x$;

2. $\frac{e^{cx}(c\cos ax+a\sin ax)}{a^2+c^2}$.

(43) 1. $\frac{\pi a^2}{2}$; 2. $\frac{\pi a^2}{4}$.

CXXV.

(3) 1. $-\frac{1}{2}\left\{\frac{\sin(a+b)x}{a+b}-\frac{\sin(a-b)x}{a-b}\right\}$; 2. $x^2\sin x+2(x\cos x-\sin x)$;

3. $x+\frac{1}{a-b}\left\{a^2\log(x-a)-b^2\log(x-b)\right\}$.

(4) 1. $\frac{1}{a}.\tan^{-1}\left(\frac{x}{a}\right)$; 2. $\frac{1}{2a^2}.\tan^{-1}\left(\frac{x^2}{a^2}\right)$;

3. $x^3\sin x+3x^2\cos x-6x\sin x-6\cos x$.

(6) $ab=2c^2$.

(7) 1. $\frac{a^m+x}{\log a}$; 2. $\frac{1}{a}.\log\frac{x}{\sqrt{x^2+a^2}+a}$; 3. $\frac{1}{3}(1+x^2)^{\frac{1}{2}}(x^2-2)$.

(8) $6a$.

(12) 1. $x+\frac{a^2\log(x-a)}{(a-b)(a-c)}+\frac{b^2\log(x-b)}{(a-b)(b-c)}+\frac{c^2\log(x-c)}{(a-c)(b-c)}$;

2. $\frac{2}{25}\log\frac{x-3}{x+2}-\frac{3}{5(x-3)}$.

(15) 1. n, even, $\frac{(n-1)(n-3)(n-5)\ldots1}{n(n-2)(n-4)\ldots\ldots2}.\frac{\pi}{2}$;

n, odd, $\frac{(n-2)(n-4)(n-6)\ldots2}{(n-1)(n-3)(n-5)\ldots3}$; 2. $\frac{\sqrt{\pi}}{2}$.

(16) 1. $\frac{2bx-a}{2a^2x^2}-\frac{b^2}{a^3}\log\frac{(a+bx)}{x}$; 2. $-\frac{1}{3}(1-x^2)^{\frac{1}{2}}(x^2+2)$;

3. $\frac{1}{2}(x-\sin x\cos x)$; 4. $\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}$.

(18) πa.

(19) 1. $\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left\{\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right\}$; 2. $\log\{x+\sqrt{x^2+c^2}\}$;

3. $\sin\theta-\theta\cos\theta$. 4.

(21) 1. $\log\sqrt{\frac{a+x}{a-x}}$; 2. $\frac{1}{5(a^2-x^2)^{\frac{1}{2}}}$; 3. $\frac{2}{3}\tan^{-1}\left(\frac{\tan x}{2}\right)-\frac{x}{3}$;

4. $\frac{1}{\sqrt{2}}\log\frac{x}{2+x+\sqrt{4+4x+2x^2}}$.

(22) 1. $x+\log\frac{x-3}{x-2}$; 2. $2\tan^{-1}x-\frac{1}{2}\tan^{-1}\frac{x}{2}$;

3. $\log\sqrt{x+2}.\sqrt{x^2+4}+\frac{1}{2}\tan^{-1}\frac{x}{2}$;

4. $\frac{x^4}{4}\left\{(\log x)^2-\frac{1}{7}\log x+\frac{1}{8}\right\}$.

(23) 1. $\frac{q}{p+q}x^{\frac{p+q}{q}}$; 2. $-\frac{1}{2}\log(a^2-x^2)$; 3. $\frac{1}{a}\sec^{-1}\frac{x}{a}$.

(25) 1. $\frac{1}{2}\sin^{-1}\left(\frac{x^2}{a^2}\right)$; 2. $\log\sqrt{\frac{(x+3)^3}{x+1}}$; 3. $\frac{1}{2}\log\tan\left(\frac{\pi}{4}+\theta\right)$;

4. $(\log\theta+\sin\theta)$.

(26) 1. $\sqrt{ax+x^2}+\frac{a}{2}\log\left\{\sqrt{x^2+ax}+x+\frac{a}{2}\right\}$; 2. $x+\frac{3}{4}\log\frac{x-2}{x+2}$;

3. $\frac{e^{ax}(a\sin cx-c\cos cx)}{a^2+c^2}$.

(27) $\frac{(\pi-2)a}{2}$. (28) $\frac{4a^2}{5}$.

(29) 2 $a^2\left(\frac{1}{3}+\frac{\pi}{4}\right)$.

(30) 1. $\frac{1}{a}\tan^{-1}\frac{x}{a}$; 2. $\log\cot\left(\frac{\pi}{4}-\frac{x}{2}\right)$; 3. $\log\tan x$.

(31) 1. $x+\log\frac{x-2}{x+2}$; 2. $\frac{1}{2}\left(x-\frac{1}{2}\sin 2x\right)$.

(33) 1. $\frac{x^4}{4}$; 2. $-\cos x$; 3. $\log\tan x$; $\log\{x+\sqrt{x^2-a^2}\}$.

(34) 1. $\frac{x^5}{5}\left(\log x-\frac{1}{5}\right)$; 2. $\frac{1}{a}\tan^{-1}\frac{x}{a}$; $\frac{1}{a\sqrt{2}}\tan^{-1}\frac{x}{a\sqrt{2}}$

$\frac{1}{2}\log\frac{x^2-x+1}{x^2+x+1}$.

(36) 2. $\frac{\pi}{2}\log\frac{1}{2}$. 3. $\frac{3\pi}{16}$.

(38) 2 $\left(\frac{\pi a^2}{4}-\frac{2a^2}{5}\right)$.

(39) 1. $\mathrm{Sin}\,x$; 2. $a^x\log^a e$; 3. $\tan^{-1}\frac{x}{a}$; 4. $\sin^{-1}\frac{x}{a}$.

(40) 1. $\log\sqrt{\frac{\tan\left(\frac{\pi}{4}+x\right)}{\cos 2x}}$. 2. $-(1+x)^{-1}$.

(41) 1. $-x^3\cos x+3x^2\sin x+6x\cos x-6\sin x$;

2. $\frac{e^{cx}(c\cos ax+a\sin ax)}{a^2+c^2}$.

(43) 1. $\frac{\pi a^2}{2}$; 2. $\frac{\pi a^2}{4}$.

MIXED MATHEMATICS.

I.

(1) EL. MEC. P. 19.

(4) 126° 52′ 11″, 143° 7′ 49″, 90°.

(6) 65·27 and 44·647 lbs.

(7) EL. MEC. P. 900; 1 : 2.

(8) 1. $\frac{Pg}{P+Q}$; 2. $\frac{P\sqrt{P^2+Q^2}}{(P+Q)^2}g$.

(9) Velocity $=\sqrt{v^2-4g^2}$; $\tan\phi=\frac{1}{v}\sqrt{\frac{v^2-16g^2}{3}}$;

space $=\frac{v\sqrt{3}}{4g}(v-\sqrt{v^2-16g^2})$.

(10) 1. 45°; 2. $\tan^{-1}\frac{1}{\sqrt{2}}$.

(12) $w=69$ oz., s.g. $=6·9$.

II.

(2) $2\sqrt{5}$; inclined 63° 26′ to the force 5.

(4) R $=80$, coincides with force 40.

(6) $\mathrm{Cos}^{-1}\frac{-Q}{P}$.

(7) EL. MEC. P. 185.

(9) $\tan\alpha=\frac{2h\pm\sqrt{4h(h-b)-a^2}}{a}$; the pt. being (a, b).

(10) 20·39 ft.

(11) 7·2 in. edge.

III.

(1) (2) EL. MEC. PP. 7, 29, 31.

(3) $2\frac{1}{12}$ ft. from wt. 9 lbs.

(4) $T=\frac{W\sqrt{3}}{4}$.

(5) $24\frac{1}{2}$ lbs. nearly.

(6) 30°; $t=10·392''$.

(7) ·838 mi.; $t=10·062''$.

(8) $\frac{b^2}{4a}$.

(9) 10° 30′; $1\frac{1}{2}$ miles.

(10) EL. HYD. P. 97.

(11) 17·55.

(12) $\frac{r}{2}$

IV.

(3) R $=\sqrt{3}$; $\angle=90°$.

(4) Pres. on B, vertical, $=81·65$ lbs.; Pres. on A $=$ Pres. on C $=70·29$ lbs. inclined $-7\frac{1}{2}°$ to horizontal.

(5) $7\frac{2}{7}$ ft. from given pt.

(6) EL. MEC. P. 36.

(7) $\sqrt{\frac{2R\tan i}{g}}$.

(9) $29\frac{1}{8}$ hrs.

(10) $\frac{2a}{3}$ from fixed end.

(11) EL. HYD. P. 123.

(12) 10 ft. nearly.

V.

(1) EL. MEC. P. 15. (3) $\frac{1}{2}P(\sqrt{6}-\sqrt{2})$. (4) $80\sqrt{3}$.
(5) $R=9\sqrt{2}$, acting parallel to AC; $CE=\frac{3}{4}CD$.
(6) MECH. PHIL. P. 23. (7) $\frac{7}{13}$.
(8) EL. MEC. P. 165. (9) $v : v_1 :: 5 : 3$.
(10) $Ll=120750$ ft.; $PS=40250$ ft.
(11) EL. HYD. P. 49. (12) 14000 grs.

VI.

(3) 11·76 lbs.; 131° 24′. (4) 135 lbs.
(5) EL. MEC. P. 88. (6) 13·06.
(8) $52\frac{10}{2}$° to horizon. (9) $\frac{(2-3e)v}{5}$.
(10) $144\frac{26}{37}$ ft. (11) EL. HYD. P. 123. (12) 45·059 in.

VII.

(1) EL. MEC. P. 22. (2) $R=10\sqrt{4+\sqrt{3}}$. (3) 9 ft.
(4) 15° and 45°. (5) $\sqrt{3} : 1$.
(6) $R=\sqrt{29+20\sqrt{2}}$, inclination to CA, $\tan^{-1}\frac{1+\sqrt{2}}{2}$.
(8) 20 ft. a second. (9) MECH. PHIL. P. 22.
(10) 134400; $8\frac{8}{65}$. (11) EL. HYD. P. 71. (12) $11\frac{1}{2}$ lbs.

VIII.

(1) EL. MEC. P. 114. (2) $\tan^{-1}\frac{(\mu+\mu_1)a}{2a+(\mu_1-\mu)c}$; c = dist. of $c.g.$ from plane, $2a$ = dist. between upper and lower feet.
(3) $2\frac{1}{4}$ ft. from greater sphere.
(4) TODHUNTER'S STAT. P. 200; 11 : 18. (5) MECH. PHIL. P. 130.
(6) 2 cwt. (8) 144·9 ft. (9) EL. MEC. P. 204.
(10) $s=41·92$ ft.; $t=2·79''$. (11) 16 oz. (12) 12 in.

IX.

(1) EL. MEC. P. 70. (2) $\frac{2^2}{2^n-1}\{2^{n-1}(n-2)+1\}$.
(3) PRAC. MEC. P. 100. (4) $W=\frac{wr\tan^{-1}\mu}{b-r\tan^{-1}\mu}$.
(5) $\cos\epsilon : \cos\epsilon' :: 1 : 2$. (6) EL. MEC. P. 194.
(7) $v=320$; $t=10''$.
(8) $\alpha=\tan^{-1}(2\pm\sqrt{3})$; and $\tan^{-1}\left\{2\pm\sqrt{3-\frac{8gh}{v^2}}\right\}$.
(9) (10) EL. MEC. PP. 234, 236; 88·0634 in.
(11) $\left(320-\frac{9w}{17}\right)$ lbs.; w = wt. of air in bell at first.

X.

(1) EL. MEC. P. 40. (2) $\alpha=\tan^{-1}\frac{\mu W+\mu_1 W_1}{W_1+W}$.
(3) $\frac{h(3r^2+2rr_1+r_1^2)}{4(r^2+rr_1+r_1^2)}$. (4) $5\frac{14}{15}$ lbs.

(5) R = 12 lbs.; 8 in. from greater force. (6) 644 ft.
(7) 57960 ft. (9) $\frac{13}{15}$.
(10) 29 : 1, nearly. (11) 1·005. (12) $3000\sqrt{2}$ oz.

XI.

(1) EL. MEC. P. 85. (3) $\frac{w}{3}$ and $\frac{2w}{3}$.
(4) $10\frac{10}{11}$ in. from end without weight. (6) 849·28.
(7) (8) MEC. H PHIL. PP. 251, 256.
(9) 1. 292·075 ft.; 2. 267·925 ft.; 3. 1324·6 ft.
(10) 15·316″. (12) 1548 lbs.

XII.

(1) TODHUNTER'S STATICS, P. 93. (2) $\frac{5h}{18}$ from base.
(3) Dist. from centre of greater sphere $= \frac{dr^2}{r_1^2 + r^2}$; d = dist. between centres. (4) 6.
(5) (6) MECH. PHIL. P. 155.
(7) V^y. of A $= \frac{v\sqrt{3}}{2}$; $\angle 90°$; V^y. of B $= \frac{v}{2}$, $\angle 0°$.
(10) $\sigma : 2\sigma + \sigma_1$. (11) EL. HYD. P. 37. (12) $\frac{p+q}{mp+nq}$.

XIII.

(1) EL. MEC. P. 77. (4) $\frac{w}{4}\frac{h^2+4r^2}{h^2+r^2}$; $\frac{3w}{4}\cdot\frac{h^2}{h^2+r^2}$.
(5) 2·5 in. (6) 10 lbs. (7) EL. MEC. P. 191.
(8) 500 ft. (9) 10·372″. (12) 5819·18 lbs.

XIV.

(1) EL. MEC. P. 124. (2) $\frac{1}{2}$. (3) 8.
(4) 1. $\frac{a}{8}$; 2. $\frac{3\sqrt{2}-2}{18}a$. (5) $5\frac{13}{15}$ ft. (6) $\sqrt{gh}$.
(7) $\frac{gt^2}{2}\left\{\frac{v\sqrt{2}-gt}{v-gt\sqrt{2}}\right\}^2$ (8) EL. MEC. P. 228.
(9) 1956 ft. (10) 36·88 lbs.
(11) EL. HYD. P. 44. (12) 1. Depth $= \frac{2h}{5}$; 2. Depth $= r(1-\cos\theta)$; θ being found from equation $\theta - \frac{1}{2}\sin 2\theta = \frac{2\pi}{5}$.

XV.

(1) EL. MEC. P. 56.
(2) $W_1 : W_2 :: \sin\beta : \sin\alpha$; $R_1 : R_2 :: \tan\beta : \tan\alpha$.
(3) Distance of point of support from greater wt. $= \tan^{-1}\frac{3+\sqrt{3}}{6}$.

(5) 1 : 6. (6) (7) EL. MEC. PP. 133, 148.
(8) $v_1=100$; $v_2=500$; $g=32{\cdot}2$.
(9) $\frac{h}{3}$ from vertical plane through summit of given plane.
(10) $5\sqrt{6}$. (11) EL. HYD. P. 49. (12) $38\frac{39}{155}$ oz.

XVI.

(1) EL. MEC. P. 126. (2) Four times st. line joining mid. pts. of diagonals.
(3) $\bar{x}=\frac{P-R}{Q-R}\cdot l$; $w=\frac{2(QR-P^2)}{2P-(Q+R)}$.
(4) $2-\sqrt{3}$. (5) EL. MEC. P. 92. (6) 12 ft.
(7) EL. MEC. P. 238. (9) $s=2672$ ft., $t=1'\ 31''$.
(10) MECH. PHIL. P. 243; 47·543.
(11) 60° 2′. (12) 7 ft.

XVII.

(1) EL. MEC. PP. 127, 130.
(2) R = 2·1283556 cwt.; P = ·72794 cwt. (3) 1·0546 lbs.
(8) $\frac{3}{4}\sqrt{gl}$. (9) 37·18 ft.; 269 ft. (10) 70·4.
(11) EL. HYD. P. 122. (12) ·6.

XVIII.

(2) 45°; $56\sqrt{2}$. (4) EL. MEC. P. 103.
(5) $2\sqrt{5}$ lbs.; inclination to A B 63° 26′; pt. of application $\frac{1}{5}$ of side from C, in D C produced.
(6) 12 lbs. (7) EL. MEC. P. 194.
(8) 3·1″. (9) EL. MEC. P. 152.
(10) Vel. of A $=-\frac{5a}{3}$; of B $=\frac{7a}{3}$.
(11) 368·5 grs. (12) 5·78 grs.

XIX.

(1) EL. MEC. P. 99. (2) 20 oz.
(3) $\text{Tan}\,\theta=\frac{1}{2}\cdot\frac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$ (4) Inclination of B C to horizon $=\tan^{-1}\frac{bw_2+aw_1\cos\alpha}{w_1\sin\alpha}$, where a and b are the distances of the centres of gravity from B.
(5) (7) (9) EL. MEC. PP. 103, 173, 236; 88° 16′; 33·06 lbs.
(10) 11. (11) (12) EL. HYD. PP. 4, 25, 145.

XX.

(1) TODHUNTER'S STAT. P. 179. (2) 94⅔ lbs.
(3) Dist. of c.g. from right angle = 1·972a. (6) 2 P and P.
(7) (8) EL. MEC. PP. 165, 199; P : Q :: 5 : 3.
(9) $s=\frac{v^2}{2g\sin\alpha}$; 2. $s=\frac{v^2}{2g(\sin\alpha+\mu\cos\alpha)}$.
(10) EL. MEC. P. 238; SP = 64·4 ft., inclined to horizon at an $\angle\ 2\alpha-90°$.
(11) EL. HYD. P. 37. (12) 20 in.

XXI.

(1) 50, $50\sqrt{3}$. (3) $33\frac{1}{3}$ lbs. (4) $n+1$.

(5) Dist. fr. centre of larger circle $= \frac{dr_1^2}{r^2 - r_1^2}$.

(6) (7) EL. MEC. PP. 177, 220. (8) 70° 58′.

(9) MECH. PHIL. P. 257. (11) Depth $= \frac{3h}{7}$. (12) 53·875 c.i.

XXII.

(1) EL. MECH. P. 121.

(2) Four times the line joining the mid. pts. of A B and C D.

(3) 5·633 in. (4) $\text{Tan}^{-1}\frac{\sqrt{3}}{2}$ from greater wt.

(5) EL. MEC. P. 112. (6) After ·5″; 13·315″.

(7) 15698 ft.; 1000·52 ft. (10) $v\sqrt{\frac{wl}{ge}}$. (11) EL. HYD. P. 78.

XXIII.

(1) (2) EL. MEC. PP. 58, 132. (3) $\mu = \frac{Hr}{\sqrt{W\rho^2 - H^2(r^2-\rho^2)}}$.

(4) MECH. PHIL. P. 142. (5) $5\sqrt{2}$ ft. fr. vertex.

(6) $14\frac{2}{3}$ in. fr. wt. 4 lbs. (7) MECH. PHIL. P. 22.

(8) 38. (9) 3741 ft. (10) 4·8 lbs, nearly.

(11) 2 : 1. (12) $\frac{875}{1296}\sqrt{3a^3}$ oz.

XXIV.

(1) EL. MEC. P. 117. (2) $50(2-\sqrt{3})$ lbs.

(4) 1. 170 lbs.; 2. 238 lbs. (5) 30°.

(6) ·3659. (7) 81·5 ft.

(9) $\tan^{-1}\frac{5}{8}$. (10) Equal.

(11) 9·938 lbs. (12) EL. HYD. P. 44.

XXV.

(1) (2) (3) MECH. PHIL. PP. 58, 147. (5) EL. MEC. P. 91.

(6) 99·38 ft. (7) 2·78 lbs. (9) 272·5 ft.

(10) EL. HYD. PP. 18, 21. (11) $\frac{1}{2}w$. (12) $\frac{21}{101}$.

XXVI.

(6) 3220. (7) $t = \frac{\sin(\beta-\alpha)}{g}\{v\sec\alpha - u\sec\beta\}$.

(8) $r = 9{\cdot}75$ in.; $t = {\cdot}499''$. (10) EL. HYD. P. 125.

(11) 18. (12) 1 : 817.

XXVII.

(4) TODHUNTER'S STAT. P. 200.

(7) Velocity $= 32{\cdot}2\sqrt{21}$: $\phi = \pm\tan^{-1}\frac{\sqrt{3}}{5}$; $s = 80{\cdot}5\sqrt{3}$, or $322\sqrt{3}$.

(8) $58\frac{6}{11}$ miles. (10) (11) EL. HYD. P. 77. (12) 17 ft.

XXVIII.

$_1-h_2$) from base. (2) (3) EL. MEC. PP. 90, 91.
$^{-1}\left(\frac{\tan(\alpha-\phi)}{\cos\phi}\right)$. (5) W sin $(\alpha+\phi)$; tan ϕ being
oef. of fraction. (6) 2 lbs.
1393 in. (9) MECH. PHIL. P. 243.
. HYD. P. 93. (12) ·1314 in.

XXIX.

1. from force 2 lbs. (3) $\epsilon=-14°\ 2'$; P=27·5 lbs.
. MEC. P. 129. (5) $7\frac{3}{8}$ ft.
1. (7) 20·125 ft.
MECH. P. 208; 39284 ft. (9) 3″ and 57″. (10) 15″.
. HYD. P. 93. (12) 4·5.

XXX.

1·81 and 965·925 lbs. (3) $\frac{r}{3}(3+\sqrt{6})$.
. MEC. P. 65. (7) 15528 ft.
$\sqrt{2g(\sqrt{l^2+h^2}-h)}$, $2h$=height, $2l$=length of block; $v=\sqrt{2g\mu a}$.
ual. (11) 33 : 29.

XXXI.

4 in. from end. (3) $r\sqrt{3}$.
CH. PHIL. P. 147. (7) 28·78″.
65 ft. (9) 7° 43″. (12) EL. HYD. P. 91.

XXXII.

1. fr wt. 8 oz. (2) $\frac{2h}{9}$ from base.
. MEC. P. 138. (4) TODHUNTER'S STAT. P. 28.
$n^{-1}\frac{1}{3}$. (8) 322 ft.
. MEC. P. 188. (10) MECH. PHIL. P. 158.
. HYD. P. 12. (12) $\frac{40aw}{b}$.

XXXIII.

164. (3) ·18334 in. (8) 35 lbs.
·1 ft.; h=3882 ft. (10) EL. MEC. P. 224.
. HYD. P. 56. (12) 5·6 lbs.

XXXIV.

. MEC. P. 112. (2) 112·5 lbs. (3) EL. MEC. P. 141.
$\frac{\sqrt{3}}{4}$. (5) $\frac{1}{4}$. (6) MECH. PHIL. P. 250.
50 ft. (9) 32·2 ft.; 4·8 oz. (11) 20.

XXXV.

(1) MECH. PHIL. P. 155.

(2) R = 10; distance fr. 8 lbs. = 3·0217 ft.; $\tan\theta = \frac{24+7\sqrt{3}}{37}$.

(3) $\frac{1}{8}\{(OC-OA)^2+(OB-OD)^2+2(OC-OA)(OB-OD)\cos\theta\}$.

(4) $2\frac{2}{3}$ ft. from end. (5) R = −P; pt. (−3).

(6) 23·67; T = $\frac{1}{17}$ lb. (7) ·00906 in.

(8) 18917·5; 231840. (11) 432·95 lbs. (12) 3141·6 oz.

XXXVI.

(2) $2(2+\sqrt{3})$ ft. (3) 89·6, 67·2, 53·76 lbs.

(4) $\frac{4a}{7}$. (5) $\frac{W\sqrt{2}}{6}$. (6) 21548522 units.

(7) 409600 units. (8) 3938 ft.

(9) $268\frac{1}{2}$ ft. (10) 116·23.

(11) Distance from vertex = $\frac{r\tan\theta}{(4+\tan^2\theta)\sin\theta}$

XXXVII.

(2) 5 in. from end; 6 lbs. (3) $\frac{11h}{8(3+\sqrt{3})}$ from base.

(6) $\frac{75}{161}$. (7) EL. MEC. P. 181. (9) 2 : 3.

(10) $v = 583{\cdot}12$ ft.; $h = 353{\cdot}69$ ft. (11) 6.

(12) MECH. PHIL. P. 281; Theoretical discharge, ·35 c. f.; Coef. of Vr. ·89.

XXXVIII.

(1) EL. MEC. P. 53. (3) $4r\cot a$; $2r\cot a$.

(5) 32 oz. (7) 292·4. (8) 4·1″.

(9) $t = \frac{3v}{4g}$; $h = \frac{15v^2}{32g}$. (10) 313·42.

(11) 13·2 mi. an hr. (12) 8·6107 oz.

XXXIX.

(1) TODHUNTER'S STATICS, P. 146. (2) $\frac{5\sqrt{3}}{3}$.

(3) W rests at a pt. which divides the string into two parts,

$$a\left\{1 \pm \frac{c\sin a}{\sqrt{a^2-c^2\cos^2 a}}\right\}; \quad T = \frac{W}{2}\cdot\frac{\sqrt{a^2-c^2\cos^2 a}}{a\sin a}.$$

(4) 1·95 cwt., and 3·55 cwt. (8) 19·05 oz.

(9) $\frac{8gmn}{25}$ ft. (11) 21 : 1, nearly.

XL.

(2) 2 or 5 lbs. (4) $2\frac{1}{4}$ ft. from 30 lbs.

(5) 4 ft. from P. (8) 5″ and 35″.

(11) 589·05 lbs. (12) EL. HYD. P. 73.

XLI.

(1) 1. 3230 lbs.; 2. 3350 lbs. (2) $\tan^{-1}(\frac{4}{3})$.
(3) 1584·3 lbs. (5) MECH. PHIL. P. 92.
(6) $\tan\theta = \frac{a-\mu\mu_1 b}{\mu(a+b)}$. (8) 19·6 lbs.
(9) (10) MECH. PHIL. P. 255; $3\frac{1}{2}$.

XLII.

(1) EL. MEC. P. 78. (2) Through the centre.
(3) Inclination to horizon $= \cos^{-1}\frac{a+\sqrt{32a^2+r^2}}{8r}$, r = rad. of sph., a = dist. of c. g. of beam from lower end.
(4) Dist. from rt. $\angle = \sqrt{2}$. (6) 3·26 ft., nearly.
(7) 18·11″; 60; $v = 17·8$ ft. (8) MECH. PHIL. P. 212.
(9) 187. (10) $s = 21\frac{7}{15}$ ft.; $T = \frac{4P}{3}$.
(11) 248510 lbs. (12) 2592 lbs.

XLIII.

(2) EL. MEC. P. 129. (3) 1 lb. (7) 2″ and 8″.
(8) $67\frac{1}{12}$ ft. (9) $4·56t$.
(10) $u' = \frac{u}{\sqrt{2}}$, $\theta = 90°$; $v' = \frac{u}{\sqrt{2}}$, $\phi = 0°$. (11) $\frac{9m}{4}$, m = mass of plate.

XLIV.

(1) TODHUNTER'S STATICS, P. 36. (2) 45°; 1.
(5) $\frac{2a}{7}$ from centre, a = side. (6) (7) EL. MEC. PP. 134, 236
(8) R = 38·82 ft.; $h = 5·2023$ ft., $t = ·804$″. (9) 179·67 ft.
(10) $v = \sqrt{\frac{gr}{\mu}}$. (11) EL. HYD. P. 11. (12) 10751 lbs.

XLV.

(1) TODHUNTER'S STATICS, P. 36. (3) $\frac{a\sqrt{3}}{20}$. (5) 40 h. 47·5 m.
(6) $v = 20$ ft.; direction inclined at $\angle$ of 36° 52′ to ship's course.
(8) 78. (10) $x = ·043593$; $k = \frac{4\sqrt{3}}{3}$

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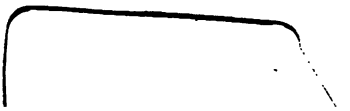
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